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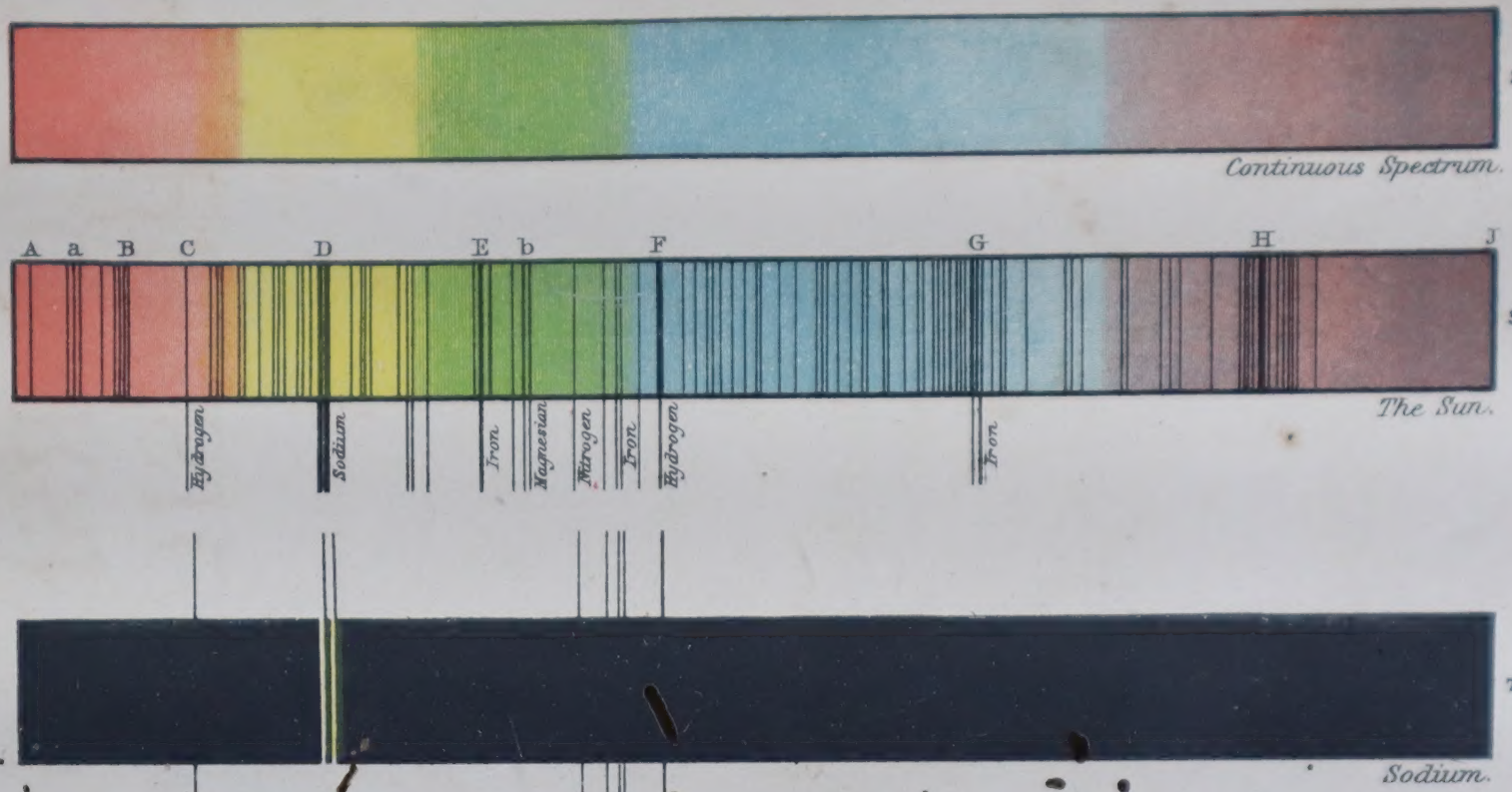

Fig. 334.—EXAMPLES OF SPECTRA

A COURSE OF PHYSICS

PRACTICAL AND THEORETICAL

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BY

CHARLES H. DRAPER, B.A., D.Sc.

Author of "Heat and the Principles of Thermodynamics"
"Light, Heat, and Sound" &c.

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PREFACE

A course of work in Physical Science is now generally regarded as a valuable element in school education. But in order that the special training which such study is calculated to afford may be acquired, it is necessary that the method employed should be largely experimental. Laboratory work is essential. Each pupil's eyes and hands should be actively engaged in observing, and in making experiments; and his mind in thinking over what he has himself seen and done. The exercises set him should be regarded as problems which he is to solve with so much guidance and help as is necessary —and no more. That is the first essential.

But as much of the work of a beginner is unavoidably imperfect, and certain fundamental experiments are for him impracticable, it is necessary to give precision and completeness to his efforts by discussion and comparison of class results, by explanations and demonstrations, by examples drawn from outside experiences which cannot be introduced into a laboratory, and to some extent by descriptions of experiments that are beyond the scope of school practice.

On these principles this book is based. It consists mainly of instructions for exercises intended to be performed by the pupils themselves, additional exercises of a confirmative character being also provided for the use of pupils who work faster than their fellows.

The exercises are both qualitative and quantitative. In the

judgment of the author it is not wise to make the laboratory work wholly quantitative. Experience indicates that where this is done there is a distinct danger of the pupil becoming so immersed in the details of the measurements that he fails to realize their purport as a whole, and the principle that lies behind them. He cannot see the wood for the trees. In some sections, therefore, qualitative exercises are freely introduced. These deal with principles which should be thoroughly realized.

But the book is not limited to laboratory instructions. Necessary descriptions, definitions, and explanations, experiments suitable for class demonstrations, and sections dealing with general phenomena whose action depends on the principles arrived at in the laboratory have been included.

In carrying out the individual practical work it is assumed that each pupil will keep a notebook in which the apparatus is sketched, the operations and observations recorded, and the results stated in the pupil's own words without reference to the book. In quantitative work inaccurate measurements should be rejected, and results insisted on that are within reasonable limits of experimental error. The results obtained by a class should be compared, and questions, suggestions, and difficulties discussed in lessons devoted to such reviews.

Graphic methods are freely used. In all such work very accurate drawing should be insisted on. Where observations are set out in a graph it is also necessary that the graph should not be employed to smother the errors of a bad set of measurements.

A large number of illustrations have been introduced with the purpose of affording help in the selection and setting up of apparatus, to enable the descriptions of apparatus to be readily followed, and to guide the operations. Much of the apparatus may be devised from simple materials; the par-

ticular form illustrated will afford an indication of the object to be accomplished.[1]

To meet the conditions of varying schemes of study the work is also issued in its several sections, namely:—

ELEMENTARY PHYSICAL SCIENCE (Mensuration and Properties of Liquids and Gases).

MECHANICS.

HEAT.

SOUND AND LIGHT.

MAGNETISM AND ELECTRICITY.

An edition in two volumes is also available, namely:—

Volume I.—MENSURATION. PROPERTIES OF LIQUIDS AND GASES, MECHANICS, AND HEAT.

Volume II.—SOUND, LIGHT, MAGNETISM, AND ELECTRICITY.

CHARLES H. DRAPER.

June, 1912.

[1] The thanks of the author and publishers are tendered to Messrs. P. Harris & Co., Messrs. J. J. Griffin & Sons, Mr. G. Cussons, and Messrs. F. E. Becker & Co. for the loan of blocks and permission to copy illustrations from their catalogues. These illustrations are the copyright of the firms named. The following are the illustrations referred to:—From Messrs. P. Harris & Co., figs. 9, 15, 17, 18, 22, 30, 53, 59, 68, 75, 81, 85, 109, 126, 133, 145, 173, 198, 202, 206, 215, 217, 221, 229, 237, 258, 268, 273, 307, 358, 360, 362, 367, 369, 370, 373, 374, 376, 396, 403, 405, 411, 413, 414, 418, 423. From Messrs. J. J. Griffin & Sons, figs. 29, 89, 104, 122, 124, 145, 147, 167, 178, 187, 204, 218, 236, 249, 287, 335, 342, 343, 353, 419, 430. From Mr. G. Cussons, figs. 31, 137, 166, 179, 182, 183. From Messrs. F. E. Becker & Co., figs. 50, 180, 406.

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3	27, 28, 30	51–53, 56	29	54, 55	—	—
4	31–41	57–76	42–45	77–84	—	—
5	46–59	85–100	60–62	101–105	—	—
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PART I.—MENSURATION

CHAPTER I

Length

1. THE METRE SCALE.—Examine a metre or half-metre scale, a portion of which is shown full size in the upper part of fig. 2.

Centimetre.—Each of the divisions marked 1, 2, 3, &c., is a centimetre. In scientific laboratory work lengths are expressed in centimetres (written **cm.**).

Millimetre.—Each of the very small divisions is a millimetre (written **mm.**). Count the number in a centimetre.

Decimetre.—A decimetre (written **dm.**) contains 10 cm. How many millimetres?

A **metre** (written **m.**) contains 10 dm. How many centimetres? How many millimetres?

A **kilometre** (written **Km.**) contains 1000 m.

Ex. 1.—Write out a complete table of lengths thus:—

10 mm. = 1 cm.	1 m. = dm.
10 cm.	
&c.	&c.

As the multiple is in each case 10, lengths are converted from one of these units to another by adding or taking away ciphers, or by moving the decimal point, thus: 127·6 cm. = 1276 mm. = 12·76 dm. = 1·276 m.

Scales suitable for many measurements may consist simply of strips of squared paper (fig. 12).

Ex. 2.—(1) Express in centimetres the following lengths:—24 mm.; 56 mm.

38 mm.; 73 mm.; 1 dm. 7 cm.; 6 dm. 3 cm.; 5 dm. 9 cm.; 2 dm. 5 cm. 3 mm.; 7 dm. 2 cm. 8 mm.

(2) Express in metres 40 dm.; 40 cm.; 40 mm.; 567 cm.; 1295 mm.

(3) On plain paper, using the rule as a straight-edge, draw lines of the following lengths:—4 cm.; 7 cm.; 53 mm.; 25 mm.; 6 cm. 2 mm.; 3 cm. 8 mm.; 5·9 cm.; 8·2 cm.; 4·4 cm.; ·8 dm.; 1·15 dm.: ·39 dm.; ·06 dm.

2. THE FOOT RULE.—Examine a foot rule, a portion of which is shown full size in the lower part of fig. 2.

Inch.—Each of the divisions marked 1, 2, 3, &c., is an inch (written **in.**). Each inch is divided into ten parts. These have no separate name. They are simply called tenth-inches ($\frac{1}{10}$ in.).

Foot.—Twelve inches make one foot (written **ft.**). Longer measurements are usually expressed in feet; shorter ones in inches; very short ones in tenth-inches.

A **yard** (written **yd.**) is 3 ft.

A **mile** is 1760 yd.

Ex. 3.—(1) Write out a table of lengths in English units.

(2) On plain paper, using the rule as a straight-edge, draw lines of the following lengths:—2 in.; 3·6 in.; 1·7 in.; 4·8 in.

(3) How many feet are there in 94 in.? 25 in.? 76 in.? 1000 in.? 15 yd.? 27 yd.?

3. HOW TO MEASURE WITH A RULE OR SCALE—PARALLAX.—Measurements are useless unless made accurately. Every instrument requires to be used in a proper way.

Ex. 4.—Make two dots a few centimetres apart on a sheet of paper, and place the scale flat on the paper in line with the dots, as shown in fig. 1. Look

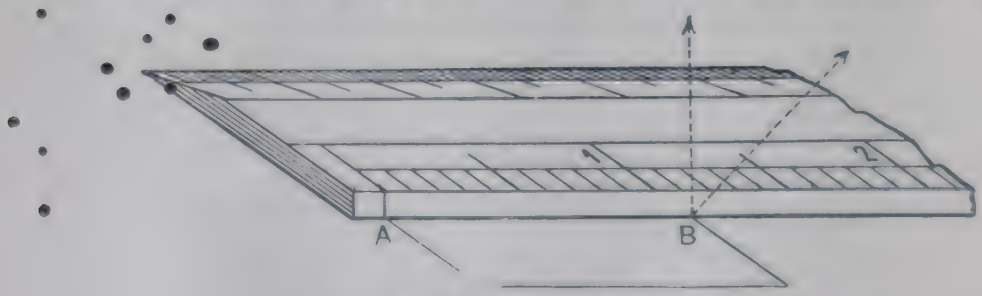

Fig. 1.—Effect of Parallax

at the dots and the scale at the same time. Move your head from side to side. What do the dots appear to do? In which direction do they appear to move as you move your head?

Hold your head immediately over the scale, and look at the scale and the dots with one eye only—first with the right, and then with the left. Do they appear to occupy different positions when thus viewed?

This effect is called **parallax**. If you look through a window

at a window bar and a tree outside, the tree appears to move to the right or left of the window bar as the head is moved to the right or left.

As you run along a road, objects close to your side seem to move in a direction opposite to yourself, while those farther off seem to move in the same direction as yourself.

Errors due to parallax are liable to be made in every department of Physics, and must be carefully avoided. In Exercise 4, the space between the dots and the graduation marks on the scale causes the difficulty. The scale must therefore be placed, as shown in fig. 2,

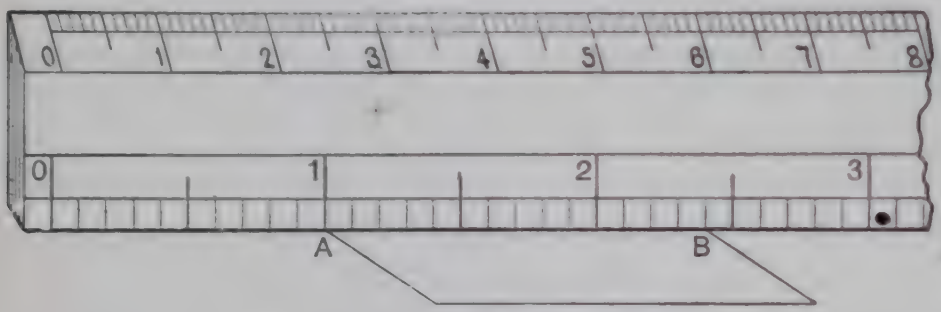

Fig. 2.—How to Use a Measuring Scale

with the graduation marks close up to the points between which the distance is required.

Scales made of thin steel offer much less risk of such an error being made. Sometimes scales made of glass are used. Such scales are placed on the surface with the graduated face downwards, and thus there is no liability to error from parallax.

Ex. 5.—Draw a long straight line on paper, and at intervals along the line make several small dots, marking them A, B, C, D, &c.

Place a measuring scale along the line and measure the lengths AB, BC, &c., successively, and record them thus:—

	Length.	
	In cm.	In in.
Line AB	3·7	...
„ BC	2·9	...
and so on	...	...
Sum of all the lengths	...	...

Add up the numbers in the columns. Measure the total length with the scale. Does the result quite agree with the sum of the parts? Why not?

The error is in some degree due to the fact that the dots did not fall exactly opposite to the division marks on the scale.

4. ESTIMATION OF SHORT DISTANCES BY THE EYE.

Ex. 6.—On the *edge* of a narrow strip of paper mark off the centimetres (centimetres only) from your scale. On another sheet of paper rule a long straight line, put several dots on the line, and mark them A, B, C, D, &c., as before.

(1) Placing one of the graduation marks on A, measure the distance AB with the paper scale. Then B does not come on one of these marks. Now estimate (guess as accurately as you can) on which millimetre mark the dot B would fall if the millimetres had been marked on the paper scale.

Measure the remaining distances BC, &c., in the same way.

(2) Measure the distances AB, BC, &c., with the correct millimetre scale. Place the results in a table, thus:—

Line.	Estimated Length.	True Length.	Error.
AB	3·2 cm.	3·3 cm.	−1 mm.
BC	4·8 „	4·6 „	+2 „
CD	...	...	...
Totals	...	...	...

Ex. 7.—Repeat the previous exercise, using a scale marked in inches and tenth-inches, estimating the short distances that are over in hundredths of an inch. Thus AB = 1·26 in., &c. Make a table as before, and see how much error has been made in the total length.

5. RULES TO BE OBSERVED IN MEASURING.

—Exercises 4 to 7 have shown that much care is necessary to avoid errors in measurements. If the same measurement is made several times the results obtained are generally not quite the same. Thus three persons measure the length of a room; their answers are 475 cm.; 472 cm.; 474 cm. What is the true length of the room? We do not know. But the most correct result will probably be got by adding up the three answers and dividing by three. This is called taking the **mean**, or as it is sometimes called the **average**. The length of the room is probably very nearly 473·7 cm.

In all measurements—

(i) Avoid error of parallax.

(ii) Count first the large divisions, feet or metres, and *write the result down*; next the smaller divisions, inches or centimetres, and write them down; and so on, recording the smallest divisions last.

(iii) Repeat every measurement and write each result down, taking the mean as the true measurement.

Before taking the mean, examine the numbers; if one or more of them differ very much from the rest, reject these as wrong and take another measurement.

6. MEASUREMENT OF LONGER DISTANCES. — The measurement of longer distances and of lines that are not straight is conveniently done by the use of a measuring tape, such as is shown in fig. 3; or in outdoor work by a chain whose links are all of the same length. It is not ordinarily necessary to make such measurements with minute accuracy, and therefore the smaller subdivisions marked on the laboratory scales are not marked on chains or tapes.

Such lengths may be measured approximately with lengths of string, which are afterwards compared with the scales.

Fig. 3.—A Tape Measure

Ex. 8.—By means of tape, chain, or string, measure such distances as the length of a table, a room, a wall, a corridor, a courtyard.

7. USE OF DIVIDERS.—Sometimes it is impossible to use a scale to measure the distance between two points, if, for example, the two points are inside a small cupboard or box, or have some obstacle between them. In some of these cases the measurement can be made by employing a pair of compasses or dividers.

Ex. 9.—Find the distance between two points on paper between which there lies a pencil or stone or other obstacle, or between two dots on the inside of the lid of a tin. Open the legs of the dividers until the points are about the required distance apart. Place one leg on one dot, and adjust the other leg until both are exactly on the dots, as shown in fig. 4.

Remove the dividers, place the ends of the legs on a millimetre scale, and read off the distance.

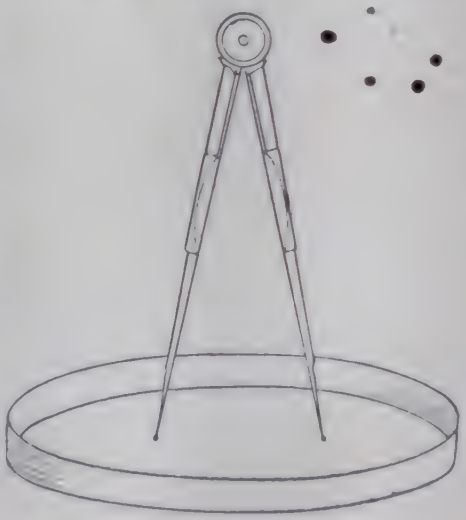

Fig. 4.—Dividers

Make several measurements of this character, such as the inside of a small box, the distance between two points on the figure on a coin, the wards of a key, &c.

8. CURVED LINES.—Curved lines may be measured in various ways—

(i) **By a thread.**

Ex. 10.—Draw a curved line on your paper, and measure its length by means of a piece of thread.

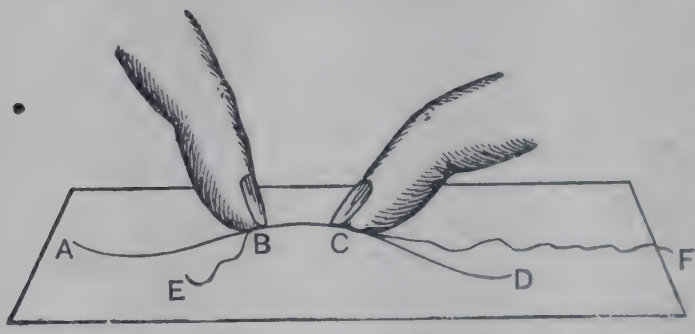

Fig. 5.—Measurement of a Curved Line by a Thread

Either one end of the thread or a knot in it may be placed at one end of the curved line. Then the thread is lightly placed along the curved line in short steps in the manner indicated in fig. 5. It may be cut off at the end of the curve, or if it is a white thread a small ink mark may be made on the thread at that point.

Place the thread on the scale, and read off the length between the points marked on it.

Repeat the exercise with more difficult curves.

(ii) **By an opisometer.** The opisometer is a wheel, generally made of metal, that has for axle a screw passing through its centre (fig. 6). The axle is mounted on a framework provided with a handle, and at the end of one of the axle bearings is a sharp point. The wheel is generally provided with small teeth to prevent slipping as it rolls along a surface.

Fig. 6.—An Opisometer

Ex. 11.—Turn the wheel of the opisometer back on its axle as far as it will go towards the end where the point is. Then the wheel is at its *zero* or starting-point.

Place the sharp point just opposite one end of the curved line, and carefully run the opisometer wheel along the line. When you get to the end make a fine ink mark on the bottom of the opisometer wheel. Put the opisometer wheel on a millimetre scale with the ink mark at the starting point of the scale, and run the opisometer wheel along the scale back to its zero. The position of the sharp point on the scale then gives the length of the line. (Rub off the ink mark.)

In the absence of a mounted wheel such as described, a disk of cardboard with divisions marked on its circumference may be used. In preparing such a disk it is best to make the diameter of the disk 31·8 mm.; then if the circumference of the disk be divided into 10 equal parts each of these divisions is a centimetre (Art. 15).

(iii) By dividers.

Ex. 12.—Open the dividers a short distance. Place one leg on one end of the curved line. Then by turning the dividers over step along the line. Count the number of steps. Measure with a scale the piece that remains "over" at the last step.

Place the dividers on a millimetre scale, and take the same number of steps as before along the scale. Record the length. Add on the length that remained over—the result is approximately the length of the curved line.

Why should the "steps" be short?

Repeat the measurement with shorter steps.

Which is the more accurate result?

Ex. 13.—Measure the length of a curved line by all the methods. Make each measurement twice and take the mean of each pair.

Do the results agree? If they differ much the work has been inaccurately done and should be repeated.

9. DIAMETERS OF CIRCLES.

Ex. 14.—Describe some circles on paper and measure their diameters. As there are no two definite points to measure between, one end of the scale must be slightly shifted about until the longest length is obtained. Estimate the fractions of scale divisions carefully. For each circle measure several diameters and take the mean.

10. SOLID OBJECTS.—The diameter of a disk or cylinder may be measured by a scale, as in the previous exercise, but it is better to place the disk between two squared blocks.

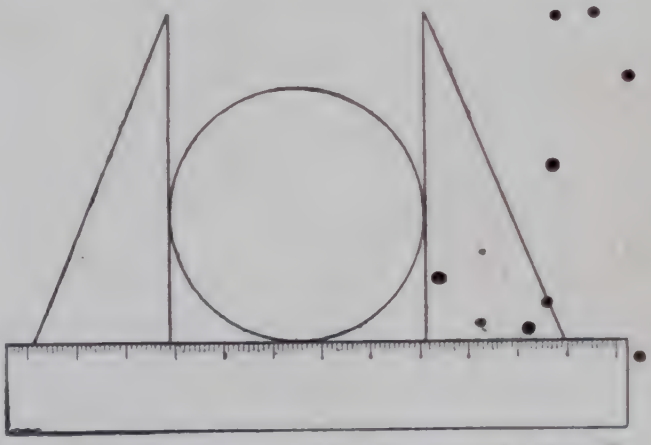

Fig. 7.—Measurement of the Diameter of a Disk

Ex. 15.—Place the disk or cylinder flat on a table between two set squares, one of the edges of each of which is placed along the scale in the manner shown in fig. 7. Then the diameter is read off on the scale.

Ex. 16.—Measure the diameter of a sphere or similar body, such as a flask. The same method may be used if the set squares be replaced by solid blocks which are higher than the radius of the sphere. Fig. 8 shows the arrangement.

Repeat the measurement with the

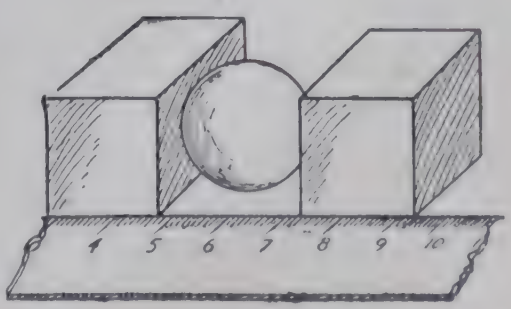

Fig. 8.—Measurement of the Diameter of a Sphere

blocks turned the other way up, but making the same faces of the blocks touch the sphere, and take the mean of the values. Why is it necessary to do this?

Ex. 17.—By the same method, using blocks, measure the following:—

(1) The height of a retort stand. The base of the stand is placed flat against one block and the top of the rod is against the other block, so that the base is vertical and the rod horizontal. The scale is placed on the table. Repeat, as before, with the blocks turned the other way up, and take the mean.

(2) The height of a pyramid and a cone. This means the vertical height as they stand on the table. Place the solid with its base vertical and flat against one block; then bring up the other block against the vertex.

11. CALLIPERS.

—A convenient way of measuring the lengths of small objects, and the diameters of small disks and tubes, is by means of callipers, which consist of two curved pieces of metal hinged like a pair of scissors, as shown in fig. 9.

Fig. 9.—Callipers

Ex. 18.—(1) Measure the lengths of small cylinders, the diameters of coins and cylinders, and of glass tubes or metal pipes by means of callipers. Place the object between the curved jaws and close them. Pass the object once or twice through the jaws to see that they touch both sides of the object. Place the jaws of the callipers on a millimetre scale and read off the distance.

(2) Measure the internal diameters of glass tubes, iron pipes, cylindrical tins, the holes in retort stand fittings, &c., using the other end of the callipers, shown in fig. 9.

Insert the jaws in the hole and open them until the points touch both sides. Draw them out carefully, and measure the distance between the points on a scale as before.

(3) Measure the thickness of the glass in glass tubes by obtaining the external diameter and internal diameter as above described, and taking the difference.

Is this difference the thickness of the wall of the tube? What remains to be done?

12. THE CONE GAUGE.

Ex. 19.—(1) From a piece of squared paper cut off a strip which is ten times as long as it is broad. The strip may be any length (20 cm. is convenient), and may be cut from paper divided into either millimetre or tenth-inch squares. Gum the strip on to stiff paper, and cut it across diagonally so as to make two triangles like that shown in fig. 10, which is half of a strip 100 mm. long and 10 mm. broad. Number the scale divisions as in fig. 10. The thin end of the wedge is the zero or starting-point.

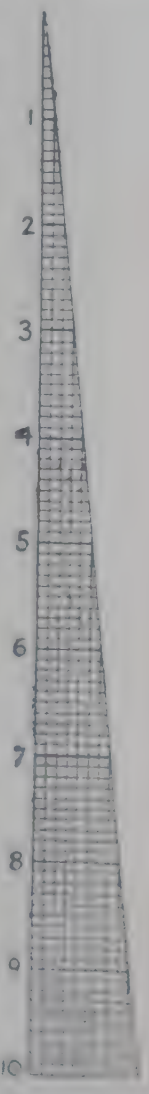

Fig. 10.—The Cone Gauge

Such a thin wedge is called a *cone gauge*. They may be obtained made of thin metal.

(2) Referring to fig. 10, or your own gauge, find the width of the strip at the points marked 10, 9, 8, 7, &c.

What is the width of the gauge at these distances from the point, 8·4 cm.? 6·7 cm.? 3·8 cm.?

What is the relation between the width of the gauge at any point and the distance of that point from the zero end?

Ex. 20.—Measure the internal diameters of glass tubes, pencil cases, small iron pipes, &c., by the cone gauge. Push the gauge gently into the tube so that it is flat, and straight across the tube. Then, if it enters for a distance of 5·7 cm., the diameter of the tube is 5·7 mm. Measure in this way the internal diameters of several tubes and the widths of cracks between blocks or between boxes placed close together on the table.

13. CIRCUMFERENCES.

Ex. 21.—Describe some circles on paper and measure their circumferences by the three methods described in Art. 8. It will be noticed that if the circles are small the operation is neither easy nor accurate. The accurate method is to measure the diameter and then calculate the circumference (Art. 15).

Ex. 22.—Measure the circumference of a thin disk such as a coin. Against a squared block of wood place a scale flat on the table. Place the disk with its edge on the scale and its face flat against the block, as shown in fig. 11.

With the finger make the coin revolve exactly once along the scale and read off the distance. If necessary, make an ink mark on the coin to ensure that an exact revolution is made. For a small coin make two revolutions. With care the coin will not slip, and an accurate result may be obtained.

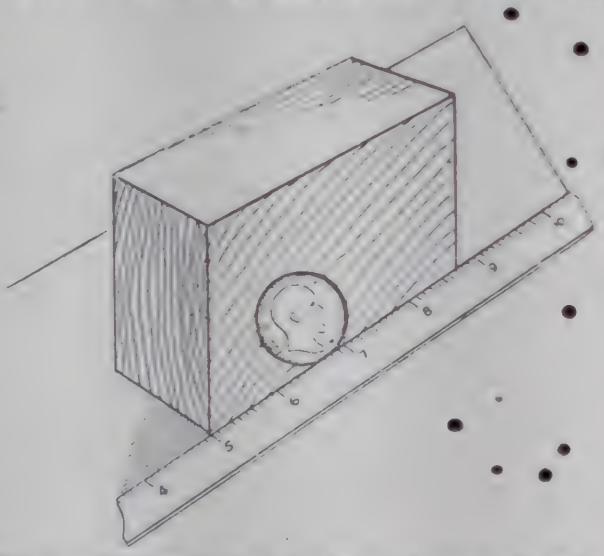

Fig. 11.—Measurement of the Circumference of a Disk

Ex. 23.—(1) Measure the circumference of a cylinder. Wrap a strip of paper round the cylinder until it overlaps, and prick through the paper with a pin on the overlapping part. Measure with a scale the distance between the two pin holes. Or a flexible scale, such as a strip of millimetre paper, may be wrapped round; but *no instrument in systematic use should have marks made on it which cannot be removed.*

(2) Measure the circumference of spheres and flasks in the same way.

14. RATIO: GRAPHS.—Many experiments in Physical science have for their object the determination of the relation between two

quantities, or, as it is often called, the **ratio** between them. This process is often much helped by setting out the observations on squared paper (fig. 12), i.e. making a **graph**. The next four exercises will illustrate these matters.

Ratio of centimetre and inch.

Ex. 24.—(1) Take a foot scale and measure on it several lengths with the centimetre scale as accurately as possible. Record thus:—

							Sum.
Length in inches ...	2	3	4	5	6	7	27
Length in centimetres ...	...	...	...	...	...	...	...

As indicated above, add up the inches and the centimetres; divide the sum of the latter by the sum of the former to three decimal places. How many centimetres are there in one inch?

(2) Take a sheet of ruled foolscap or exercise paper. Measure various distances marked out by the lines with the foot scale and the centimetre scale. Choose long distances several spaces apart for greater accuracy. It is better to employ dividers and a diagonal scale (Art. 16), but failing these estimate the fractions of a scale division carefully.

Make six or eight measurements and record as above. Add, and find as before how many centimetres make one inch.

Compare with the previous answer. The correct value is 2·54.

Ex. 25.—From all the above observations make a graph.

Take a sheet of squared paper. Along the base line OX (fig. 12) let the centimetres represent inches, and starting from the *origin* O, number them 1, 2, 3, &c. Along the vertical line OY take centimetres to represent centimetres, and number them similarly.

To obtain the point B. It was found that 2 in. = 5·08 cm. Along the inch line (the base line OX) find the point marked 2: run the eye up the vertical (the centimetre line) that passes through this 2, for a distance 5·08 cm. That is the point B. Place a small dot there and mark it B.

Every pair of measurements is treated in the same way. Thus 4 in. = 10·16 cm. Reference to the graph shows the point D to be on the line marked 4 at the base, at a distance 10·16 above the base line.

The point E, corresponding to 5 in., and its value in centimetres is the last that can be shown on this page.

From every pair of your measurements obtain thus a point on the paper.

Place a straight-edge along the line of points.

Are they exactly in a straight line?

Probably not. In this case the divergence from a straight line marks the extent of the errors made.

Draw from O a thin straight line that passes fairly through the dots, leaving some on one side and some on the other. Such a line is a *graph*. Each dot represents a measurement.

This graph may now be used to obtain an answer to such a question as this:—

How many centimetres are there in 2·7 in.? Look along the base (the inch) line OX until the distance 2·7 is reached, marked F in fig. 12. Run the eye up that line until the graph line is reached. How far up? 6·85 cm. Then 2·7 in. = 6·85 cm.

How many inches are there in 11·6 cm.? Run the eye up the centimetre line OY until the distance 11·6 cm. from O is reached, then along that line until the graph line is reached. The point G required is 4·57 in. divisions from the line OY; thus 11·6 cm. = 4·57 in.

Ex. 26.—By means of the graph in fig. 12, find out how many inches there are in 6 cm.; 2·9 cm.; 9·5 cm.; 12·3 cm.; and how many centimetres there are in 1·5 in.; 2·7 in.; 3·3 in.; 4·6 in.

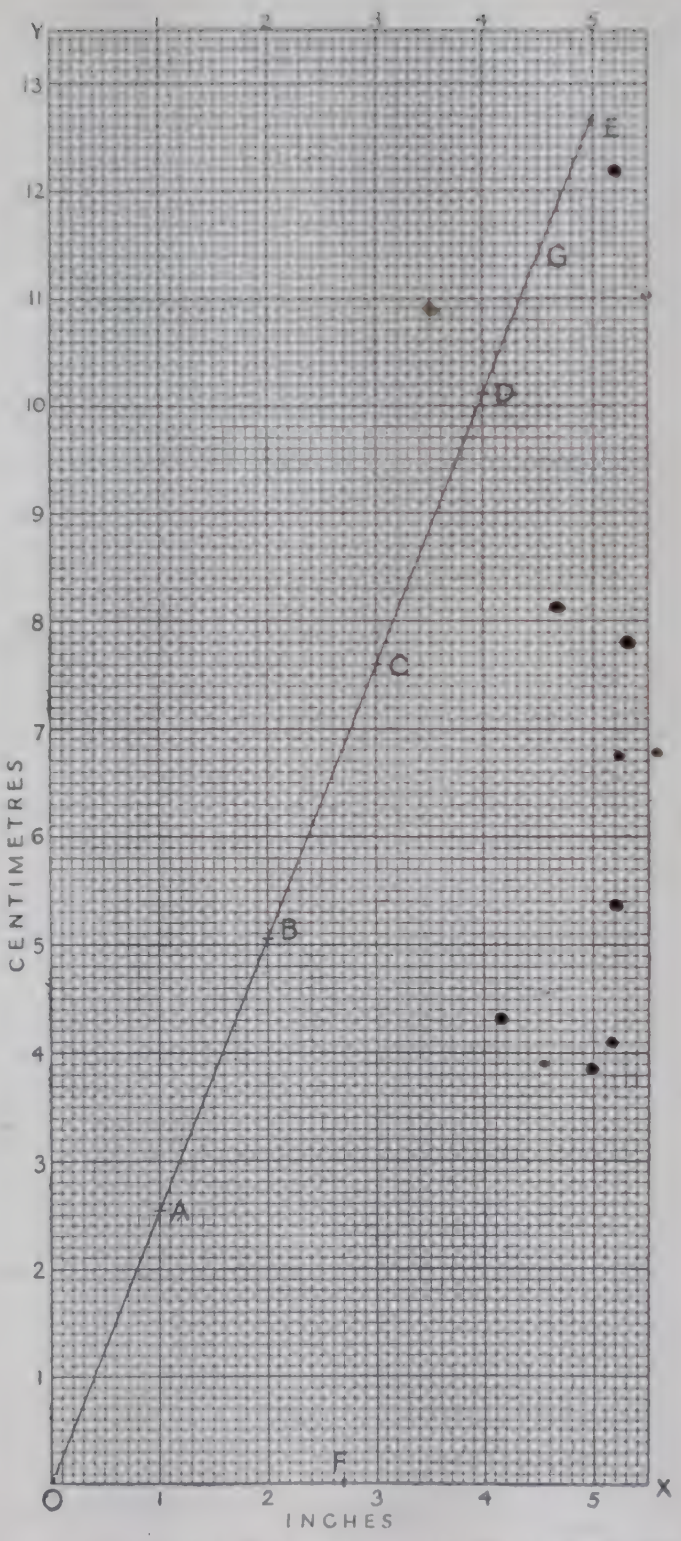

Fig. 12.—Graph showing the Relation between Centimetres and Inches

15. RATIO OF CIRCUMFERENCE AND DIAMETER OF A CIRCLE.

Ex. 27.—Find the values of the circumference and diameter of a number of circles (on paper); and of cylinders and disks; or, take those measured in previous exercises. Record them thus:—

	Circumference.	Diameter.	Circumference divided by Diameter.
1st Circle ...	19·8 cm.	6·3 cm.	...
2nd Circle ...	...	...	...

The numbers in the last column show the ratio between the circumferences and diameters. Are they nearly the same?

If one differs much from the rest, repeat the measurements for that circle.

Continue to test them until they are as correct as you can make them.

Then take the mean of all the values of this ratio. This number is called π (pi).

$$\pi = 3{\cdot}1416.$$

Ex. 28.—On millimetre squared paper make a graph showing the relation between the diameter and the circumferences. Thus if the diameter and circumference of the first circle are 4 cm. and 12·53 cm. respectively, the point on the graph will be 4 cm. from the left, and 12·53 cm. from the bottom.

Similarly for the other pairs of measurements.

The graph must be drawn fairly through the points.

Allowing for inaccuracies of measurements, this graph also is a straight line. Graphs are not always straight lines. When the graph comes out to be a straight line, one of the quantities is always, as in these two examples, a simple multiple of the other. The two quantities are then said to be **proportional** to one another, or to be in a simple **ratio** to one another. The circumferences of circles are in the ratio of their diameters.

Ex. 29.—Use the graph made in Exercise 28 to find out the circumferences of circles of known diameters, and the diameters of circles of known circumferences.

Ex. 30.—Taking $\pi = 3{\cdot}14$, calculate—

(1) The circumference of circles whose diameters are respectively 4 cm.; 6 m.; 4 m.; 5·4 yd.

(2) The diameters of circles whose circumferences are respectively 20 cm.; 2 ft.; 3·2 m.; 1 mile.

Ex. 31.—Miscellaneous Examples:—

(1) Three persons measure the length of a street. Their results are: 157 yd. 1 ft.; 156 yd. 2 ft.; 158 yd. What is the probable length of the street?

(2) Measure the length of one of the leaves of a book. How far would all the leaves stretch if they were placed on end?

(3) Measure the lengths of two pieces of pencil or other small objects and find out their difference, and how many times one is contained in the other.

(4) On millimetre paper take a length 1 dm. to represent 1 mile. Mark off the distances that represent ¼ mile; 88 yd.; 200 yd.

(5) Draw a triangle and measure the lengths of its sides. Is the sum of the lengths of two sides greater than the third? Repeat the measurements for several triangles.

(6) Draw a line whose length = 3·2 + 0·4 + 1·7 in., and measure it in mm.

(7) Calculate the ratio between a mile and a kilometre.

(8) How would you measure correctly the height of a flask?

(9) Describe how by means of a bicycle the distance between two towns could be ascertained (opisometer).

(10) If the large wheel of a locomotive is 1·75 m. in diameter, how many times would it revolve in travelling 200 Km.?

(11) Using the scale given on the map of a country and a pair of dividers, find the length of the coast line of the country, and of its principal rivers.

(12) Measure the curved distance between two marks on a flask or beaker.

(13) Measure the length of the curved back of a chair, or the distance between two marks round the curved corner of a table.

(14) Put a number of shot in a row beside a scale, and find the mean diameter in that way.

The remainder of the exercises in this chapter are of a less elementary character, and involve the use of more accurate instruments.

16. THE DIAGONAL SCALE.—More exact measurements of length may be made by the use of a diagonal scale, such as is shown in fig. 13.

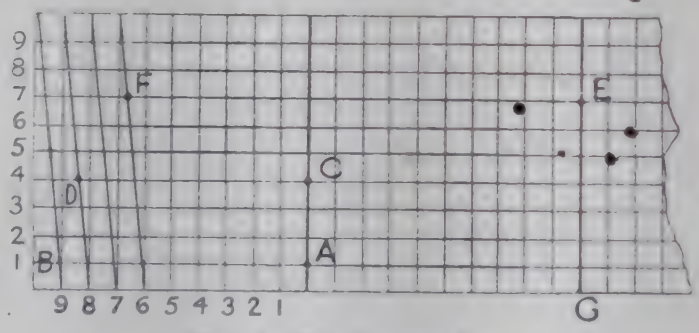

Fig. 13.—The Diagonal Scale

Ex. 32.—(1) On squared paper ($\frac{1}{10}$ in. is a convenient size) draw diagonal lines, as shown in fig. 13, so as to divide the first large square into twenty wedges, each ten times as long as the width of its base. Number the division lines as shown in fig. 13. Then on the principle explained in Exercise 19 the distance AB = ·91 in.; CD = ·84 in.; EF = 1·67 in.

(2) On your scale find lengths of 1·54; ·38; 2·72, &c., units.

(3) Make a number of dots on a piece of paper; measure the distance between them with dividers, as in Exercise 9; transfer the dividers to the diagonal scale, and read off the length. One leg of the dividers must be placed on one of the principal vertical division marks of the scale (AC, GE in fig. 13), and on one of the long lines (CD, EF in fig. 13). The other leg must come exactly on one of the points where the diagonal crosses these length lines (B, D, F in fig. 13). The correct length line to be used must be found by trial.

If paper divided into millimetre squares be used. lengths may be measured correct to $\frac{1}{10}$ mm. or $\frac{1}{100}$ cm.

(4) Repeat Exercise 9, using a diagonal scale.

17. THE VERNIER.

Ex. 33.—(1) Cut out two strips of cardboard AB and CD (fig. 14).

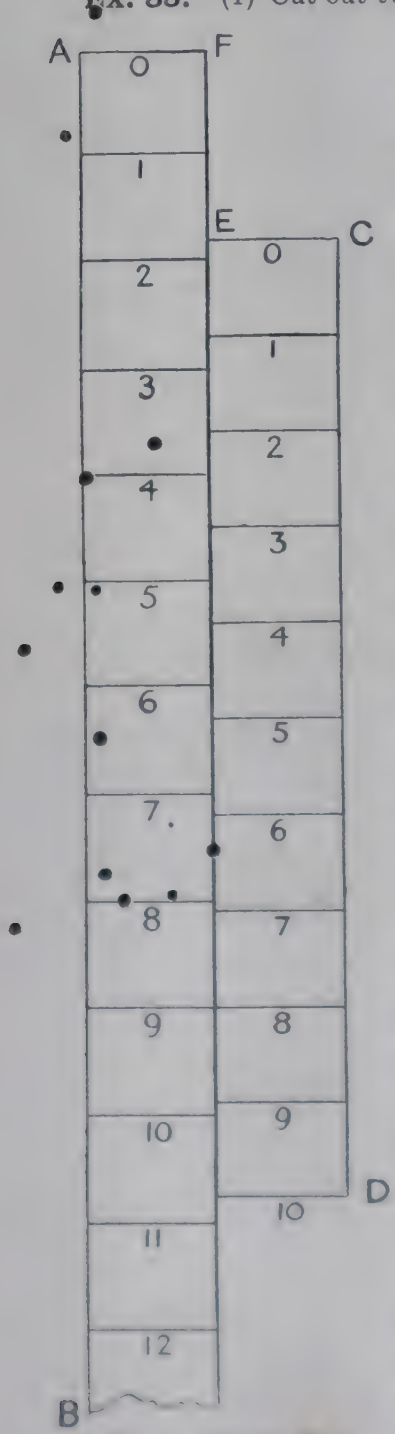

Fig. 14.—The Vernier

On AB mark divisions 0, 1, 2, &c., exactly 1 cm. apart up to about 15 cm. Cut off CD exactly 9 cm. long, and by a scale or millimetre paper divide it accurately into 10 equal parts, each 9 mm. long. Number the division marks 0, 1, . . . to 10.

Place the two strips side by side as in fig. 14. AB is called the scale and CD the vernier.

Place the two zeros in a line. Which division on the vernier coincides with a division on the scale?

Slide CD along AB so that the vernier 0 coincides with scale division 1. What other division lines now coincide?

Slide CD a little further so that its zero is halfway between scale divisions 1 and 2. Which vernier mark now coincides with a scale mark?

Slide CD 3 mm. further along (measure the 3 mm.). Which divisions now coincide? This position is illustrated in fig. 14; the length EF is 1·8 cm.

(2) Move the vernier CD along the scale AB so as to make EF these lengths: 2·3 cm.; 2·9 cm.; 3·1 cm.; 3·6 cm.

(3) Make two dots on a piece of paper. Place the scale so that one dot is at F and the other along the edge FE; place the vernier in position so that its zero line CE runs into the second dot. What is the distance between the dots? Verify by a scale.

Make several measurements of this kind.

Ex. 34.—In the same manner as the distance FE above was measured, by means of your vernier measure the lengths of small objects, such as small cylinders, pencils, pieces of glass tube, blocks of wood.

If exact coincidence in the marks is not obtained, take the mean between the two nearest. This gives a further decimal point in the answer, e.g. a distance may be 3·65 cm.

The same principle may be applied to any unit of length. Verniers are often put on instruments, such as callipers and barometers, to enable readings to be taken more accurately.

18. THE SLIDE CALLIPERS (fig. 15) is an instrument arranged on the principle of fig. 7—a scale having two straight jaws attached to it, the inner edges of the jaws being exactly at right angles (Art. 22) to the scale. One jaw is fixed and the other slides

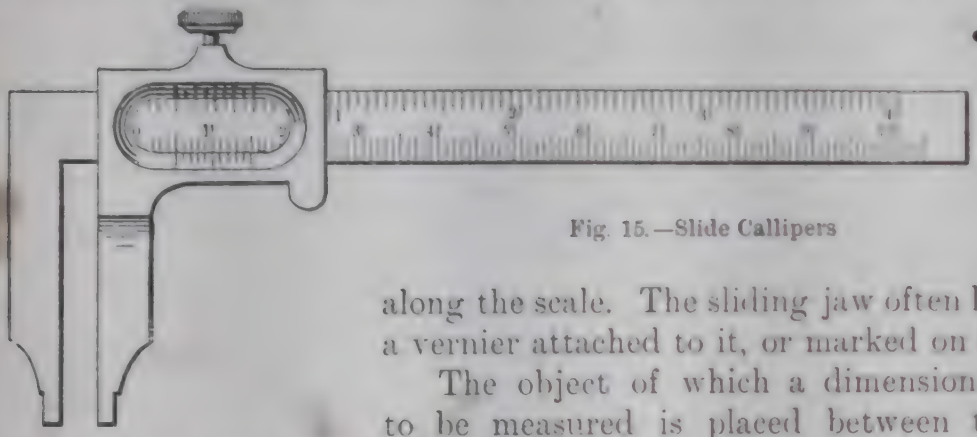

Fig. 15.—Slide Callipers

along the scale. The sliding jaw often has a vernier attached to it, or marked on it.

The object of which a dimension is to be measured is placed between the jaws and the length read off on the scale.

Ex. 35.—By means of the slide callipers find the lengths of objects, the diameters of disks, spheres, tubes, and wires.

19. THE SCREW GAUGE.—The dimensions of small objects, such as the thickness of disks and the diameters of wires, may be accurately measured by instruments that contain a screw.

Ex. 36.—Examine a screw, such as one of the large brass screws that are used for the terminals of electrical instruments (fig. 16). Note the thread of the screw, and the thread inside the nut B through which the screw works.

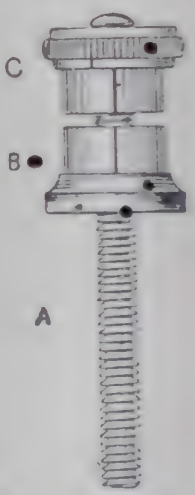

Fig. 16.—Screw

Measure accurately the length (l mm.) of the screw, and count the number (n) of turns of the thread in that distance. Calculate $\frac{l}{n} = p$ mm. the *pitch* of the screw, i.e. the distance between two successive turns of the thread.

Note that when the head C is turned through one revolution the screw advances by a distance p.

Turn the screw home firmly against its nut, and rule a line along the head and nut as shown in fig. 16. Call this the zero line.

Turn C away from B and place a knitting needle or a pencil in the gap so that the object is just gripped. Estimate the fraction of a complete revolution between the two parts of the zero line. Remove the object and turn the screw home. Thus, suppose the value of p is 1·17 mm., and the gap between the jaws was such that it required 2·3 turns to close it, then the thickness of the plate is $2{\cdot}3 \times 1{\cdot}17 = 2{\cdot}7$ mm.

The arithmetic is simpler if the pitch of the screw is exactly 1 mm.

A screw gauge is shown in fig. 17, where A is the screw, B the

nut, and C the head. The head is a hollow cylinder which is attached to the hidden end of the screw and works up and down outside the nut. The nut is connected by a curved piece of metal D to a jaw E, to and from which the screw moves. The object of which a dimension is to be measured is placed between E and A.

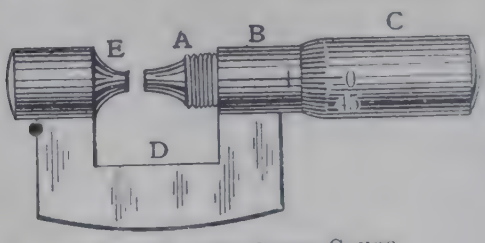

Fig. 17.—The Screw Gauge

A scale is engraved on the nut, the distance between the divisions being the same as the pitch of the screw, or in some cases double that distance. The circumference of the head is divided generally into 50 equal parts.

Ex. 37.—Examine the screw gauge. *Never turn the screw up tightly.*

Into how many divisions is the head graduated?

One complete turn of the head advances or withdraws the screw by p mm. Find p, as in Exercise 36.

Turning the head through one of its divisions advances or withdraws the screw by y mm. Find y.

By how much is the width of the gap altered when the head is turned through 3 complete turns + 22 head divisions? by 2 complete turns − 6 head divisions?

Turn the screw home. Is the head at zero? If not, allow for this zero error in making measurements.

Ex. 38.—Measure the diameters of small spheres, of coins, of thick wires, and then of thin wires by the gauge or the simple screw.

20. THE SPHEROMETER.—The spherometer (fig. 18) is like the screw gauge in principle although different in form. The screw works through the centre of a three-legged table. The mode of graduation is the same as that of the screw gauge.

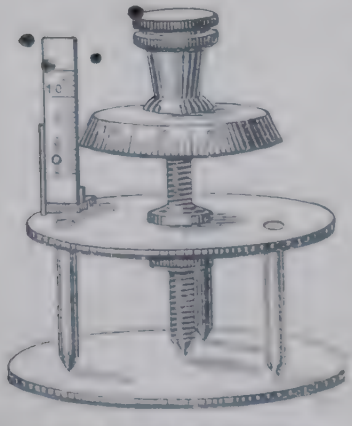
Fig. 18.—The Spherometer

Ex. 39.—Put the spherometer on a sheet of plate glass. Turn down the screw till the instrument rocks and the feet rattle. By trial adjust the screw so that it just touches the glass without any rattle when the instrument is touched. Then examine the zeros of the scales, and note the error if any.

Turn the milled head back through a complete revolutions, and see how far p mm. on the vertical scale. Hence calculate the pitch of the screw.

Find the quantity b of Exercise 37.

Ex. 40.—(1) Place a thin slab of glass, such as a microscope cover glass, on a large glass plate and under the screw. Adjust as before and take the reading

Repeat the whole operation, and take the mean of the readings. Hence find the thickness of the glass. The following are a set of three readings:—

Zero readings: − ·0075; − ·00625; − ·00875. Mean, − ·0075.
Readings with glass inserted: ·24; ·245; ·2475. Mean, ·2442.
Thickness of glass = ·2442 + ·0075 = ·252 mm.

(2) **Find the radius of curvature of a lens.**

Find the zero reading on plane glass as before.

Place the spherometer on the lens and adjust the screw till all four legs touch the lens. How much has the screw been raised (x mm.)? Take the mean of three readings.

Then, in fig. 19, if ACB represents a central section of the portion of the lens dealt with, ACBF represents a section of the sphere of which the lens is a detached part. CD is the distance x above. DB is the radius of the circle whose circumference passes through the bottom of the three legs of the spherometer, and whose centre is the bottom of the screw. Measure DB accurately on the spherometer.

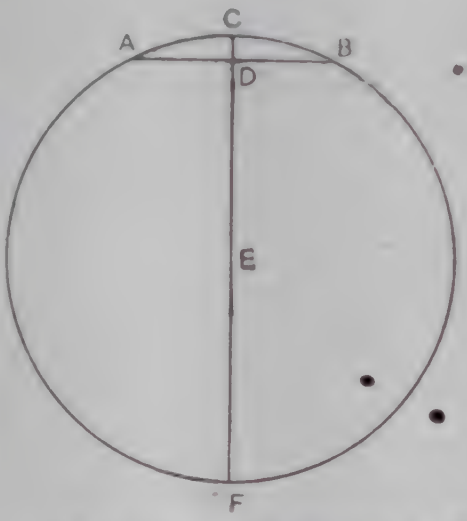

Fig. 19.—To find the Radius of a Spherical Surface

Then, by geometry, CD × DF = DB². This equation gives the value of DF, whence obtain CF and CE.

Example: x = CD = 1·511 mm. Mean distance between the bottom of the screw and the legs on a plane surface = DB = 2·3 cm.

$$\therefore 1{\cdot}511 \times DF = (23)^2, \therefore DF = 350 \text{ mm.}$$

and the radius required = 17·5 cm.

CHAPTER II

Angle

21. VERTICAL AND HORIZONTAL.

Ex. 41.—(1) Tie a piece of metal to one end of a thin string, and holding the other end of the string suspend the mass in the air. Such an arrangement is called a **plumb bob.** The direction in which the string hangs is said to be **vertical.**

There are many edges of objects in the room which are nearly vertical. Test the verticality of these lines by holding the suspended plumb bob exactly between them and the eye, using only one eye. If any line thus tested appear to coincide with the string throughout its whole length that line is vertical.

Fig. 20.—Vertical and Horizontal

(2) Place a vessel of water on the table, preferably a flat shallow vessel. The surface of the water is exactly **horizontal**.

Support the plumb bob on a stand with the bob just below the surface of the water. Blacken the water with a little ink. From various positions examine the string and its reflection in the water. One line appears to be the exact prolongation of the other. This is an exact test for verticality and horizontality.

(3) Replace the vessel of water by a mirror. If the string and its reflection do not appear to be exactly in the same straight line the mirror is not horizontal.

A straight line drawn to a surface in any direction other than perpendicular to it is said to be **oblique** to that surface.

22. RIGHT ANGLE.—An angle is a corner. If you are travelling along a line, and turn out of that line into another line, that is, turn a corner, you have turned through an angle.

Think of the plumb bob hanging in water, and imagine a straight line drawn along the surface of the water through the thread, then the angle between the line of the plumb bob and the line on the water surface is called a **right angle**, and the two lines are said to be at right angles to each other.

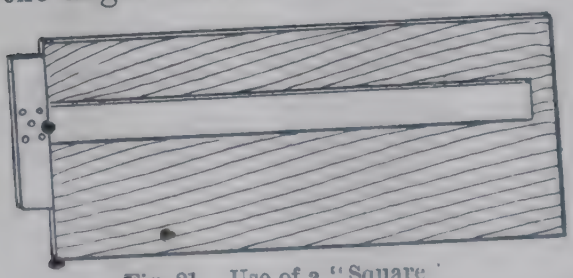

Fig. 21.—Use of a "Square"

There are probably many pairs of lines in the room at right angles to each other. Probably the corners of the window panes and of the room itself are right angles.

In Exercise 15, fig. 7, set squares were used. They were used because it was desired to obtain two lines each at right angles to the scale. Why was it necessary to do that?

In fig. 21 is shown a T-square, which is used in drawing to obtain lines at right angles to each other.

Builders use a similar "square".

Ex. 42.—By means of such squares—

(1) Test the right angles on common objects.

(2) Draw an accurate right angle; an angle greater than a right angle, which is called an *obtuse* angle; and an angle less than a right angle, which is called an *acute* angle.

Note that the length of the lines has nothing to do with the size of the angle.

23. DIVISION OF A RIGHT ANGLE.

Ex. 43.—Draw a large circle on a sheet of paper which is not too thick to allow lines to be seen through the paper.

Fold the paper over so that one half of the circumference of the circle exactly covers the other when the paper is held up to the light. Mark the **diameter** thus obtained. Fold the paper again so that the two halves of this diameter exactly cover each other; this gives a second diameter exactly at right angles to the first. Mark them both carefully. The circle is now divided into four equal parts each **called a quadrant.**

By trial with dividers (Art. 7) divide each quarter circumference into three equal parts corresponding to the hours on a clock face, and draw diameters from each of the points thus obtained. You have now got 12 equal angles at the centre of the circle.

Protractor.—Fig. 22 shows a semicircle in which the process of division has been carried further than you can accurately carry it. Such an instrument is called a *protractor*, and is used for measuring angles. In fig. 22 lines are shown drawn from the centre, dividing each right angle into 9 equal parts; and round the circumference ten short lines are drawn between each of the divisions thus obtained. If each of these short lines were prolonged to the centre of the circle, each right angle would be divided into 90 very small angles. Each of these very small angles is called a **degree.** The lines last mentioned are not drawn in the figure, because they would all blur into one another at the centre.

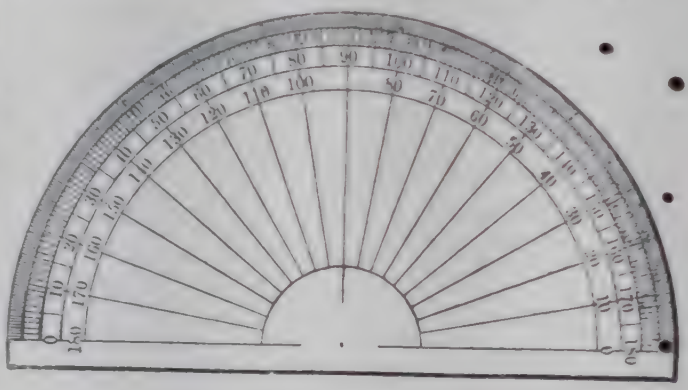

Fig. 22.—**The Protractor**

A degree is the $\frac{1}{90}$th part of a right angle, and is marked (°). This small angle of 1° is the **unit angle**. All other angles are measured by saying how many degrees they contain. How many degrees are there in each of the angles drawn in Exercise 43?

Ex. 44.—Continue the subdivision of your circle until it marks angles of 10 degrees (10°) or 5 degrees (5°).

Ex. 45.—(1) By means of a protractor draw lines including angles of 50°; 20°; 70°; 45°, and (by estimation if necessary) 22°; 84°; 63°.

(2) Draw a *large* triangle. Measure the three angles as accurately as you can, and add up the number of degrees. Repeat with another large triangle of different shape. What is the sum of the three angles of a triangle?

(3) By means of a protractor measure the angles of your set squares.

(4) By measurement with protractor verify the following propositions i geometry:—

(*a*) Alternate angles are equal.

(*b*) Any two angles of a triangle are less than two right angles.

(*c*) The greater side of every triangle is opposite to the greater angle.

24. THE SEXTANT.

Ex. 46.—(1) On a piece of stiff cardboard graduate a semicircle in divisio of 10° or, better, of 5° Cut it out a little larger than the semicircle, as in fig. 2

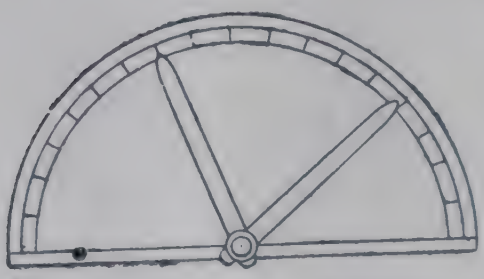

Fig. 23.—An Elementary Sextant

Cut out two narrow strips of cardboa for movable arms. By means of a pin f one end of each arm at the centre of t semicircle, and let the other end mo round the graduations.

Hold the instrument with the cent against the eye, and place the arms so th one points to one side of a window a the other to the other side of the wi dow. Read the angle between the arm The breadth of the window is said *subtend* that angle at the eye.

Move nearer to the window and measure the angle again.

Move further away and measure the angle again.

(2) Measure the angle subtended at the eye by various objects.

The apparent size of objects as seen by us is estimated accordi to the angle they subtend at the eye. The instrument used for t purpose is called a sextant; that shown in fig. 23 is a primitive fo of sextant.

25. SIMILAR FIGURES.

Ex. 47.—(1) Draw any two straight lines AB, AC (fig. 24), making with other any angle BAC.

Fig. 24.—Similar Triangles

In the line AB take several points D, &c., and through these points draw lines, &c., parallel to each other, cutting AC in points K, L, &c. It is better to draw a l figure.

Measure accurately the lengths AD, AE &c., and the lines DK, EL, FM, &c., and re them in tabular form.

(2) Make a graph showing the relatio tween these two sets of lengths. Using sq paper, set off the distance AD along the line, and the distance DK along the ve line: this gives one point on the graph.

Next set off the distance AE along the base line, and the distance EL alon vertical line: this gives the next point on the graph.

Proceed thus for each pair of lines. When several points have been obtained on the graph, draw a line through the points.

Ex. 48.—Draw two straight lines AB, AC at right angles to each other, and with A as centre describe several quarter circles, as in fig. 25.

Measure the radius AD and the quarter circumference DK. Similarly with AE and EL, and the other pairs. From the two sets make a graph as in the last exercise, measuring the radii along the base line, and the quarter circumferences along the vertical line. Compare the two graphs.

In fig. 24 all the triangles ADK, AEL, &c., are similar figures.

Since the graph drawn above comes a straight line, we see that in pairs of similar triangles the lengths of corresponding sides are proportional: AD bears the same ratio to DK that AF does to FM.

The same remark applies to the quarter circles in fig. 25. The portion of the circumference FM bears the same ratio to DK as the radius AF bears to the radius AD. Hence the circumferences of circles are proportional to their radii.

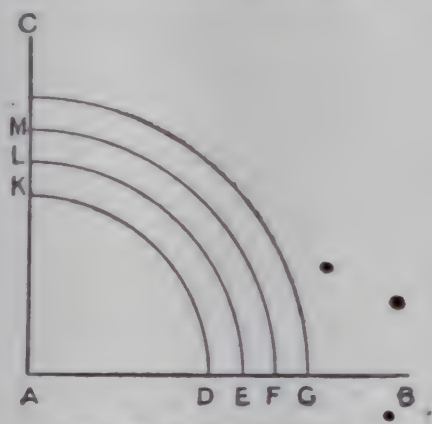

Fig. 25.—Similar Figures

These two examples show that in similar figures corresponding lengths are proportional.

26. TANGENT AND SINE OF AN ANGLE.

Ex. 49.—(1) Draw a right-angled triangle ACB (fig. 26). Think of the angle B. Then

the ratio $\frac{AC}{CB}$ is called the tangent of the angle B;

the ratio $\frac{AC}{AB}$ is called the sine of the angle B.

Measure the lengths of the sides, and measure the angle B in degrees, and find the above ratios. Record thus:—

Tangent of 30° (written tan 30°) $= \frac{10}{17{\cdot}32} = {\cdot}577.$

Sine of 30° (written sin 30°) $= \frac{10}{20} = {\cdot}5.$

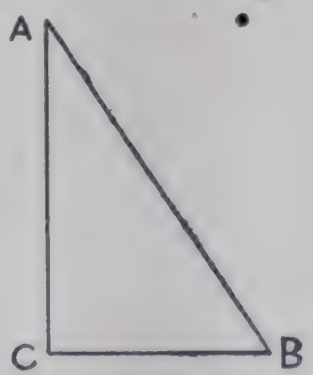

Fig. 26

(2) By this means find the values of the sines and tangents of angles of the following sizes: 30°; 60°; 45°. Draw first the line CB; then CA at right angles to CB; then make the angle at B the required size; then produce BA to meet CA.

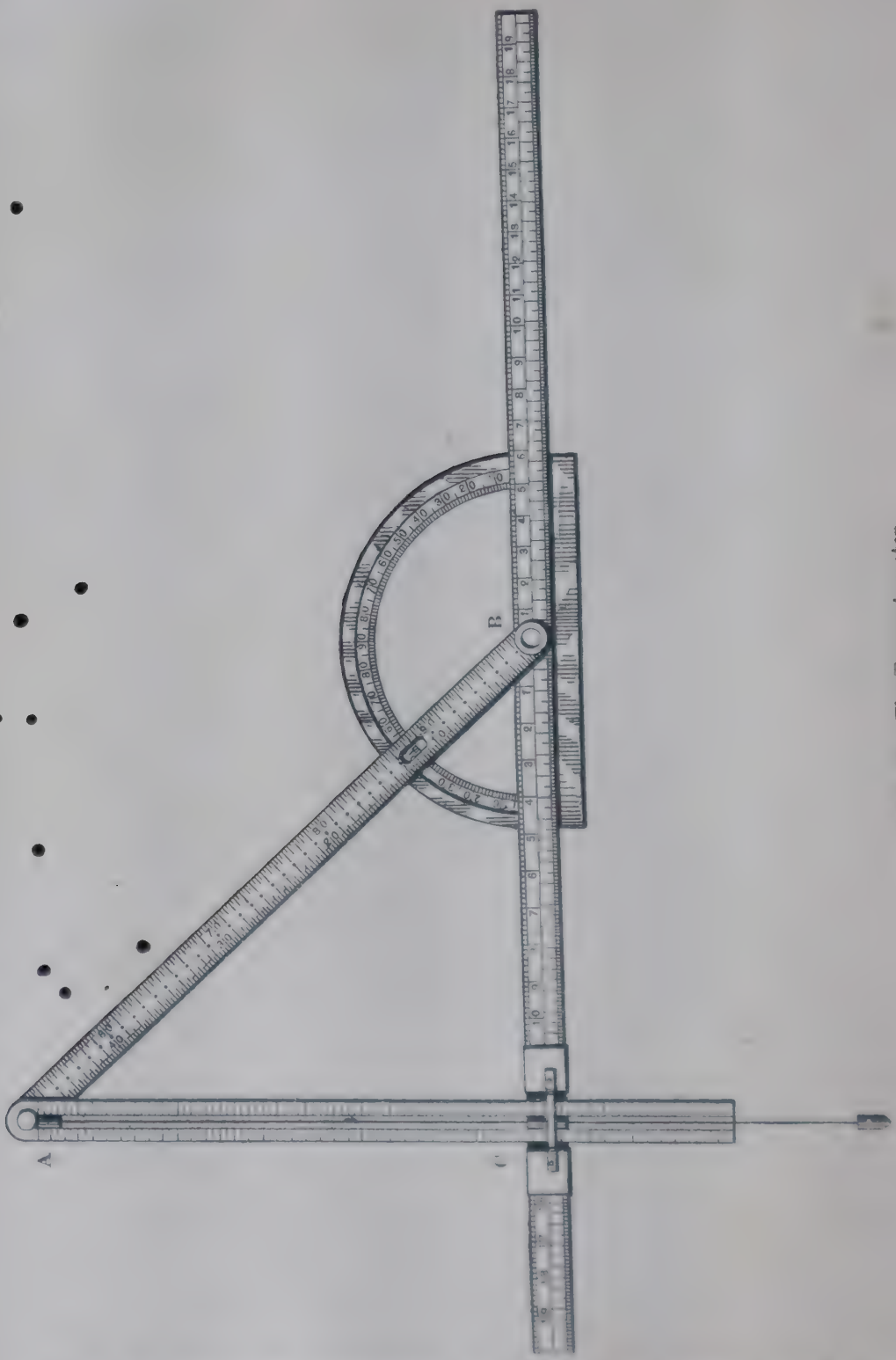

Fig. 27.—The Tangentometer

Fig. 27 shows an apparatus by which the tangent and sine o
any angle can be obtained. Compare it with fig. 26. At the poin
corresponding to C there is a slider which moves along the horizonta

arm CB. The arm AC must be kept vertical, and is provided with a plumb bob. The arm AB is hinged at B, the centre of a semicircular protractor.

An apparatus to answer this purpose can be readily made, and the sines and tangents of angles read off on the scales.

Ex. 50.—Estimation of Angles.—Draw as nearly as you can, without the use of instruments, an angle of 45°. Measure it with a protractor, and find the error. Repeat successively with angles of 30°; 60°; 20°; 70°; 10°; 80°.

CHAPTER III

Mass: The Balance

27. The **mass** of a body is the quantity of stuff or material that there is in it.

Ex. 51.—Cut out two pieces of muslin, or similar thin material, of the same size. In each piece tie up an equal number (100 or 200) of lead shot of the same size. You thus have two equal masses of matter. We will call them A and B.

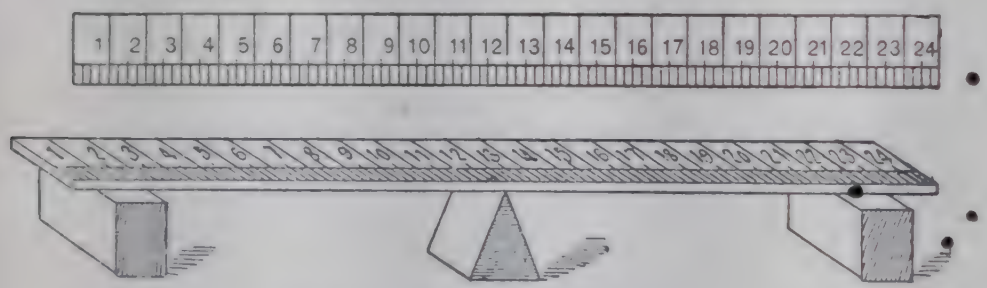

Fig. 28.—An Elementary Balance

Support a metre or half-metre scale at its middle point on one of the edges of a wedge, as in fig. 28. This edge is called a knife-edge.

Adjust the scale so that it rests horizontal, by placing, if necessary, a loop of string or wire on the end that rises. Under each end of the scale place a block of wood so that the ends cannot descend far, and thus upset the apparatus.

Such an apparatus is called a **balance**. It is a lever with equal arms (Art 139). This lever is called the beam of the balance.

By means of threads suspend the equal masses A and B on the scale in such positions that they balance, and the scale remains horizontal.

Let the positions of A and B be nearly at the ends of the scale. Read carefully the distances of the threads from the point of suspension of the scale.

Move A two or three centimetres nearer to the middle of the scale, adjust the position of B until balance is obtained, and read the distances as before. Do this

for several positions of A and B, but do not go too near to the centre, as mistakes are then more likely to be made.

When the set of observations is concluded, exchange the positions of A and B and repeat the whole of the operations. Record thus:—

Distances from the Knife-edge.	First Set.				Second Set.				Sum of Distances
Mass A	...	...	...	...	...	...	...	...	...
Mass B	...	...	...	...	...	...	...	...	...

Examine the two rows of figures.

Allowing for the errors in the experiments, what relation does there appear to be between the distances of A and B from the centre?

In the right-hand column add up all the distances obtained for A, and all the distances for B, and write them one under the other. This process (see Exercise 6) eliminates some of the errors.

The sum of the distances of A is seen to be equal to that of B. When the balance is accurately made, and the whole experiment carefully conducted, each pair of distances is the same.

Now we know that the mass of A is equal to the mass of B. Hence this experiment shows that two equal masses suspended from the two equal arms of a balance leave the balance undisturbed, or, as it is called, **in equilibrium**.

This is the test for determining whether two masses are equal. It is all the same whether the two masses are of the same kind of stuff or of different kinds. If B were a lump of brass, or wood, or cotton, or any material whatever, we say that the mass of material in B is the same as the mass in A when the two masses balance each other in the manner described above.

A balance, then, compares masses; it enables us to obtain one mass equal to another mass.

Every mass of matter that we deal with is heavy. It is pulled downwards by the earth, and when the masses on the two equal arms of a balance are pulled downward equally, so that the balance is in equilibrium, we say that the masses are equal.

This process is called **weighing**, and generally the masses suspended from one of the arms are called **weights**.

The matter is explained further in Chapter XI.

28. UNIT OF MASS.—Just as to measure lengths some length had to be chosen as a unit, and other lengths compared with this unit; and to measure angles, a unit angle was chosen; so to measure mass some quantity of matter has to be taken and called the **unit**

of mass, and any other mass is measured by determining how many of these units it will balance.

Different countries have chosen different units. In England the unit of mass in common use is the **pound** (written **lb.**). In France, and in scientific work generally, the unit of mass is the **gramme** (written **grm.**). Pieces of metal that represent these units are kept in London and in Paris, and other pounds and grammes are made by balancing them against those *standard* masses in the manner described above.

These units of mass are subdivided into smaller parts, just as the centimetre is divided into millimetres. A set of such masses is usually called a set of weights.

Examine a set of weights, such as those shown in fig. 29.

Fig. 29.—A Set of Gramme Weights

The smallest shown in that figure is a gramme. All ordinary laboratory weights are expressed in grammes.

The largest shown is 500 grm.

A set of weights up to 100 grm. generally includes weights of 100 grm.; 50 grm.; 20 grm.; 10 grm.; 10 grm.; 5 grm.; 2 grm.; 2 grm.; 1 grm.

From these all the weights up to 200 grm. can be made.

Thus 189 grm. is made up of 100, 50, 20, 10, 5, 2, 2.

If there is only one 10-grm. and one 2-grm. weight, a larger weight must be used, and the excess counterbalanced by a small weight on the other arm of the balance.

Fractions of a gramme are usually reckoned in milligrams (mg.).

$$1 \text{ mg.} = \tfrac{1}{1000} \text{ grm.}$$

Fig. 30 shows a set of milligram weights by which any fraction of a gramme can be made up.

The English pound is divided into 16 equal parts, each called an **ounce** (**oz.**). These weights are not often used in a science laboratory.

Ex. 52.—Using only the sets shown in figs. 29 and 30, which weights would you use for—

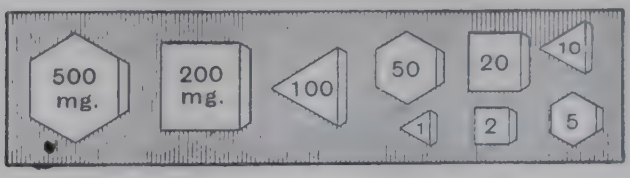

Fig. 30.—A Set of Milligram Weights

(*a*) 16 grm.? 27 grm.? 34 grm.? 48 grm.? 71 grm.?
(*b*) 19 mg.? 246 mg.? 9 mg.?
(*c*) 18·75 grm.? 44·03 grm.? 79·79 grm.?

Ex. 53.—Suspend by thin threads equal "weights" from the (equal) arms of the balance, thus—

(i) 20 grm. on each side;
(ii) 2 grm. on each side;
(iii) 50 on one side, and 20 + 20 + 10 on the other side.

Do you obtain equilibrium in each of these cases?
Any failure in exact equilibrium is probably due to faults in the balance.

29. THE PHYSICAL BALANCE.—Balances are made in various forms.

Most of the weighings required in elementary work may be made by a balance set up as shown in fig. 31, which is the same in principle as the simple lever of fig. 28, except that the beam is more carefully mounted.

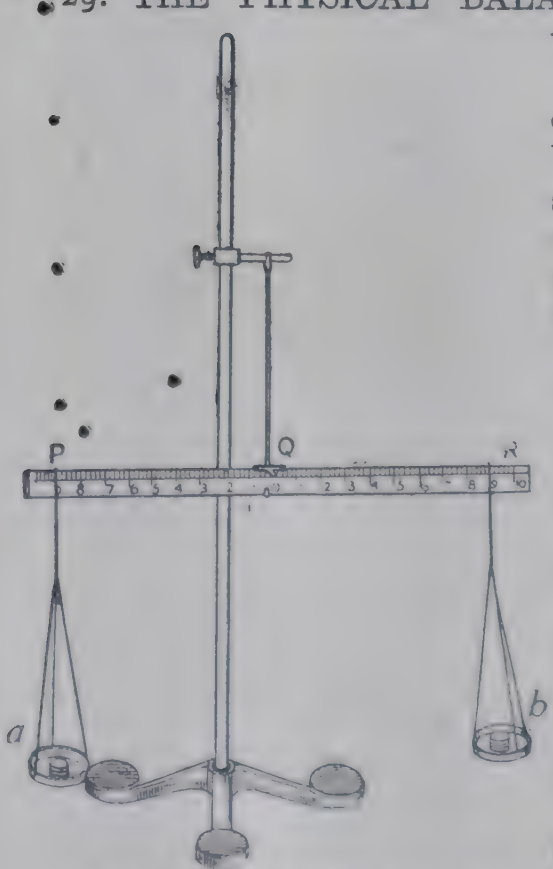

Fig. 31.—A More Accurate Balance

But to obtain accurate results with small masses, it is necessary to use a balance such as that shown in fig. 32. Such instruments are usually made of brass. The **beam** rests exactly at its middle point on a sharp knife-edge, and at each end of the beam a pan hangs, also suspended from a knife-edge. These knife-edges are hard and sharp, being usually made of steel or agate, and their distances are accurately adjusted.

The central wedge on which the beam turns has the knife-edge turned downwards, and rests on

the top of a tall central column. Arrangements are made by which, when the balance is not in use, this knife-edge is lifted off the surface, on which, when in use, it rests. The handle shown at the bottom of the instrument effects this. When that handle is turned towards the right the beam is resting on the central knife-edge, and the balance can be used. With the handle in the position shown in fig. 32 the beam is not resting on the knife-edge, and does not move if weights are placed in the pans.

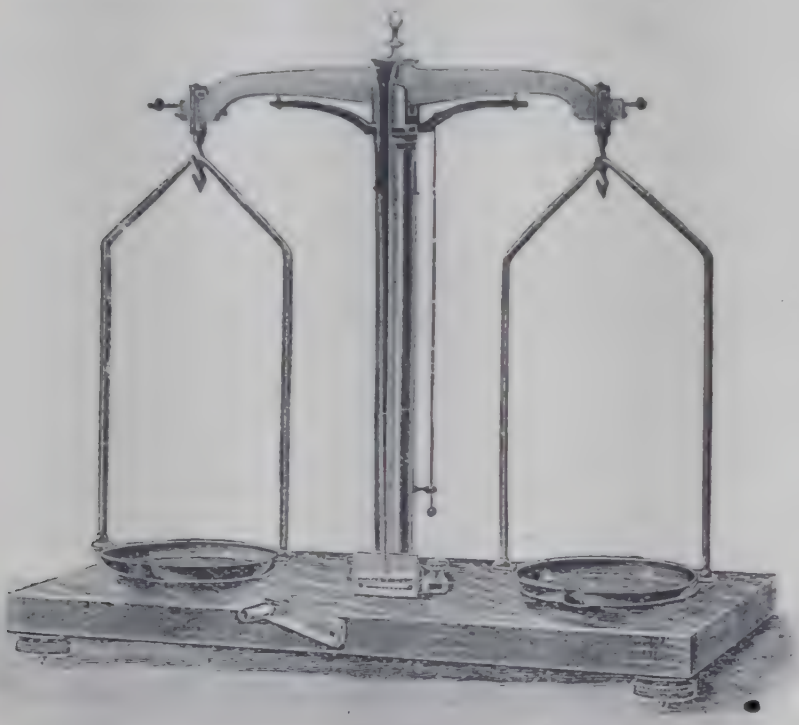

Fig. 32.—A Physical Laboratory Balance

The beam carries from its centre a long pointer, seen in front of the instrument. The end of the pointer moves over a scale, and by watching this pointer on the scale the least movement of the beam is seen.

Such an instrument is delicate, and is soon spoiled by rough or improper usage.

Rules for the Use of a Physical Balance.

1. The instrument must not be jerked, nor the pans swung to and fro, nor dislodged from their knife-edges.
2. The pans must be kept clean and dry.
3. The beam should be kept on its knife-edge only while weigh-

ing is actually proceeding. Turn the handle gently, and stop if both pans do not rise.

4. Put the weights in the right-hand pan, and the mass to be weighed in the left. Before adding or subtracting weights, lower the pans to their resting-places. Put the larger weights in the middle of the pan.
5. Move the weights with forceps, not with the fingers. Do not put them down anywhere except in their box or in the pans.
6. Never put on the balance a heavier mass than that for which it was made. [The maker always states this mass.]

Ex. 54.—(1) Weigh in grammes and milligrams the packets of lead shot previously used, and some common articles, such as a glass stopper, a piece of metal, a roll of paper, a block of wood.

(2) Weigh a bottle or beaker empty. Pour in water up to a mark and weigh again. Hence find the weight of the water. [Carefully dry the outside of the beaker so as not to wet the balance.]

(3) Cut off and measure the length of a piece of wire; weigh it. Weigh also an unknown length of the wire. From these quantities calculate the length of the second piece of wire.

(4) Compare the two systems of weights by weighing some English weights in grammes.

By the method of Exercise 24, find the number of grammes in a pound.

Ex. 55.—Weigh a body by the method of substitution.

Place the body in the right-hand pan. Put a sheet of paper over the left-hand pan. On this sheet put shot or sand or any suitable masses until balance is nearly obtained. Exact balance may be got by adding small pieces of paper. In this operation the body is said to be counterpoised.

When exact balance is obtained remove the body, and place weights (grammes and milligrams) in the right-hand pan until balance is again obtained. Then these weights give the correct mass of the body.

This method should be employed with balances which are not accurately made, because it prevents errors due to inequality in the arms.

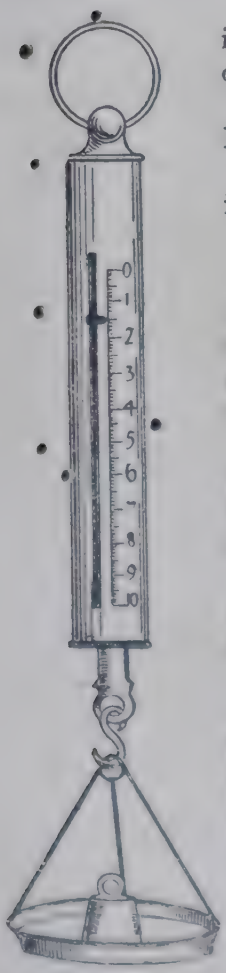

Fig. 33.—The Spring Balance

30. THE SPRING BALANCE. — For many purposes where great accuracy is not required a spring balance (fig. 33) is useful. This is a steel spring enclosed in a case. When masses are hung on the spring it extends, and moves a little index over a scale marked on the case.

Ex. 56.—(1) On a spring balance hang known weights and observe the position of the index for each weight, thus verifying the scale.

(2) By means of the spring balance weigh objects that can be suspended from it, and compare the results with those obtained by an ordinary balance.

CHAPTER IV

Area

31. **Area** is extent of surface.

Ex. 57.—(1) On plain paper draw a thin straight line, and on it mark two small dots A and B 1 cm. apart. At A and B, by means of a square (Art. 22), set up perpendiculars to AB. On these perpendiculars measure distances AD and BC, each 1 cm. long. Join CD.

The figure ABCD is a **square centimetre**.

(2) Draw a **square inch** in the same way.

For small areas these are the **units of area**, and other surfaces are measured by saying how many of **these units** they contain.

Fig. 34 shows a square centimetre (sq. cm.), and a square inch (sq. in.).

The square centimetre is divided into small equal squares. Each of these is a square millimetre (sq. mm.). How many square millimetres are there in 1 sq. cm.?

The square inch is shown similarly divided, but the small squares have no special name. Each is $\frac{1}{100}$ of a square inch ($\frac{1}{100}$ sq. in. or ·01 sq. in.).

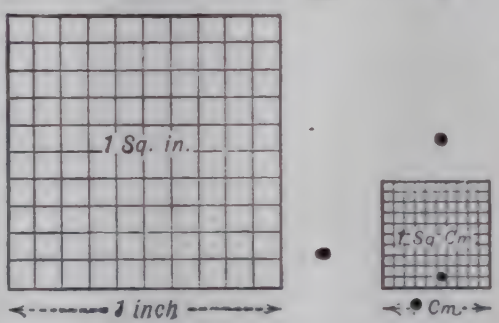

Fig. 34.—A Square Inch; a Square Centimetre

Ex. 58.—Find out the number of—

(1) Square centimetres in 1 sq. dm.
Square decimetres in 1 sq. m.
Square centimetres in 1 sq. m.
Square millimetres in 1 sq. dm.

(2) Square inches in 1 sq. ft.
Square feet in 1 sq. yd.
Square yards in 1 sq. mile

Ex. 59.—On a sheet of squared paper mark out squares the lengths of whose

sides are respectively 1 cm.; 2 cm.; 3 cm., . . ., and count the number of square centimetres in the squares thus marked out. Record thus:—

Length of side in centimetres	1	2	3	4	5	6	and so on.
Area of square in square centimetres	...	...	...	16	...	...	,,

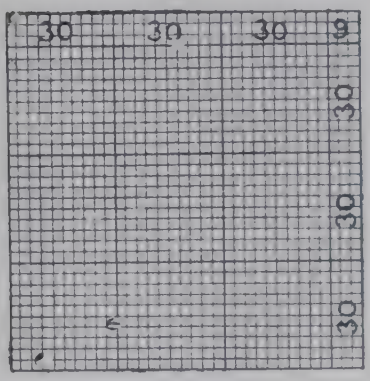

Fig. 35.—Area of a Square

The numbers in the second line are called the **squares** of those in the first line; the numbers in the first line are called the **square roots** of those in the second line. Such numbers are often required. The table should be learned.

Fig. 35 shows a square whose side is **33 mm. or 3·3 cm.** By counting the squares its area is seen to be 9 sq. cm. + 189 sq. mm. = 10·89 sq. cm. The ·89 sq. cm. means 89 sq. mm.

Ex. 60.—(1) What is the meaning of 1·43 sq. cm.? ·49 sq. cm.? ·81 sq. m.? 2·89 sq. dm.?

(2) On millimetre squared paper mark off squares the lengths of whose sides are respectively 1·5 cm.; 2·5 cm.; 3·7 cm.; ·4 cm.: 6 mm.; 9 mm.; and by counting the squares find the areas marked out. Record as above.

From what you have done, write down the rule for calculating the area of a square when you know the length of its side. In ordinary practice areas are not found by seeing how many squares will cover them, but by measuring the lengths of the sides and then calculating the area according to the rule.

(3) On plain paper draw a square. Measure the length of one side, and hence calculate its area. Do this in centimetres and in inches.

(4) The areas of some squares are respectively 81 sq. ft.; 16 sq. mm.; 25 sq. in.; 289 sq. yd.; 900 sq. in.; 1·44 sq. in.; ·81 sq. dm.; 1·96 sq. cm. What is the length of the side of each square?

(5) Draw squares whose areas are 25 sq. in.; 2·25 sq. cm.; ·25 sq. dm.; 6·25 sq. cm.

(6) The perimeter (distance round) of a square courtyard is 96 metres. Find its area. In its centre is a square plot of grass covering a quarter of the whole area. What is the width of the path round the plot of grass?

32. ESTIMATION OF SMALL AREAS.—In using squared paper it is often necessary to estimate fractions of the squares.

Ex. 61.—On squared paper draw an irregular closed figure about a square inch in area. (See fig. 44.) Find the area of the figure.

First count the complete squares and write the number down. Next, take each of the interserted squares in turn and estimate the portion included (thus

·7 ·25, &c.); write each one down and add up. Add the result to the number of complete squares.

Ex. 62.—(1) Draw a large square, and either by counting the centimetre and millimetre squares enclosed or by calculation, determine its area (*a*).

Draw a diagonal of the square, and on this diagonal describe a second square. Determine its area (A).

Divide A by *a*. What is the ratio between the area of the square on the diagonal and the area of the original square?

(2) Draw on squared paper a right-angled triangle ABC (fig. 36) whose two shortest sides are 3 and 4 units. On each side describe a square. Determine the area of each square. What is the relation between the area of the largest square and the sum of the areas of the two smallest squares?

Test whether the relation holds for the squares on the sides of any right-angled triangle.

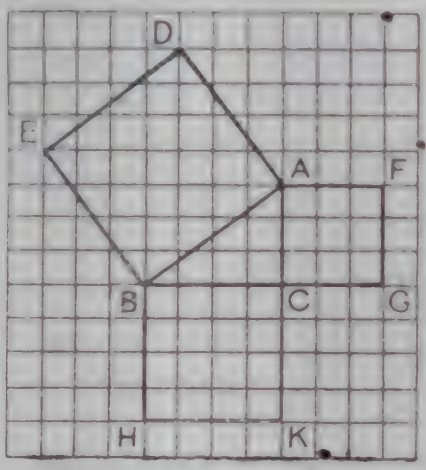

Fig. 36.—Squares on the Sides of a Right-angled Triangle

(3) A square has an area of 1 sq. dm. Find the area of a square inscribed in it. Each corner of the second square is at the middle point of the side of the first square.

33. TO COMPARE THE SQUARE CENTIMETRE AND SQUARE INCH.

Ex. 63.—On millimetre squared paper draw a square whose area is 4 sq. in. Count the square centimetres and square millimetres, and divide by four.

1 sq. in. contains . . . sq. cm.

Repeat the measurement with a square whose side is 3 in.

Compare the two answers. Take their mean. The correct answer is, 1 sq. in. contains 6·45 sq. cm.

What is the relation between this number 6·45 and the ratio (2·54) found in Exercise 24?

34. THE RECTANGLE.

A rectangle is a four-sided figure whose angles are all right angles. Fig. 37 shows a rectangle whose length is 3 cm. and breadth 2 cm. Count the centimetre squares to find its area.

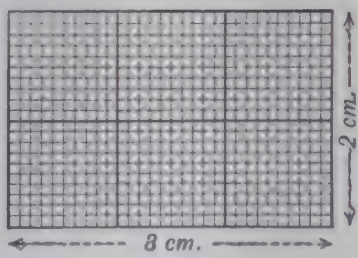

Fig. 37.—Area of a Rectangle

Ex. 64.—(1) Trace out other rectangles on squared paper, and find their areas.

(2) Write down the rule for calculating the area of a rectangle.

(3) Draw on plain paper rectangles whose areas are: 12 sq. in.; 10 sq. in.; 24 sq. cm.; 16 sq. cm.; 120 sq. mm.; 70 sq. mm.; 21 sq. mm.

Fig. 38 shows a rectangle whose length is 3·9 cm. and breadth 2·4 cm. By counting the squares its area is seen to be

6 sq. cm. + 336 sq. mm. = 9·36 sq. cm.
The product of 3·9 × 2·4 = 9·36.

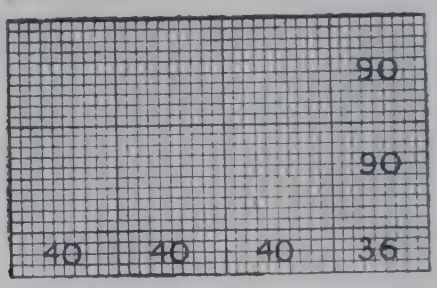

Fig 38.—Area of a Rectangle

Thus the rule obtained above for the simplest case is always true.

Area of rectangle
= length × breadth.

Care must be taken as to the units employed. Thus 3·9 cm. × 2·4 cm. makes 9·36 sq. cm., but 3·9 cm. × 2·4 mm. does not make 9·36 squares at all. Only multiply together lengths measured in the same units. Thus—

3·9 cm. × 2·4 mm. = 39 mm. × 2·4 mm. = 93·6 sq. mm.
= ·936 sq. cm.

Ex. 65.—(1) Find the area of rectangles whose lengths and breadths respectively are: 5·7 cm. and 1·3 cm.; 4·6 in. and 2·9 in.; 3·8 m. and 1·7 m.; 9·6 yd. and 4·8 yd.; 15 mm. and ·9 cm.; 2·8 cm. and 3·6 mm.

(2) The perimeter of a figure is the sum of the lengths of its sides. Draw a square whose area is 16 sq. cm., and a rectangle of the same area. Measure their perimeters and compare them. Repeat for squares and rectangles whose areas are 36 sq. cm. and 6·25 sq. cm.

If a square and a rectangle have the same area, which has the greater perimeter?

(3) Show by drawing on squared paper that if a straight line be divided into any two parts a and b, the square on the whole line is equal to the squares on the two parts together with twice the rectangle contained by the parts: i.e.

$$(a + b)^2 = a^2 + 2ab + b^2.$$

35. INVERSE PROPORTION.

Ex. 66.—A rectangle has to be drawn whose area is 60 sq. cm. Using whole numbers only, write down in two columns the length and breadth of every rectangle whose area is 60 sq. cm.

Make a graph from the two columns of figures, setting off the lengths of the rectangles along the horizontal axis, and the corresponding breadths along the vertical axis. The result is a regular curve like fig. 116.

Since the area of the rectangles is always the same, as the length diminishes the breadth increases in the same ratio. This is expressed by saying that one quantity is *inversely proportional* to the other. The curve obtained is always of the same shape when this statement is true of the quantities that it represents.

36. THE PARALLELOGRAM.—Two lines which are at the same distance apart throughout their whole length, like the lines ruled on foolscap paper, are said to be **parallel** to each other, and any four-sided figure whose opposite sides are parallel is called **a parallelogram.**

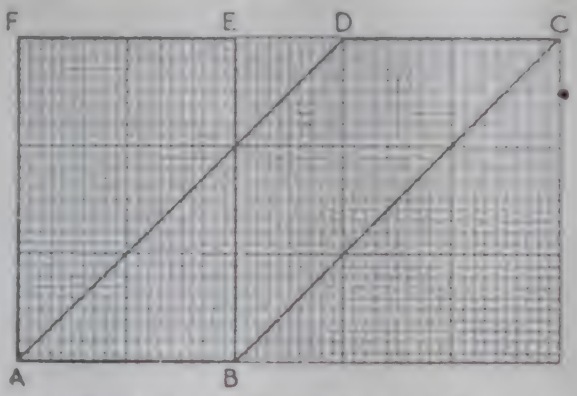

Fig. 39.—Areas of Parallelogram and Rectangle Compared

Fig. 39 shows a rectangle ABEF and a parallelogram ABCD drawn on squared paper.

The base is 2 cm. long, and the **altitude**, which means *perpendicular height*, is 3 cm. for each figure. ABCD is drawn so that its two longer sides bisect the centimetre squares.

Ex. 67.—(1) What is the area of the rectangle ABEF?

What is the area of the parallelogram ABCD that has the same base and altitude as the rectangle?

(2) On plain paper draw a large figure like fig. 40, making ABEF a rectangle, and ABCD a parallelogram. Cut the figure out.

Cut off the triangles DFA and CEB, and fit them one on the other. Are they equal or unequal?

Put DFA back in its former position. What figure have you got?

Take away DFA and put CEB in its former position. What figure have you got?

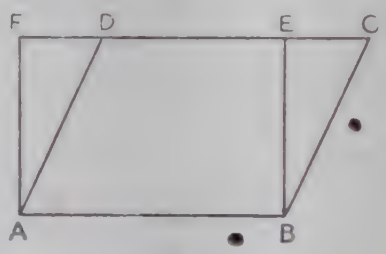

Fig. 40

(3) From the two preceding examples, write down the rule for finding the area of any parallelogram?

Ex. 68.—(1) On a base of 6 cm. construct an oblique parallelogram whose area is 18 sq. cm.

(2) On a base of 4 in. describe two parallelograms, the area of each being 12 sq. in.

(3) The lengths of two adjacent sides of a parallelogram are 6·3 cm. and 4·9 cm., and the angle between them is 45°. Draw the figure and find its area.

37. THE TRIANGLE.—Fig. 41 shows three triangles and three parallelograms standing on equal bases and having the same altitude.

Ex. 69.—(1) Draw figures similar to fig. 41, and either by cutting out, as in Exercise 67, or by counting the squares, find the relation between—

(*a*) The area of a triangle and the area of a parallelogram, having the same base and altitude.

(*b*) The areas of different shaped triangles, having the same base and altitude.

(2) Write down the rule for finding the area of a triangle when its base and altitude are known.

(3) Draw a triangle whose area is 6 sq. cm. (Draw a parallelogram whose area is 12 sq. cm., and bisect it by a diagonal line.)

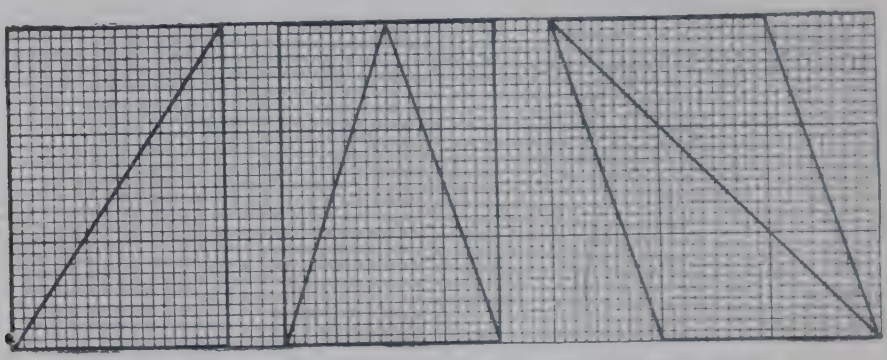

Fig. 41.—Areas of Triangle and Parallelogram Compared

Similarly, draw triangles whose areas are respectively 3 sq. in.; 8 sq. in.; 12·5 sq. cm.

(4) Draw a line 4 cm. long. On this line as base draw three triangles, each having an area of 12 sq. cm.

(5) Draw a triangle and measure its sides. In turn take each side as the base, and measure the altitude of the triangle from that side. Compare the three answers thus obtained for the area of the triangle.

38. THE TRAPEZOID.—A four-sided figure having two sides parallel, such as ABCD (fig. 42), is a trapezoid.

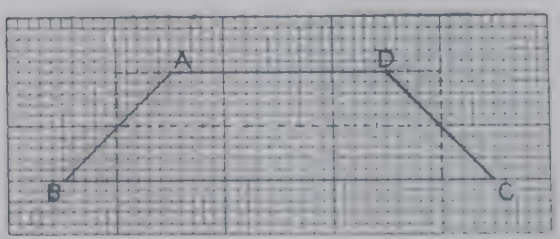

Fig. 42.—Areas of Trapezoid and Parallelogram Compared

Ex. 70.—(1) Examine fig. 42 carefully, and from the hint afforded by the dotted lines write down the rule for finding its area.

(2) The lengths of the parallel sides of a trapezoid are 5·6 mm. and 2·9 cm., and the distance between them 8·6 mm.; find its area.

39. POLYGONS.—Rectilineal figures having many sides may be divided into parallelograms and triangles, and the total area found by adding the areas of the parts.

Ex. 71.—(1) Draw a circle, and two diameters at right angles to each other. By joining the four points thus obtained on the circumference, a square is

obtained which is said to be *inscribed in the circle*. Find the area of this inscribed square.

Find also the area of the square described on the radius of the circle.

Compare the areas of the two squares by dividing the area of the larger by the area of the smaller.

The ratio is a simple one. What is it?

(2) Taking the figure of Exercise 71 (1), draw a diameter of the circle parallel to each of the sides of the square. This gives eight points on the circumference of the circle. Join them. The eight-sided figure thus formed is an *octagon*: it is divided into eight equal triangles. Find the area of one triangle, and hence of the octagon.

(3) In a similar way, find the area of a *hexagon* (a figure of six equal sides) inscribed in a circle whose radius is 6 cm. (Each side of the hexagon is equal to the radius of the circle.)

Into how many triangles is the hexagon divided? What kind of triangles are they?

(4) ABCDEF is the plan of a small field (fig. 43). If each millimetre on the plan represents 1 yd., find the area of the field—

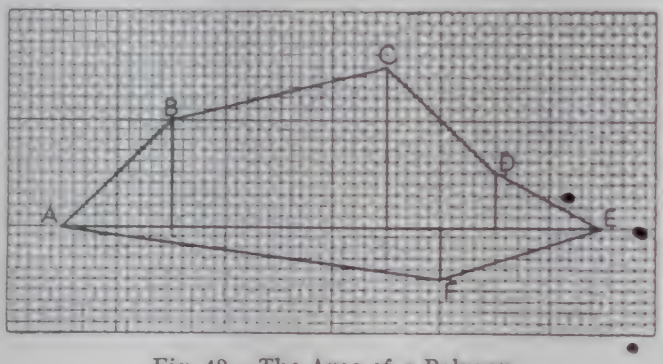

Fig. 43.—The Area of a Polygon

(*a*) By calculating the areas of the parts into which it is shown divided, and finding the sum of those parts.

(*b*) By counting the squares and parts of squares. (Estimate the parts of squares in tenths.)

Compare the results.

The lines shown in fig. 43 show how such a figure may be conveniently divided.

40. IRREGULAR FIGURES.—The areas of irregular figures having curved outlines may be found by drawing their outline either full size or on a reduced scale on squared paper and counting the squares. Fig. 44 represents the outline of the main island of Ceylon, copied from a map, with the scale appended. According to the scale a length of 33 mm. on the plan represents 400 Km., so that 1 sq. mm. represents 147 sq. Km. The number of squares included in the outline is 435. Hence the area is $435 \times 147 = 63945$ sq. Km. approximately.

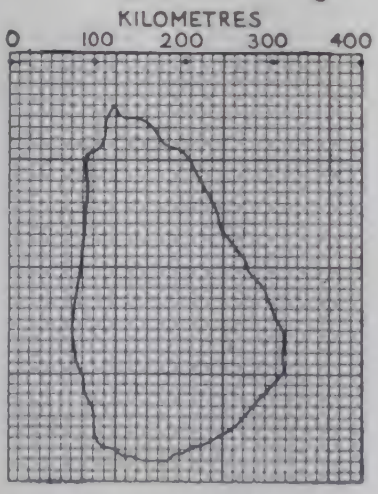

Fig. 44.—Estimation of the Area of an Irregular Figure

Ex. 72.—(1) Draw an outline of your hand on squared paper, and approximately determine its area.

(2) By the method indicated above, find the area of a country from a map.

41. THE CIRCLE.

Ex. 73.—(1) On squared paper describe a quarter of a large circle whose radius is 10 units (fig. 45). Make the lines thin.

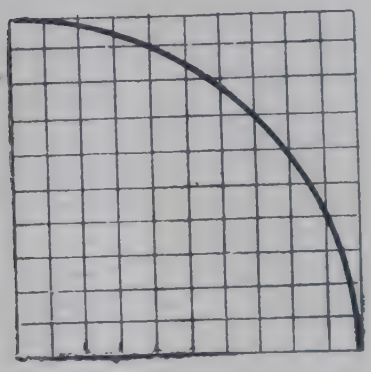

Fig. 45.—Estimation of the Area of a Quarter Circle

Count the number of squares included, the intersected squares being carefully estimated. If the work is accurately done the value obtained will be close to 78·5 square units.

Therefore, approximately, we have:—Circle of radius, 10 units, has an area 314 units, and area ÷ square of radius = 3·14.

(2) Repeat with circles of 6, 8, and 12 units, in each case finding the ratio between the area and the square of the radius. Take the mean of these ratios.

(3) Make graphs showing (*a*) the relation between the radius and the area; (*b*) the relation between the square of the radius and the area.

The two graphs (on different scales) are shown in fig. 46. AB shows the relation between the square of the radius (horizontal) and the area, and as the graph is a straight line it shows that the areas of circles are proportional to the squares of their radii. CD shows the relation between the radius and the area. The horizontal measurements in the curve CD are ten times as great as in the line AB. To represent them both on the same scale would require a sheet larger than the page. This makes no difference to the *character* of the curve.

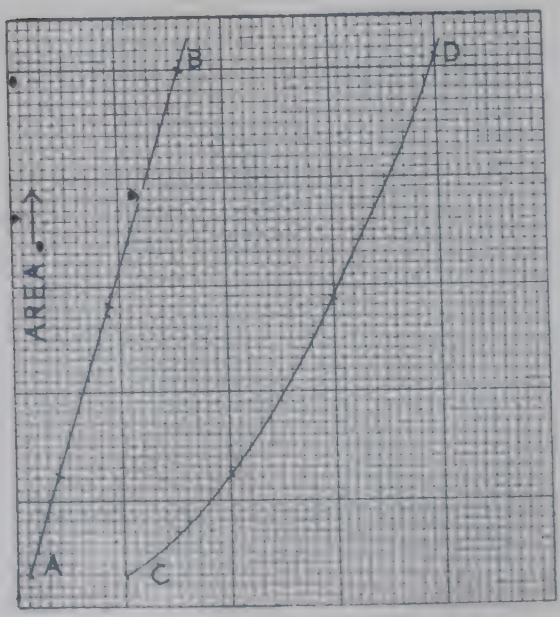

Fig. 46.—Graphs: AB showing the Relation between the Area of a Circle and the Square of its Radius. CD showing the Relation between the Area of a Circle and its Radius

Ex. 74.—From your graph find the areas of circles whose radii are different from those from which the graph was made.

The correct value of the ratio obtained in Exercise 73 is 3·1416, which is the number called π obtained in Art. 15.

Thus we see that the area of a circle is π times the area of the square on the radius r:

$$\text{Area of circle} = \pi r^2.$$

Ex. 75.—On plain paper describe a large circle.

By the methods previously used mark out the circumference into 24 equal parts by 12 diameters (fig. 47). Carefully cut out the circle and divide it into 24 sectors (Art. 42) by cutting along the diameters. Paste the sectors on paper side by side, as shown in fig. 47.

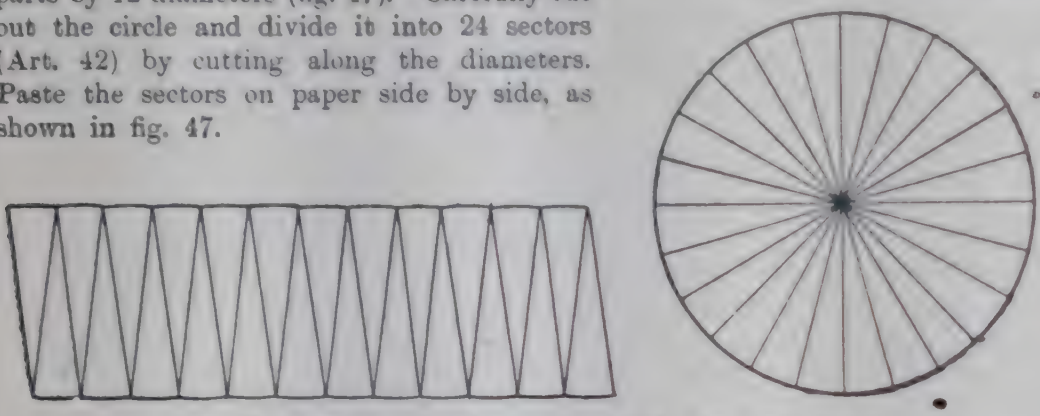

Fig. 47.—A Circle reduced approximately to a Parallelogram

The circle has been cut up into parts, each of which is nearly a triangle, and these triangles make a figure which is nearly a parallelogram. The greater the number of sectors the nearer the figure approaches that of a parallelogram.

Examination of the figure shows that the length of this parallelogram, if the number of sectors is very great, would be half the circumference of the circle; its altitude is the radius (r) of the circle. Therefore

$$\text{Area of circle} = \text{half-circumference} \times \text{radius};$$
$$\text{but (Art. 15) half-circumference} = \pi r,$$
$$\text{therefore Area of circle} = \pi r^2,$$

the same result as was obtained in the last exercise.

Ex. 76.—Miscellaneous Examples.

(1) Draw a circle. By stepping with dividers measure the semi-circumference; measure also the radius. Hence calculate the area. Why is the result more accurate if the steps are small?

(2) The diameter of a round table is 1·46 m. What is its area?

(3) The area of a circular pond is 1256 sq. yd. What is its diameter? (Take $\pi = 3{\cdot}14$.)

(4) How many circles each 1 cm. in diameter could be described on 1 sq. dm.? How much of the square decimetre would not be included in the circles?

42. SECTOR AND SEGMENT OF A CIRCLE.—In fig. 48 ABDC is a sector of a circle, and BDCE is a segment of a circle.

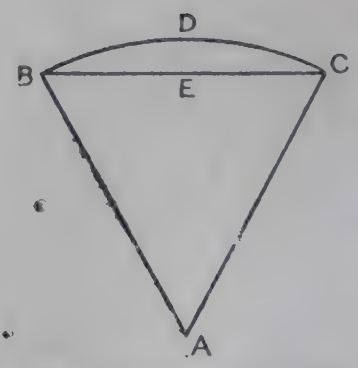

Fig. 48.—Sector and Segment of a Circle

Ex. 77.—Draw such a figure, and determine the areas of the segment and sector, thus:—

(1) Measure the radius AB = r. Calculate the area of the whole circle. Measure the angle A in degrees; then the area of the sector is $\frac{A}{360}$ of the area of the circle. Why? (Art. 23.)

(2) Measure the length of the curved line BDC; call it a; then $\frac{1}{2}ar$ = area of sector. Compare with result of (1).

(3) Make the necessary measurements and determine the area of the triangle ABC; then sector − triangle = segment.

43. AREAS OF SIMILAR FIGURES.—Squares and circles are similar figures. It has been found that the areas of squares are proportional to the squares of the numbers representing the lengths of their sides, and that the areas of circles are proportional to the squares of their radii.

Ex. 78.—(1) On squared paper mark out the following rectangles:—

(a) 3 cm. × 2 cm.; (b) 6 cm. × 4 cm.; (c) 9 cm. × 6 cm.

These are similar figures; their shape is the same, (b) being twice as long is also twice as wide as (a), and so on. Find their areas, and record them thus:—

(a) Base = 3 cm.; area = 6 sq. cm.
(b) „ = 6 „
(c) „ = 9 „

Compare the two columns of figures. The bases are in the ratio of the numbers 1, 2, 3; the areas in the ratio 1, 4, 9. What is the ratio between the areas of similar rectangles?

(2) A number of similar rectangles have their corresponding sides of lengths: 1·1 in.; 3·7 in.; 2·4 in.; 1·8 in. Write down four numbers showing their *comparative* areas.

The triangles in fig. 24 are similar; they have one angle the same, and the two sides containing that angle proportional.

Ex. 79.—Mark out on squared paper a number of triangles, as in fig. 24. Measure the length of the *corresponding* sides—*one* side for each triangle. Find the areas of the triangles. Record thus:—

Length of Side (l).	Value of l^2.	Area of Triangle (A)	Ratio $A \div l^2$

What is the relation between the areas of similar triangles and the lengths of corresponding sides?

Ex. 80.—Write down the relation between the following quantities:—

(1) The sides of two squares whose areas are in the ratio of 16 to 49; and of the sides of two other squares whose areas are in the ratio of 2·25 to 1·96.

(2) The corresponding sides of two similar rectangles whose areas are in the ratio of 289 to 100.

(3) The radii of two circles of which one is 25 times as large as the other.

(4) The corresponding sides of two similar triangles of which one is twice as large as the other.

44. DETERMINATION OF AREAS BY WEIGHING.—

The areas of figures described on paper may be compared by cutting them out and comparing their weights. The same kind of paper or cardboard must be used, and the figures should be as large as possible.

Ex. 81.—Cut out a piece of cardboard whose area is 100 sq. cm. (or 16 sq. in.). Weigh it as accurately as possible, and then calculate to two places of decimals the mass of 1 sq. cm. (or 1 sq. in.). If the mass of unit area is m, then the area of any other piece of such cardboard is obtained by weighing it, and dividing its mass by m.

Describe three or four circles the radius (r) of which you know, e.g. 5 cm., 6 cm., &c. Cut out the circles, weigh them separately, and thus find the area (A) of each.

Radius of Circle.	Value of Square of Radius r^2.	Mass of Cardboard Circle.	Area (A) of Circle.	Value of $A \div r^2$.
5 cm.	25 sq. cm.	...	...	...
6 cm.	36 sq. cm.	...	...	...
.	...	...	...	...

Find the mean value of $A \div r^2$ obtained in this way? What is the correct value? Compare with Art. 41.

Ex. 82.—By the same method find the area of a country or a town from a plan or map to which a scale of length is attached.

45. AREAS OF THE SURFACES OF SOLIDS.

[It may be more convenient to take this Article after Articles 46 to 52 have been studied.]

The areas of the surfaces of regular solids generally consist of figures whose areas have been found in the previous exercises. The area of each face is found separately, and the areas of the faces are then added up.

Ex. 83.—Find the total area of the surface of—

(1) A cube (fig. 50). Measure the length of an edge (x cm.): then the area of one face $= x^2$ sq. cm.; and there are 6 such faces.

(2) A **rectangular prism** (fig. 51). If the base is square, there are 2 squares and 4 rectangles. If the base is a rectangle (fig. 51), the ends are 2 equal rectangles, and the sides 4 other equal rectangles.

(3) A **right triangular prism** (fig. 52). There are 3 rectangles and 2 triangles.

(4) A **pyramid on a square base** (fig. 53). There are 1 square and 4 triangles. The height of each triangle is measured from the apex of the prism to the middle of the side of the triangle dealt with.

(5) A **cylinder** (fig. 52). If a sheet of paper be fitted round the curved surface of a cylinder, it will be found on unrolling to be a rectangle whose length is the height of the cylinder, and whose breadth is the circumference of the cylinder.

Total area 2 circles and 1 rectangle.

Measure the diameter and height by the method of Exercise 16.

(6) A **cone**. Fit a piece of paper on the slant surface of a cone. On unfolding the paper it is seen to be a sector of a circle whose radius is the slant height of the cone. The length of the curved base of the sector is the circumference of the base of the cone. The most accurate way of finding the length of this curved line is to measure the diameter of the base of the cone, and then to calculate its circumference.

The area of the sector is then found as in Exercise 77.

The area of the circular base of the cone is to be found separately, and added to the area of the sector.

Ex. 84.—Miscellaneous Examples.

(1) Knowing that 1 yd. = 91·44 cm., calculate the number of square decimetres in 1 sq. yd.

(2) A flat ring, such as a washer, is ·7 cm. in internal diameter and 1·15 cm. in external diameter. Calculate the area of one face. [Subtract the area of the inner circle from that of the outer.]

(3) Measure the dimensions of a flat ring and calculate its area.

(4) A circle and a square each have a perimeter of 12 cm. Compare their areas as nearly as you can.

(5) Describe a square whose perimeter is 9 cm., another whose area is 9 sq. cm., and a third whose area is equal to the sum of the other two.

(6) Measure the diameter of a wire: calculate its radius, and hence the area of its cross section.

(7) Calculate the total area of the surface of—

(*a*) A cube whose edge is 2·6 cm.

(*b*) A prism on a square base whose edge is 5 ft., the height of the prism being 26 in. (Reduce all measurements to inches, so that the answer is in square inches.)

(*c*) A right triangular prism, the lengths of the edges of the base being 3, 4, and 5 cm. and height 9 cm.

(*d*) A pyramid whose slant height is 40 cm. and edge of square base 75 cm.

(*e*) A cone whose slant height is 1 ft., and the diameter of whose base is 5·85 in.

(8) Find the area of the surface of a knitting needle or pencil.

CHAPTER V

Volume

46. Volume refers to space occupied, and involves length, breadth, and thickness.

Examine a centimetre cube, such as is shown in fig. 49. Its 12 edges are each 1 cm. long, its 6 faces each 1 sq. cm. in area. In the C.G.S. (centimetre-gramme-second) system of units the centimetre cube is the *unit of volume*. The volume of any body is expressed by saying how many such *cubic centimetres* (written **cub. cm.**) would fill the space that the body occupies.

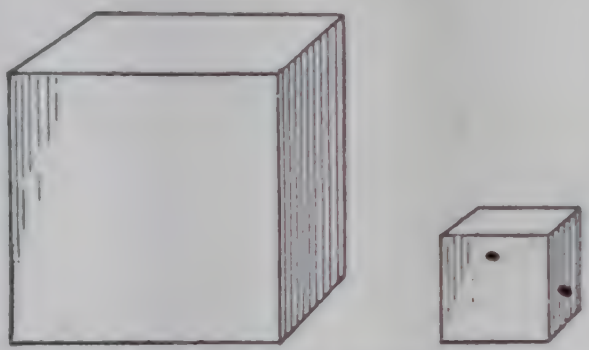

Fig 49.—A Cubic Inch; a Cubic Centimetre

Fig. 49 also shows a *cubic inch*, in which English unit small volumes are expressed.

Ex. 85.—Build up larger cubes from smaller unit cubes, taking successively the smallest number of units that will build the larger cube.

How many units are required to make the next smallest cube?

Complete the following table:—

Length of edge of cube	1 cm.	2 cm.	3 cm.	4 cm.	5 cm.	and so on.
Volume of cube ...	1 cub. cm.	8 cub. cm.	...	...	...	...

The numbers in the second line are called the **cubes** of those in the first line; those in the first line the **cube roots** of those in the second line. The first ten of these numbers should be learned.

Fig. 50 shows on the left a large cube built up of smaller cubes; and on the right one layer of the smaller cubes, one section removed from the layer, and one unit cube removed from the section. Taking the small cubes in the figure to represent cubic centimetres—

How many cubic centimetres are there in one section?

How many in one square layer?

How many in the whole large cube?

The whole put together make up a cubic decimetre or **litre**.

How many cubic centimetres make one litre?

Ex. 86.—(1) Suppose a cubic metre were built up of cubic decimetres, as in fig. 50:—

How many cubic decimetres would there be in one section?

How many in one square layer?

How many cubic decimetres in 1 cub. m.?

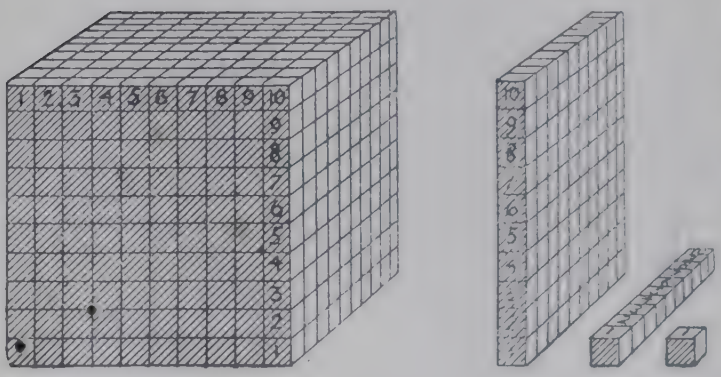

Fig. 50.—A Litre built up from Cubic Centimetres

(2) Suppose a cubic centimetre similarly built up of cubic millimetres, write down corresponding questions to those in (1) above, and their answers.

(3) Suppose a cubic foot were built up similarly from cubic inches, answer the three corresponding questions to those above.

(4) How many cubic feet make a cubic yard?

(5) From what you have done in the previous exercises, write down the rule for calculating the volume of a cube when the length of the edge is known.

In practice the volume of a body is not found by seeing how many cubes will fill the space that it occupies, but by measuring the lengths of its edges and then calculating the volume.

Ex. 87.—(1) What are the volumes of cubes whose edges are: 9 cm.? 6 mm.? 8 dm.? 12 m.? 5 in.? 7 ft.? 9 yd.?

(2) What are the lengths of the edges of cubes whose volumes are respectively 64 cub. in.? 27 cub. yd.? 729 cub. cm.? 1331 cub. mm.? 125 cub. m.?

47. Just as with areas we had fractions of square centimetres and square inches, so with volumes we may have fractions of cubic centimetres and cubic inches. Thus a cube whose edge is ·5 cm. has for the area of one face (·5 × ·5) or ·25 sq. cm., and a volume of (·5 × ·5 × ·5) or ·125 cub. cm.

Ex. 88.—(1) Write down the area of one face and the volume of each of the following cubes:—Length of edge, ·3 cm.: 1·2 cm.: ·9 cm.: 1·6 cm.: ·4 mm.: 2·6 mm.: 3·7 mm.: 14·9 mm.: ·5 ft.: ·7 in.: 11·3 m.: 3·6 yd.

(2) How many cubic centimetres are there in 3·4 litres? ·7 litre? ·03 litre?

(3) Measure the length of the edge of any cube that is available and calculate its volume.

(4) What is the volume of a cube whose edge is 2·54 cm.? How many cubic centimetres are there in a cubic inch? (See Exercise 24.)

48. THE RIGHT PRISM.—Suppose layers to be added to or taken from the cube shown in fig. 50, the solids thus formed are called **right prisms**. The word *right* indicates that the edges are at right angles to the base.

Ex. 89.—(1) Referring to fig. 50, what would be the volume of prisms of 18 layers? 4 layers? 57 layers?

(2) Thinking of a cubic foot built up as described above from cubic inches, let layers be added until the prism is first 15 in. high, then 27 in. high, then 44 in. high. What are the volumes in cubic inches of the right prisms thus formed?

(3) Taking a prism on a square base, measure the length of the side of one of its square faces; calculate the area of the square. Measure the height of the prism. Calculate its volume.

The base of the prism may be not a square but a rectangle, as shown in fig. 51. The method of finding its volume is the same.

Ex. 90.—(1) The base of the right prism shown in fig. 51 is 5 cm. long and 4 cm. broad, and the height of the prism is 6 cm. How many cubic centimetres are there in each layer? What is the volume of the prism?

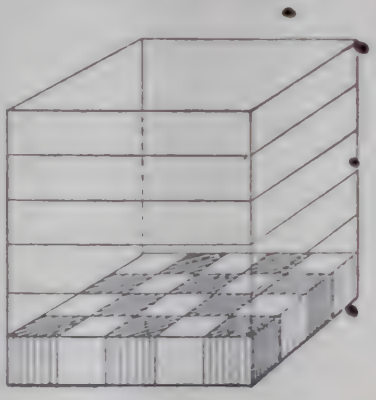

Fig. 51.—Volume of a Right Prism

(2) Find the volume of right prisms of the following dimensions:—Base, 5 cm. by 4 cm.; height, 9 cm. Base, 7 in. by 9 in.; height, 11 in. Base, 4 ft. by 6 ft.; height, 12 ft. Base, 9 m. by 6 m.; height, 8 m.

(3) A right prism is to be built up containing 144 cub. in. Write down all the arrangements you can think of by which this could be done. Thus: Base, 8 in. by 3 in.; height, 6 in.

(4) Remembering that the area of the base in square units tells you how many cubic units would be required to cover it, write down the rule by which you can calculate the volume of any block similar to those hitherto dealt with.

(5) Measure the edges of right prisms, and calculate their volumes.

(6) Taking a brick as 9 in. long, 4½ in. broad, and 3 in. deep, how many bricks would be required to build a wall 30 ft. long, 6 ft. high, and 18 in. thick?

49. Referring to fig. 50, suppose you cut off one corner by a vertical cut that bisected the corner cubes, you would have a prism on a triangular base containing twelve layers, each layer consisting of one-half a cubic centimetre. Volume . . . cub. cm.?

Fig. 52 shows a triangular prism. The rule for finding its volume may be seen to be the same as before:—

Volume of prism = area of base × height.

Ex. 91.—(1) Calculate the volume of the following triangular prisms:—

Area of base in sq. cm. ...	5	2·5	·9	1·5	7·8
Height in cm.	9	8	12	6·3	12·3

(2) Measure the dimensions of a triangular prism and calculate its volume. For area of triangular base see Exercise 69.

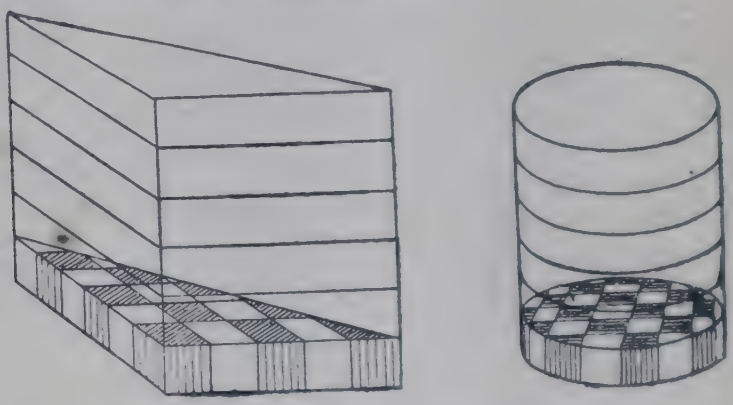

Fig. 52.—The Triangular Prism; the Cylinder

(3) The edges of the base of a prism are 6 cm., 7 cm., and 8 cm., and the height of the prism is 12 cm. What is its volume? (To find the area of the base describe a triangle on paper of the given dimensions, and determine its area from that figure.)

50. THE CYLINDER.—Fig. 52 shows a cylinder. It is a prism on a circular base. The bottom layer of cubes contains a certain number of whole cubes, and a number of fractions. Estimating these fractions of cubes would be an inaccurate process. Their number is the number of squares that they cover, and this may be accurately found by measuring the diameter of the circular base and then calculating its area.

The volume is then obtained as before by multiplying by the number of layers, that is by the height of the cylinder.

It is important to realize how it is that area of base × height gives the volume of such bodies.

Ex. 91 (*Cont.*).—(4) Calculate the volume of the following cylinders:—

(*a*) Area of base, 51 sq. cm.; height, 16 cm.
(*b*) Diameter of base, 8 cm.; height, 9 cm.
(*c*) Radius of base, 5 cm.; height, 12 cm.
(*d*) Circumference of base, 30 cm.; height, 15 cm.

(5) Find the volume of a pencil or of a knitting needle.

51. THE PYRAMID.—Fig. 53 shows six pyramids formed by cutting a cube across through three diagonals. The pyramids thus formed are exactly alike; hence the volume of a pyramid on a square base is seen to be one-sixth of a cube on the same base.

The altitude (perpendicular height) of one of these pyramids is equal to half the side of the square. Hence the volume may be expressed as the product of the area of the base by one-third of the altitude.

Pyramids may be cut in a similar way from rectangular blocks that are not cubes. The same result follows:

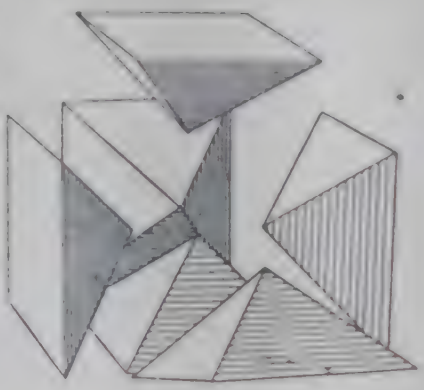

Fig. 53.—Volume of Pyramid and Cube Compared

$$\text{Volume of pyramid} = \text{area of base} \times \tfrac{1}{3} \text{ height.}$$

Ex. 92.—(1) Find the volume of a pyramid on a rectangular base. Measure the sides of the base and calculate its area. Measure the height by blocks as in Exercise 17.

(2) A pyramid whose height is 25 cm. stands on a triangular base. The sides of the base measure 5, 6, and 7 cm. Calculate its volume.

(3) A pyramid whose height is 15 in. stands on an octagonal base, each side of which measures 2 in. Calculate its volume.

52. THE CONE.—Exercise 92 (3) suggests the extension of the pyramid rule to the cone. The circular base of the cone may be regarded as a polygon with a very large number of sides (Exercise 75). Hence

$$\text{Volume of cone} = \text{area of base} \times \tfrac{1}{3} \text{ height.}$$

Ex. 93.—(1) Find by measurements and calculation the volume of a cone. (Measure height and diameter of base.)

(2) The circumference of the base of a cone is 40 cm. and its height 20 cm. Calculate its volume.

53. CAPACITY.—The interior volume of a hollow vessel is called its **capacity**. If this interior be one of the regular shapes previously dealt with, its capacity is found by taking inside measurements and calculating the volume by the rules obtained above; or practically by the methods shown below.

54. IRREGULAR BODIES.—Many solids are of irregular shape, and liquids take the shape of the containing vessel, which

is generally not one of the geometrical forms. In these cases the volume of the solid or liquid cannot be calculated from length measurements, and other methods of determining it are used. One of these methods is by weighing.

Ex. 94.—(1) Take a block of material of the same kind as a unit cube and of some regular shape.

Calculate the volume of the block from measurements.

Weigh the unit cube; mass w.
Weigh the block; mass W.

Then $W \div w$ is the volume of the block. Compare the result with the calculated volume.

(2) Using the above method find the volumes of solid bodies by comparing their masses with that of unit mass of the same material.

55. MASS OF A CUBIC CENTIMETRE OF WATER.—The most convenient substance to use in this method of determining volumes is water, because it is the most common, and there is in the C.G.S. system of units a simple connection between the unit of volume and the unit of mass.

Ex. 95.—Find as accurately as possible the mass of a known volume of water. Take a vessel of regular shape—a hollow cube or rectangular vessel, or an ordinary cylindrical tin canister. A very small vessel is undesirable. Scratch a small mark on the inside, measure the internal dimensions of the vessel, and calculate the volume up to the mark. (Suppose it is v cub. cm.)

Place it on the balance and counterpoise with shot or sand.

Fill the vessel with water up to the mark, and add weights to the other pan to obtain correct balance. (Suppose the weight of the water thus found is w grm.)

Compare w and v.

Repeat the experiment with another vessel.

If the work has been at all carefully done, the numbers w and v come out very nearly the same. In a very accurate experiment they are exactly the same. **One Cubic Centimetre of Water weighs One Gramme.** This is an important result, and should be carefully verified. It gives us a new method of finding volumes.

Ex. 96.—(1) Find the interior volume of a small flask. Place a mark on the neck by a small piece of gummed paper or otherwise.

Counterpoise the vessel on the balance.

Fill the vessel up to the mark with water; add weights (x grm.) to the counterpoise until balance is obtained. Then the volume of the flask up to the mark is x cub. cm.

(2) Take a beaker with an unknown quantity of water in it. Find the volume of the water. [Weigh the beaker; pour out the water; weigh again.]

(3) Half a litre of water is poured into a beaker whose internal diameter is 10 cm. What is the depth of the water?

56. MEASURING VESSELS.—Fig. 54 shows a graduated cylinder, on which the volumes of liquid that it contains when filled up to different points are marked on the glass.

The cylinder illustrated is graduated in steps of 5 cub. cm. up to 500 cub. cm. Smaller cylinders are made to contain 100 cub. cm. only, and on these generally each separate cubic centimetre division is marked.

Pour water into such a cylinder, and note that the liquid surface is not flat but curves up at the sides like a saucer. This curvature is called a **meniscus**, and is produced by the surface tension (Art. 78) of the liquid. Read the level at the bottom of the meniscus, i.e. at the middle part of the liquid.

Fig. 54.—A Graduated Cylinder

Ex. 97.—Using such a cylinder, deliver into a beaker counterpoised on a balance 50 cub. cm. of water, and weigh it. Add another 50 cub. cm., and add the necessary weights to balance. Continue in this way until the beaker is full.

Compare the volumes and masses of the water at each stage.

57. VOLUMES OF SOLIDS BY DISPLACEMENT OF WATER.—When a solid body is placed in water and sinks, it is clear that it must push out of the place that it occupies a volume of water just as large as itself. We call this **displacement** of the water, and speak of the water displaced by a solid immersed in it.

By using graduated vessels we can use this fact to measure the volume of solids that sink in water.

Ex. 98.—(1) Pour water into a graduated cylinder.

Take the reading of the level of the water, x.

Put in the solid whose volume is to be determined (not a very small body), and again take the reading of the level of the liquid, y.

Then $(y - x)$ cub. cm. is the volume of the solid.

(2) Find the interior volume of the flask used in Exercise 96 by filling it with water up to the mark, and then pouring the water into a graduated cylinder.

58. The Burette (fig. 55) is a more finely graduated tube which enables us to obtain conveniently any required volume of water. It is closed at the bottom by a tap or by a spring clip placed on a short piece of indiarubber tubing.

Ex. 99.—(1) Examine the graduations on a burette. Place the burette vertical on its stand. Close the tap. Fill the burette with water up to about the mark 10. Read the level (x). Place a beaker under the tap, and turn the tap a little so that water trickles out slowly. When the level has fallen to the mark 20

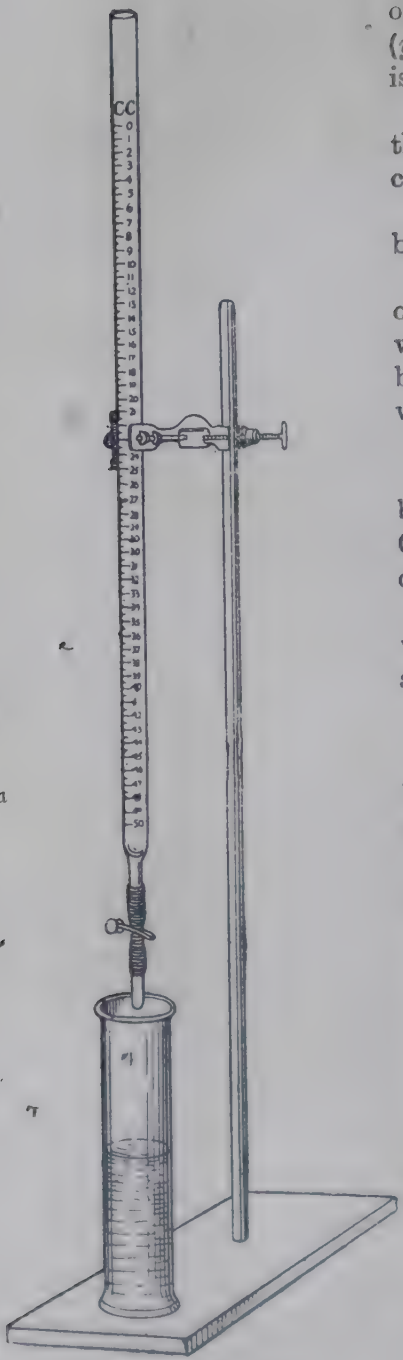

Fig. 55.—A Burette

or 30, close the tap and again take the reading (y). Find the value of $y - x$. How much water is there in the beaker?

(2) By this method obtain in a beaker exactly the following volumes of water: 10 cub. cm.; 15 cub. cm.; 30 grm.; 25 grm.; and weigh them.

(3) Graduate a vessel such as a cylinder, beaker, or flask.

Gum a strip of millimetre paper up the outside of the vessel. Allow 5 cub. cm. or 10 cub. cm. of water to trickle slowly into the vessel from a burette. Close the tap, and mark the level of the water on the paper strip.

Proceed thus carefully in equal steps.

(4) By immersion in water contained in a burette, find the volume of a sheet of tinfoil. Calculate its area from measurements. Hence deduce its thickness.

(5) By outside measurements of length of 20 volume divisions on the burette, find its sectional area and internal diameter.

59. THE OVERFLOW JAR. — It is often more convenient to obtain the volume of a solid by ascertaining how much water overflows from a jar when the solid is immersed.

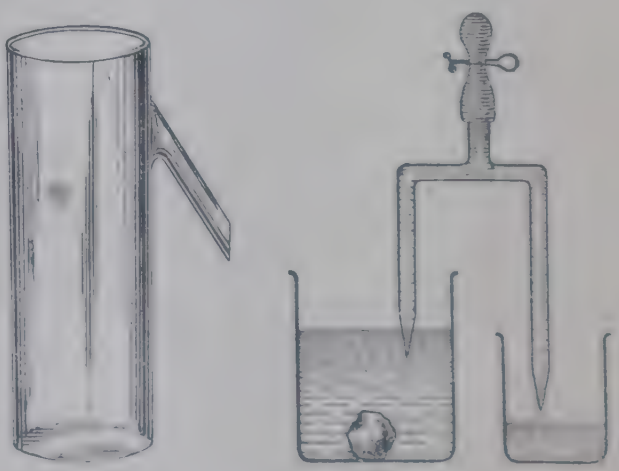

Fig. 56.—Overflow Jars

Fig. 56 shows two forms of overflow jar, one with a spout in the side, the other using a small siphon (Art. 97) attached to the side of the vessel. The ends of the siphon are drawn out to fine points. The vessel having been filled with water to a level slightly

higher than the points of the siphon, the siphon is filled with water and placed in position. Water flows over until one of the siphon points just touches the surface of the water in the vessel.

The solid whose volume is to be found is then immersed, and the overflow water caught and weighed.

Ex. 100.—(1) Weigh a beaker (x grm.).

Pour water into the jar until it overflows. When the last drop has issued from the spout, place the weighed beaker under the spout, and put a solid body gently in the jar, catching the water that overflows in the beaker. When the last drop has fallen, weigh the beaker (y grm.). Then the volume of the solid is $(y - x)$ cub. cm.

(2) To test the accuracy of the displacement method employed, find the volume of a metal cube or cylinder by this means, using one of the largest available, or several smaller ones.

Then measure the dimensions of the solid immersed, and calculate its volume. Compare the two results.

(3) Find the volumes of irregular solids by immersing them in the overflow jar.

(4) Find the volume of the glass contained in a small flask or bottle by sinking it in the jar.

(5) Measure the dimensions of a pyramid and of a cone, and calculate their volumes by the rule of Art. 51 or 52. Determine their volumes by the overflow jar, and thus verify the rule.

(6) Find by the overflow jar—

(a) The average volume of a number of lead shot, or of similar nails.
(b) The thickness of glass tubing by immersing a measured length.

(7) Find the volume of an irregular mass of wood. [Take a sufficient mass of metal to sink the wood; find the volume of the metal; tie wood and metal together, and find their combined volume.]

60. VOLUME OF A SPHERE.

Ex. 101.—Take several spheres of different diameters such as large marbles.

Of each sphere, measure the diameter d, and by displacement the volume v, and calculate the value of $v \div d^3$.

Compare the values of $v \div d^3$ obtained from the different spheres.

They will not be very different if the work is carefully done. If they differ much, repeat the measurements. When they agree fairly well, take the mean of the values of $v \div d^3$.

The answer is ·5236, which $= \frac{\pi}{6}$.

Hence, if v be the volume of a sphere, d its diameter, and r its radius—

$$v = \frac{1}{6}\pi d^3 = \frac{4}{3}\pi r^3.$$

61. Some interesting exercises may be performed with a right

cone, a cylinder, a sphere, and a hollow cube, having the following dimensions:—

Diameter of the base of the cone
Diameter of the base of the cylinder (internal)
Diameter of the sphere
Height of the cone and of the cylinder
Edge of the cube (internal)
} each = $2r$.

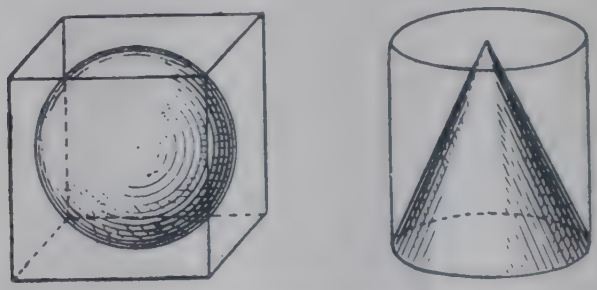

Fig. 57.—Geometrical Relations between Cube, Sphere, Cylinder, and Cone

Ex. 102.—To find the relation between the volumes of cylinder and cone, having the above relative dimensions (fig. 57).

(1) Weigh the cylinder empty ... ... = m_1 grm.
(2) Weigh the cone ... ... = m_2 grm.
(3) Weigh the cylinder full of water... ... ... = m_3 grm.
(4) Put the cone in the cylinder, fill up with water, and weigh again ... ... = m_4 grm.
Then volume of cylinder = $(m_3 - m_1)$ ... ... = a cub. cm.
Also volume of water contained in the cylinder in operation (4) = $m_4 - (m_1 + m_2)$ ... ... = b cub. cm.
$\therefore$ Volume of cone ... ... = $(a - b)$ cub. cm.
$\therefore \dfrac{\text{Volume of cone}}{\text{Volume of cylinder}}$ ... ... = $\dfrac{a-b}{a}$.

The value of this ratio is $\frac{1}{3}$.

Ex. 103.—Repeat the operations of the preceding exercise with the cylinder and sphere.

The ratio $\dfrac{\text{volume of sphere}}{\text{volume of cylinder}}$ is found to be $\frac{2}{3}$. The volume of the cylinder is known to be $\pi r^2 \times 2r = 2\pi r^3$; hence the volume of the sphere is $\frac{4}{3}\pi r^3$, the result arrived at in Exercise 101.

Ex. 104.—Repeat the operations of Exercise 102 with the cube and sphere.

The ratio $\dfrac{\text{volume of sphere}}{\text{volume of cube}}$ is found to be $.523 = \frac{\pi}{6}$. The volume of the cube is known to be $(2r)^3 = 8r^3$; hence the volume of the sphere is

$$8r^3 \times \frac{\pi}{6} = \frac{4}{3}\pi r^3, \text{ as before.}$$

62. AREA OF A SPHERE.—Fig. 58 shows part of a sphere with its curved surface divided into a number of areas, and the corners of one of these areas ABCD joined to the centre O of the sphere. Then OABCD is a pyramid with a curved base. The height of this pyramid is the radius r of the sphere, and its volume is ABCD $\times \frac{r}{3}$. If the whole surface of the sphere were thus divided into small areas, the volume of all the pyramids formed in the above manner would be area of sphere $\times \frac{r}{3}$.

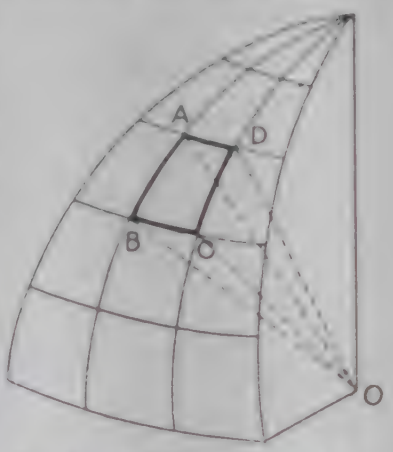

Fig. 58.—A Sphere regarded as a Number of Pyramids

Thus if v be the volume and a the area of a sphere, $v = a \times \frac{r}{3}$; but v has been found $= \frac{4}{3}\pi r^3$.

$$\therefore a \times \frac{r}{3} = \frac{4}{3}\pi r^3,$$

$$\text{and } \therefore a = 4\pi r^2.$$

Ex. 105.—Solve the following exercises by drawing:—

(1) A cube of 12 cm. edge stands on a table. A point P is marked on the top face ABCD, 2 cm. from AB and 2 cm. from AD. Find the distance from P to the middle point Q of the base of the cube. [The drawing is a right-angled triangle PNQ, of which PQ is the hypotenuse, PN is one side, having length 12 cm., and the side QN is equal to the distance of P from the middle point of the top face of the cube.]

(2) A room is 6 m. long, 5 m. wide, and 4 m. high. Find the distance from the middle point of the floor to an angle of the ceiling.

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CHAPTER VI

Density

63. DENSITY.—When we make the general statement that some materials are heavier than others, e.g. lead heavier than tin, tin heavier than wood, we are thinking of the same volumes of the different materials.

The masses of equal volumes of different materials are different.

It is usual to compare all substances with water. The **specific gravity** or **relative density** of a substance is the ratio between the mass of unit volume of the substance and unit volume of water. But 1 cub. cm. of water weighs 1 grm. Hence the simplest way to compare densities is to weigh a cubic centimetre of different materials.

Such cubes are sold by instrument makers for this purpose, or may with care be cut out from such substances as pine, cork, wax, potato with approximate accuracy.

Ex. 106.—Take such a set of cubes and weigh them each separately, and record their weights.

Thus, e.g. 1 cub. cm. of pine weighs ·6 grm.
1 cub. cm. of beeswax weighs ·95 grm.
1 cub. cm. of iron weighs 7·5 grm.

The numbers obtained for different specimens of such substances as those mentioned will not always be quite the same, because the different specimens are not all exactly alike. But whatever your specimen may be, *the mass in grammes of 1 cub. cm. of the substance is its density.*

Ex. 107.—Take blocks of different materials of some of the regular shapes dealt with in previous exercises. Measure their dimensions, and calculate their volumes in cub. cm. Weigh them and find their mass in grammes.

Record thus:—

Substance.	Volume (v) in cub. cm.	Mass (m) in grammes.	Density, $m \div v$.

Ex. 108.—Find the densities of the materials of irregular solids such as a glass stopper, a stone, a piece of metal. It is better to take a large object. Determine the volume by the overflow jar, or by the graduated jar, and the mass by weighing. Divide the mass (in grm.) by the volume (in cub. cm.).

For liquids the principle is exactly the same as for solids.

Ex. 109.—To find the density of a liquid. Take a hollow unit cube and counterpoise it on a balance. Fill the cube with a liquid—alcohol, glycerine, oil are suitable—and weigh it. Then the mass of the liquid in grammes (since the volume is 1 cub. cm.) is its density.

Generally, since the liquid must be contained in a vessel, and takes the shape of the vessel, its volume must be separately determined.

Exercise 96 showed how to find the interior volume of a flask up to a certain mark.

Ex. 110.—Using a flask whose capacity has been thus determined, find the density of salt water, and copper sulphate solution, by the method of the previous exercise.

64. SPECIFIC-GRAVITY OR DENSITY BOTTLE.—A bottle such as is shown in fig. 59 is often used for liquids.

In closing an ordinary bottle full of liquid with a stopper, some air is always shut in. This must be avoided, and for this purpose the stopper of a density bottle has a hole through it.

When such a bottle is quite filled with liquid, and the stopper gently lowered into its place, some

Fig. 59

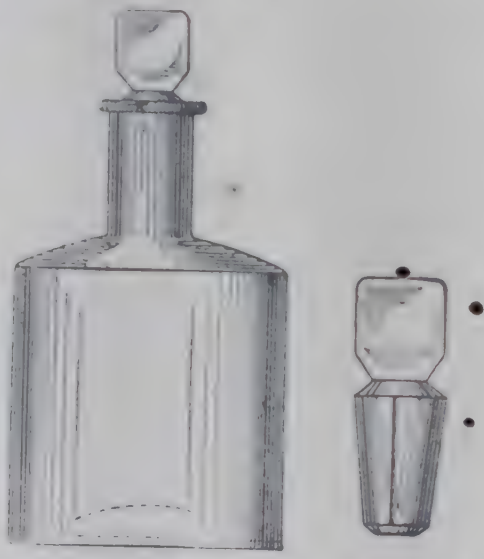

Fig. 60

The Specific-gravity or Density Bottle

of the liquid escapes through the hole, leaving the bottle quite full with no air bubble enclosed. The specific-gravity bottles sold by makers are carefully made to contain a stated mass of water.

Any bottle will answer the same purpose if it has a well-fitting stopper, along which a groove has been made with the edge of a file, as shown in fig. 60.

Ex. 111.—By the specific-gravity bottle find the relative density of some liquids, such as methylated spirit, oil, mercury, turpentine, glycerine, solution of copper sulphate, saturated solution of common salt.

See that the stopper is in the bottle in every weighing, and that the outside of the bottle is quite dry. Find these quantities:—

Mass of bottle empty	=	a grm.
Mass of bottle full of water	=	b grm.
Mass of bottle full of liquid	=	c grm.
$\therefore$ Mass of water that fills the bottle	=	$(b - a)$ grm.
and Mass of equal volume of liquid	=	$(c - a)$ grm.
Relative density of liquid	=	$\frac{c - a}{b - a}$.

Ex. 112.—To determine the density of a powder. The density of a solid (not dissolved by water) which is in the form of small pieces—such as shot, metal filings, sand—may be determined by means of the specific-gravity bottle. Proceed as follows:—Weigh—

(1) The empty bottle.
(2) The bottle with the substance in it.
(3) The bottle containing the substance and filled up with water.
(4) The bottle filled with water only.

Record as below, where, as an example, an experiment on sand is given:—

		Mass in Grammes.	Volume in Cubic Centimetres.
(i) Empty bottle	a	30·08	...
(ii) Bottle + sand	b	138·95	...
∴ Sand only $(b - a)$	c	108·87	...
(iii) Bottle + sand + water	d	196·51	..
∴ Volume of water only $(d - b)$	e	57·56	57·56
(iv) Bottle filled with water	f	130·1	..
∴ Capacity of bottle $(f - a)$ ...	g	...	100·02
∴ Volume of sand only $(g - e)$ ...	h	...	42·46

Thus the volume of the sand (h) and its mass (c) have both been found.

$$\therefore \text{Density} = \frac{c}{h} = \frac{108{\cdot}87}{42{\cdot}46} = 2{\cdot}56.$$

Ex. 113.—To find the internal diameter of a fine tube (a capillary tube). Weigh a length of the tube (m_1 grm.). Suck up into it some mercury. Weigh the tube containing the mercury (m_2 grm.). Measure the length of the thread of mercury (l cm.).

Then if r cm. is the radius of the bore its sectional area is πr^2 sq. cm., and the volume of the mercury is $l\pi r^2$ cub. cm.

The mass of the mercury is $(m_2 - m_1)$ grm., and density = mass ÷ volume;

$$\therefore (m_2 - m_1) \div l\pi r^2 = 13{\cdot}6,$$

whence r can be calculated.

Example.—A piece of tube weighed 2·75 grm., and when it contained a thread of mercury 5 cm. long, 2·88 grm.

The volume of the mercury $= 5\pi r^2$ cub. cm.
The mass of the mercury $= {\cdot}13$ grm.

$$\therefore \frac{{\cdot}13}{5\pi r^2} = 13{\cdot}6,$$

$$\therefore {\cdot}13 = 214r^2.$$

$$\therefore r^2 = {\cdot}000608.$$

$$r = {\cdot}025 \text{ cm. nearly.}$$

The diameter was ·5 mm. nearly.

UNITS.—It will be seen from the work of the preceding chapters that to express any measurement we use a unit and a number.

A length, area, volume, or mass can only be expressed by saying how many units of length, area, volume, or mass it contains.

The unit employed should never be omitted. To say that the area of a surface is 16 has no meaning: to say that the area is 16 cm. has no meaning: to say that the area is 16 sq. cm. or 16 sq. miles has meaning.

When arithmetical operations are performed with quantities, the student must always think of what is their practical meaning.

Quantities that are of entirely different kinds cannot be joined arithmetically. Thus 9 cm. + 6 grm.; 9 cm. × 6 grm.; 9 cm. ÷ 6 grm., &c., are all without meaning.

In finding density, we divided the number of units of mass of a body by the number of units of volume it contained. This does not mean that a mass has been divided by a volume, which cannot be done. It means that if 9 grm. occupy 6 cub. cm., then 1 cub. cm. has a mass of 1·5 grm.

Examples.—(i) 6 cm. × 9 means that 9 lines each 6 cm. long are placed one after the other in a line, which is therefore 54 cm. long.

(ii) 6 cm. × 9 cub. cm. is a meaningless operation.

(iii) 6 sq. cm. ÷ 9 sq. cm. means the comparison of two areas. The answer is a number, $\frac{2}{3}$. The first area is $\frac{2}{3}$ of the second.

Ex. 114.—Write out the full meaning (if any) of the following operations:—
(1) 12 cub. cm. ÷ 6 sq. cm.; 12 cub. cm. ÷ 6; 12 cub. cm. ÷ 6 cm.
(2) 12 grm. × 6 cub. cm.; 12 grm. ÷ 6 cub. cm.

PART II.—PROPERTIES OF LIQUIDS AND GASES

CHAPTER VII

Properties of Liquids

65 LIQUID PRESSURE.

Ex. 115.—Take a large thistle funnel or small bell jar, or the central piece of the apparatus shown in fig. 63, and over its mouth fasten a piece of thin sheet indiarubber, which may be firmly secured by being stuck on to the glass with indiarubber solution and tied round with thread or very thin wire. The membrane should be stretched a little, like the skin of a drum, but not too tightly.

By means of indiarubber tube connect the funnel with a long glass tube, as shown in fig. 61.

Tie the tube to a metre scale and fix it vertical with the membrane downwards. Pour in water coloured with cochineal. The membrane bulges out.

Add water in two or three successive stages, and at each stage measure the height of the liquid column and the depression of the centre of the membrane. The latter quantity may be read by allowing the membrane to touch a squared block which has one side against the scale.

Continue until the tube is filled.

Turn the funnel to the side and in various positions. The membrane bulges out even when it faces upwards.

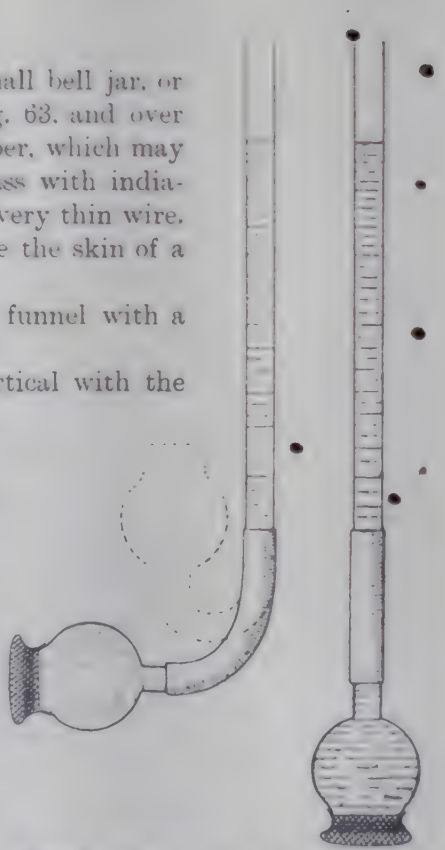

Fig. 61. Pressure of a Column of Liquid

The results indicate that the liquid exercises on the membrane a pressure which increases as the depth of the liquid grows greater.

Moreover, the pressure of the liquid is exercised not only vertically downwards, but in all directions, even vertically upward.

Ex. 116.—Using the tube shown in fig. 61, mark the height of the water on the stem. Holding the tube vertical with the membrane downwards, slowly lower it vertically into a deep vessel of water.

Note the rise of the coloured liquid in the tube, showing that the membrane is pushed inwards gradually as it gets further into the liquid.

At what depth is the membrane quite flat?

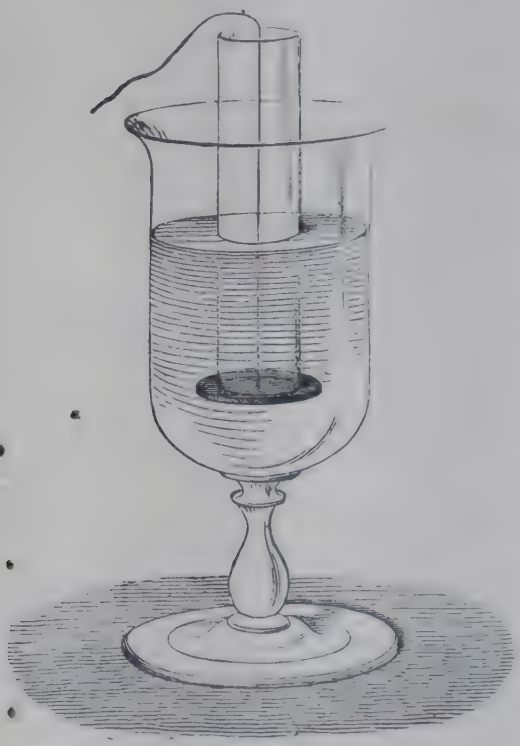

Fig. 62.—Upward Pressure in a Liquid

Thus the pressure exercised by a liquid is not only manifested in a tube, but exists in any mass of liquid.

Ex. 117.—Take a glass tube open at both ends, such as a cylindrical lamp glass, having one end ground so that its circumference is quite flat. Also a thin flat plate larger than the tube, and made of metal or of glass.

Pressing the plate in close contact with the ground end of the tube (fig. 62), lower the tube into a vessel of water in a vertical position.

When the plate is a short distance below the surface the plate will stay in its place without being held. Why?

The lower it is sunk the greater the pressure on the plate.

Gently and slowly pour water into the tube. When this water reaches nearly to the level of that outside, the plate falls off. Why?

66. LEVEL OF LIQUID IN CONNECTED VESSELS.

Ex. 118.—Join together several glass vessels by tubes in the manner shown in fig. 63. Such an apparatus is more useful if it can be taken to pieces and the arrangements varied. Fig. 64 shows a similar apparatus made from a burette and a bell jar, joined by tubes of indiarubber and glass.

Fig 63

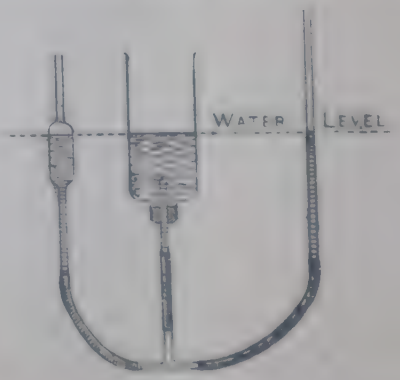

Fig 64

Level of a Liquid in Communicating Vessels

Pour a little water into one of the vessels. It rises in each tube: note the heights. Continue to add water till the vessels are full. Does the shape or size of the vessels affect the height at which the water stands?

Does it matter whether the tubes are vertical or slanting?

Pour out the water, dry the tubes, and repeat the experiment with other liquids. The experiments show that all liquids "find their own level".

Ex. 119.—Join a glass funnel, or the central vessel of fig. 63, by means of a long piece of indiarubber tube to a glass tube (fig. 65).

Fix the funnel, hold up the glass tube, and pour water into the funnel.

Raise and lower the glass tube, and note the water levels.

A tube of this kind with two arms is called a **U** tube.

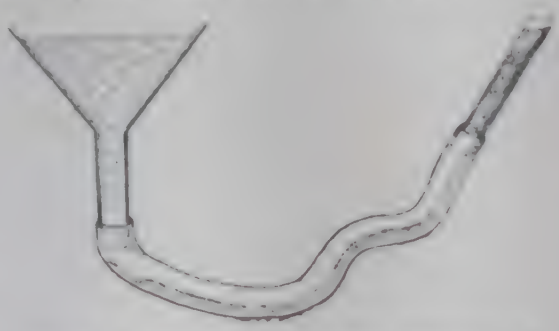

Fig. 65.—Action of an Elevated Reservoir of Water

Water from a reservoir connected by pipes to a town thus flows across valleys, and up over hills lower than the reservoir, into the pipes in the houses. Illustrate the process by bending the tube.

Ex. 120.—Replace the glass tube of Exercise 119 by a tube having a fine nozzle. Lower the nozzle below the level of the reservoir. Fountains are thus produced.

Ex. 121.—Fill the apparatus of fig. 63 half-full of water. Pour oil into one of the vessels. Are the liquid surfaces now level? Why is this?

Experiments 115 to 117 have demonstrated a characteristic property of liquids. A column of solid material only presses on its base; but a column of liquid presses not only on the bottom but also against the sides of the vessel that contains it. It also presses upwards on any solid body that is immersed in it. Moreover, this pressure is greater at greater depths than at smaller ones, increasing as the depth increases.

Experiments 118 to 121 show that if a liquid be placed in a set of vessels connected at the bottom the surfaces in the various vessels stand at the same height. The reason for this is shown by experiment 115. The liquid bulged out the membrane when it was held sideways, proving that there was a pressure towards the side. If the membrane had been pricked with a small hole when held sideways the water would have spurted out.

At the bottom of a **U** tube there is one column of liquid pressing towards the left and another column pressing towards the right. When these pressures are equal the liquid at the bottom comes to rest and remains at rest.

If more liquid is poured into the left-hand arm the pressure from left to right is greater than that from right to left, and the liquid moves round until the pressure at the bottom is the same in both directions.

If two different liquids are used the levels in the two arms are no longer the same. The lighter liquid in one arm stands higher than the heavier liquid in the other arm, because it requires a taller column of lighter liquid to exercise a pressure at the bottom equal to that of a shorter column of heavier liquid.

Moreover, neither the size of the tubes nor their direction affects the result, but only the vertical height of the liquid column.

67. FLOTATION.—Every day experience shows that some solid bodies float on water while others sink, and that the floating body is always partly immersed in the liquid.

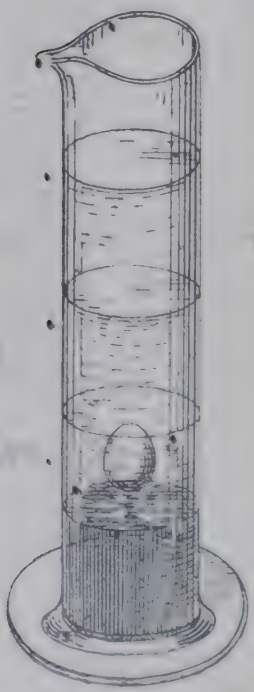

Fig. 66.—Lighter Liquids floating on Denser Liquids

Ex. 122.—Try experiments with other liquids than water.

(1) Put a piece of beeswax in water, and in alcohol, ether, or turpentine. Does it sink or float?

(2) Put wood, glass, crockery, iron, or stone in mercury. (The substances should be clean. Many substances make the mercury unfit for use afterwards.)

(3) Using only a small quantity of mercury, try fragments of zinc, copper, lead in mercury.

What is the connection between the density of the liquid and the floating or sinking of solids placed in it?

Some liquids float on water, and some sink in water in the same way that solids do. Other liquids do not remain separate, but mix with the water.

Fig. 66 illustrates this. At the bottom of the jar is a quantity of mercury; on this floats salt water, on which floats oil, on which floats methylated spirit. An egg sinks through the first two liquids, but floats nearly immersed in the third.

Mercury, a saturated solution of potassium carbonate, alcohol, and petroleum form a similar series of liquids with which such a jar may be filled.

Ex. 123.—(1) Gently place a single large drop of glycerine on the surface of water. Does it sink or float? Note the appearance.

Try a very small drop, such as may be obtained on the end of a knitting needle.

(2) Repeat the experiment with oil.

(3) Place together in a test tube larger quantities of two liquids, such as oil and water, which do not mix. The lighter floats entirely on the top of the heavier, not partly immersed, as does a solid. Note their behaviour when shaken.

68. UPTHRUST OF A LIQUID.

Ex. 124.—Take an empty beaker or bottle, and holding the mouth upward push the vessel slowly straight downwards in water. A backward push is felt.

If the closed vessel be large, all your strength will not be sufficient to push it down into the water.

Try the same experiment with mercury if possible. What difference do you notice?

This upward push or pressure exercised by liquids is called an **upthrust**. It causes some bodies to float on their surface.

Ex. 125.—(1) Hold a lump of brick, or glass, or coke, or any large heavy object, in the hand. Then lower it into water. From what you feel of the weight, does there appear to be an upthrust on heavy bodies that sink?

(2) Tie two equal masses of metal or heavy stuff to the ends of a string, and put the string over an easily running pulley wheel: see that they balance. Under one of the masses bring up a beaker containing water, so that the solid enters the water but does not touch the bottom (fig. 67). Why do the masses no longer balance?

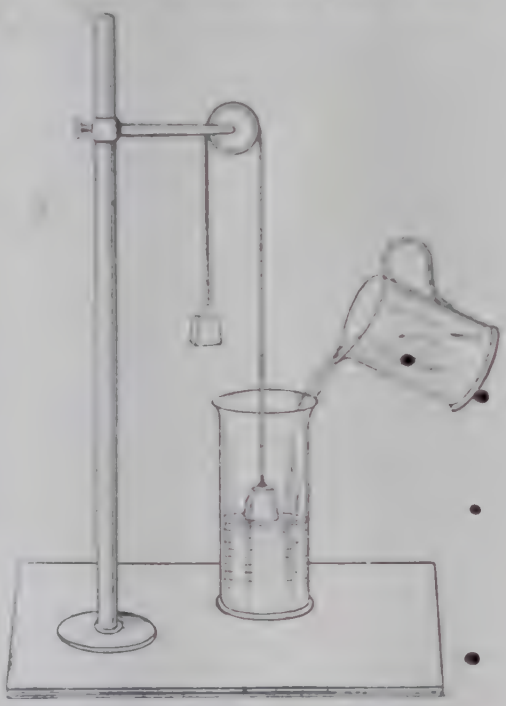

Fig. 67.—Upthrust of a Liquid

These experiments demonstrate that whether a body floats or sinks in a liquid, there is always an upthrust exercised on it by the liquid, produced by the liquid pressure.

69. CONNECTION BETWEEN DISPLACEMENT AND UPTHRUST.

Ex. 126.—(1) Pour water into a graduated cylinder, and read the level of the surface (a).

Weigh a mass of wood as large as will easily go into the cylinder (m grm.).

Place the wood in the cylinder so that it floats freely. Read the level of the water surface (b).

Then $(b - a)$ cub. cm. is the volume of the water displaced, and $(b - a)$ grm. is the mass of the water displaced.

What is the relation between $(b - a)$ and m?

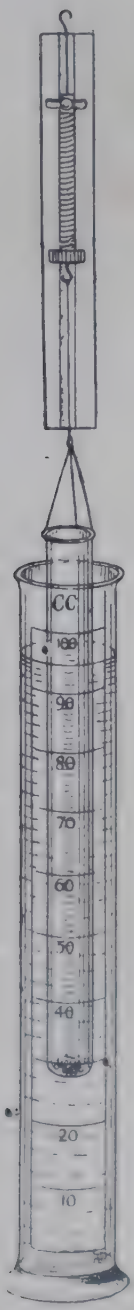

Fig. 68.—Measurement of Upthrust on a Floating Solid

Repeat the operation with other floating masses, and record in two columns:

Mass of floating body; Mass of water displaced.

Compare the two rows of figures.

(2) Using the same floating objects, immerse them in the overflow jar, making the same records as in the last experiment.

As the pairs of numbers are seen to be in a fixed ratio, add up the figures in the two columns, and thus obtain (*a*) the total mass of all the bodies examined, (*b*) the total mass of water they displace when floating.

Compare the totals, and write down the relation between the mass of a floating body and the mass of water that it displaces.

Ex. 127.—Put sufficient sand, shot, or nails into a test tube to sink it in water to about three-fourths of its length. Suspend the tube by thin threads to a spring balance (fig. 68), or to one arm of a physical balance, and weigh it in air.

Record its weight in grammes.

Put about 50 cub. cm. of water in a 100-cub.-cm. graduated cylinder, and record the height of the surface of the water. Bring up the cylinder so that the test tube just enters the water. Raise the cylinder slowly, and note the behaviour of the spring balance.

The balance shows loss of weight of the test tube as the water level rises.

Perform the operation in three or four steps, recording each time the *loss of weight* of the body in grammes, and the *volume of water displaced* by the body in cubic centimetres.

Arrange these numbers in two columns, and compare them pair by pair.

As before, all the readings of loss of weight may be added together, and all the readings of volume of water displaced may be added together. The errors in reading are then likely to cancel each other, and a more accurate result is obtained.

Remembering that 1 cub. cm. of water weighs 1 grm., what is the connection between the loss of weight of a body floating in water and the weight of the water it displaces?

These experiments show that when a solid floats in a liquid it displaces a quantity of liquid whose weight is equal to its own. The water displaced by a ship weighs just as much as the ship. This weight is called the displacement of the ship.

70. The amount of the upthrust on a body that sinks may be found in the same manner.

Ex. 128.—(1) By means of a fine thread suspend some body, such as a

cube or cylinder, whose volume (v cub. cm.) is known, and whose density is greater than that of water, to one arm of a balance (fig. 69), or on a spring balance, and weigh it. Bring up a beaker of water, so that the solid enters the water, and find the loss of weight (m grm.).

Compare m and v.

(2) Repeat the experiment with an irregular body whose volume has been found by the overflow jar. In this case take special care that no air bubbles cling

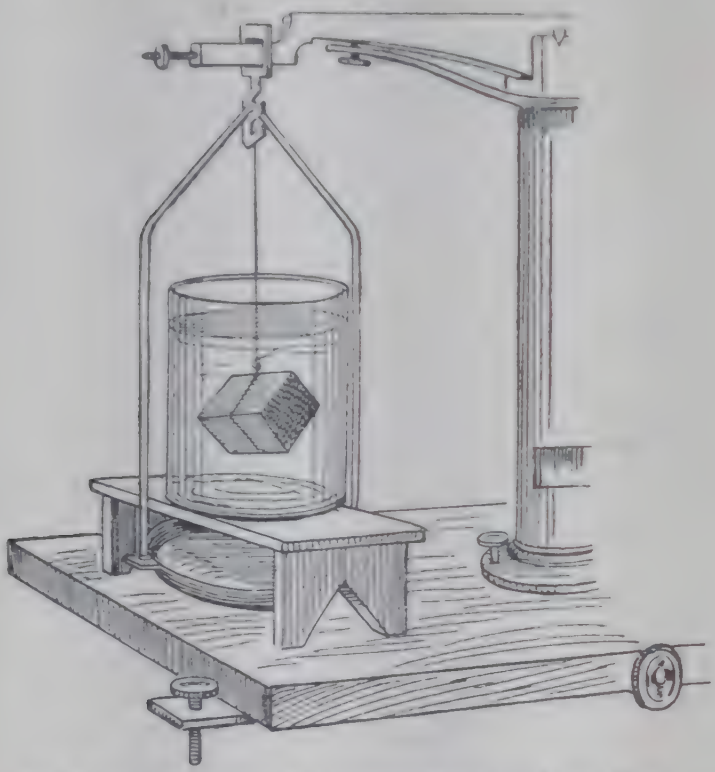

Fig. 69.—Measurement of Upthrust on a Solid that Sinks

to the body. They may generally be removed by raising the body out of the water.

Again compare volume (in cubic centimetres) with loss of weight (in grammes).

Such experiments show the same result as with floating bodies.

Thus it has been demonstrated that in all cases when a body is immersed in water it loses weight, and **the loss of weight is equal to the weight of the water displaced.** This statement is called the *principle of Archimedes.* A similar statement is true for all liquids.

71. REACTION ON THE VESSEL.

Ex. 129.—(1) Place a beaker containing water on one pan of a balance, and counterpoise it. Hold a rod so that one end is in the water. What does the balance show? How is the rod being supported?

(2) Place in the beaker the solid used in Exercise 128 (1), having previously

weighed it. How much additional weight is recorded by the balance after the solid has been put in the beaker?

What is the connection between the weight of the solid and the additional weight of the beaker and contents?

(3) Now suspend the solid in the beaker by means of a fine thread from a separate stand, as in fig. 70.

Is the beaker any heavier? Add the necessary weights, and find out by how much its weight has been increased. Compare this increase of weight with the loss of weight of the same body in Exercise 128 (1).

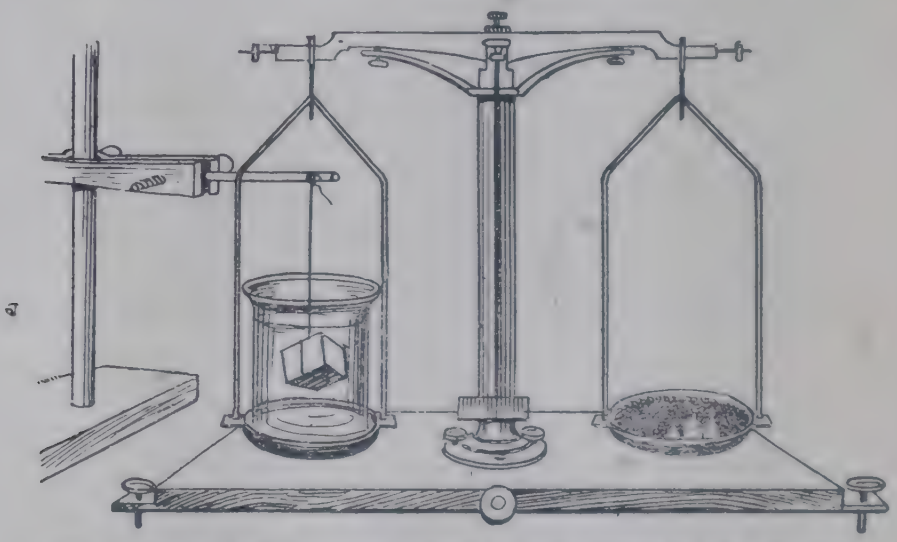

Fig. 70. – The Vessel becomes Heavier by the Amount of the Upthrust

The comparison shows that when a body is suspended in a liquid contained in a vessel, the vessel gains in weight just as much as the body loses.

*72. DETERMINATION OF DENSITY. — When a good balance is used the principle of Archimedes furnishes a very accurate method of determining volumes and densities, since the number of grammes weight that a body loses when suspended in water is the number of cubic centimetres in its volume.

Ex. 130.—Find the mass of 1 cub. cm. of water. Take a set of (suppose 6) centimetre metal cubes. Fasten them together or tie them up in a small bag of thin muslin. Suspend the cubes from one arm of a balance and counterpoise. Bring up a beaker of water under the cubes, as shown in fig. 69. Find the upthrust in grammes (suppose x grm.). Then x grm. is the mass of 6 cub. cm. of water.

Ex. 131. – (1) Find the volume of some heavy irregular solids, such as a stone, a glass stopper, a mass of metal, by weighing in air and in water.

(2) Find the average thickness (x) of a coin. Determine its volume (v) as above, and measure its diameter (d).

Then its radius $r = \frac{d}{2}$; its area $= \pi r^2$; its volume $= \pi r^2 x$, which is equal to v. $\therefore$ average thickness $x = v \div \pi r^2$.

(3) Find the diameter (d) of a piece of wire. Measure its length (l).

Determine its volume as above (v); then the volume $= \pi\left(\frac{d}{2}\right)^2 l = v$, in which equation all the quantities are known except d, which can therefore be calculated.

(4) Weigh (in air) the bodies whose volume has been determined in the previous exercises (1), (2), (3). This determines the mass. Then density $= \frac{\text{mass}}{\text{volume}}$. Thus find the densities of the substances.

Example.—The following example is from an experiment on a glass stopper:—

Weight in air, 24·13 grm.; in water, 15·13 grm.
Volume, 9 cub. cm.
Density $= 24{\cdot}13 \div 9 = 2{\cdot}68$.

73. To find the density of a solid that floats on water, a sinker must be used.

Ex. 132.—Find the density of a piece of wood.

Weigh the solid: mass $= a$ grm.

Attach the solid to the sinker, suspend from the arm of a balance, as shown in fig. 71, and bring up a beaker of water so that only the sinker is immersed. Counterpoise the suspended mass.

Add water until the wood is immersed. By means of weights (b grm.) obtain exact balance. Then b cub. cm. is the volume of the wood. Its density is $a \div b$. Thus a piece of wood weighed 1·92 grm.; the upthrust on it was 5·46 grm. Then its density $= 4{\cdot}92 \div 5{\cdot}46 = {\cdot}9$.

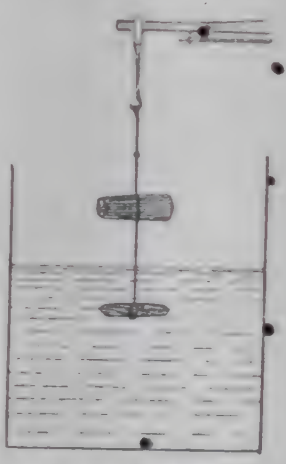

Fig. 71.—Upthrust on a Heavy Solid connected to a Light Solid

74. The density of a liquid other than water may be found by weighing in it a solid of known density. The upthrust of any liquid on a solid is equal to the weight of a volume of the liquid equal to the volume of the solid immersed in it. Hence different liquids give different upthrusts. By comparing the upthrusts we compare the densities of the liquids.

Proceed thus—the figures refer to an experiment:—

Ex. 133.—Weigh the solid (a glass stopper is suitable)—

(1) In air (w_1).
(2) In water (w_2): upthrust $= w_1 - w_2 = 1{\cdot}54$ grm.
(3) In the liquid (w_3): upthrust $= w_1 - w_3 = 1{\cdot}38$ grm.

The two upthrusts give the weight of a mass of water and of a mass of liquid each equal in volume to the solid.

$$\therefore \text{Density of liquid} = \frac{w_1 - w_3}{w_1 - w_2} = \frac{1{\cdot}38}{1{\cdot}54} = {\cdot}9.$$

Find by this method the density of turpentine, or of the liquids used in Exercise 109.

75. Density of a solid which is acted on by water.

If the solid is one that is dissolved by water, some other liquid whose density is known must be used to determine the density of the solid. The method of experiment is precisely the same as with water. But the calculations involve the density of the liquid used.

Thus, the density of turpentine being ·87, the upthrust when 1 cub. cm. of turpentine is displaced is not 1 grm., but ·87 grm.; when 1 cub. cm. of mercury is displaced the upthrust is 13·6 grm., and similarly for any other liquid.

Ex. 134.—Determine the density of copper sulphate by suspending a large crystal of copper sulphate in turpentine.

76. HYDROMETERS.

Ex. 135.—In water contained in a tall cylinder place a wooden cylindrical rod, such as an uncut pencil or a round ruler, so that it floats in a vertical position (fig. 72).

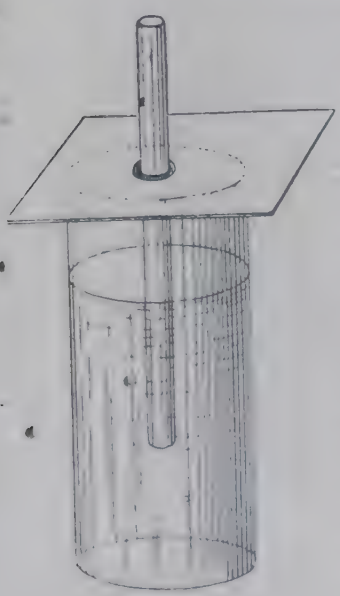

Fig. 72

Measure (*a*) the length of the rod immersed in the water, (*b*) the whole length of the rod. From (*a*) and (*b*) determine the relative density of the wood of which the rod is made. Repeat with rods of different kinds of wood.

Ex. 136.—Take *one* of the rods and immerse it in the same manner in a different liquid. Measure the distance to which it sinks in this liquid. The distance to which it sinks in water is known. From these lengths find the density of the liquid. Does the rod sink deeper into the lighter liquid? If so, why?

A floating body which is thus used to determine densities is called a **hydrometer**. A hydrometer which sinks to a different depth in different liquids is called a hydrometer of variable **immersion**, and affords a ready method of determining the relative densities of liquids.

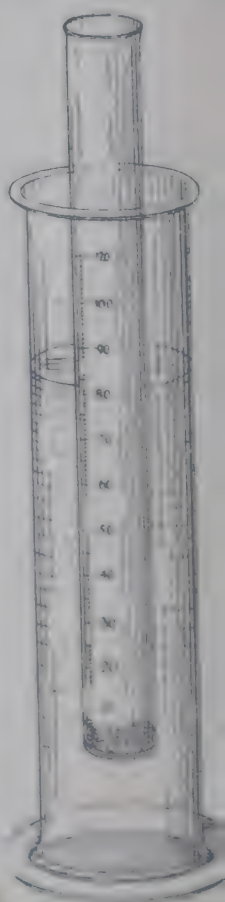

Fig. 73.—Hydrometer of Variable Immersion

A useful hydrometer may be made from a

piece of glass tube closed flush at the bottom by a small plug smeared over with sealing wax (fig. 73). A specimen tube with a flat bottom is suitable. If a test tube is used, allowance must be made for the rounded end of the tube.

Ex. 137.—In the tube fit and gum on a piece of millimetre paper on which the centimetres are clearly marked and numbered from the bottom, in such a way that the numbers show the distance from the *outside* of the bottom of tube. Put sufficient sand or shot in the tube to sink it in water to some convenient mark (preferably 100), and record the water mark. Suppose it has sunk to a depth x divisions on the scale.

Place the hydrometer gently in turpentine. It sinks to y divisions on the scale.

Why is y not equal to x?

From the numbers y and x determine the density of turpentine.

Repeat with salt water, oil, alcohol, glycerine.

77. NICHOLSON'S HYDROMETER.—This instrument, shown in fig. 74, consists of a hollow metal cylinder having at the top a thin stem supporting a pan, and at the bottom and a pan or cage.

When placed in liquid it floats vertically. A mark is made on the upper stem, and the hydrometer is always made to sink down to this mark. The mark should always be viewed from just below it through the liquid. The instrument is sensitive, and must be gently touched when floating. The stem should be cleaned before use by rubbing it with clean rag moistened with methylated spirit.

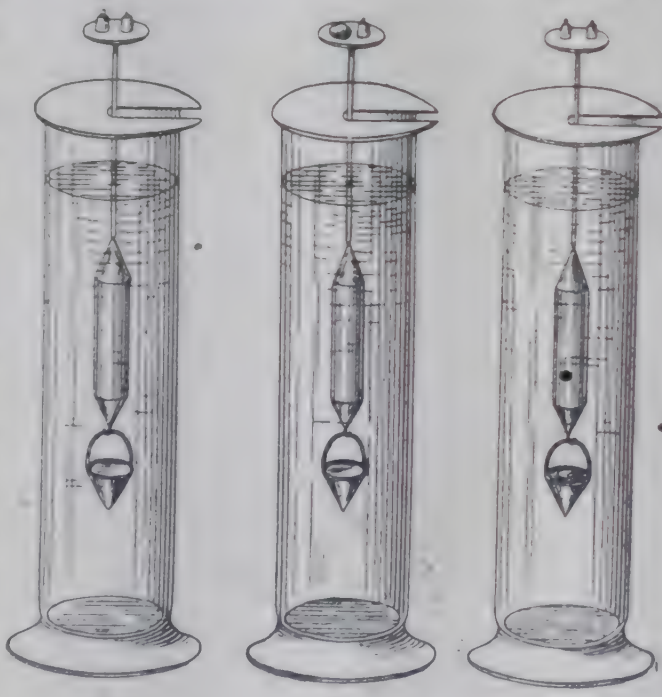

Fig. 74.—The Method of using a Nicholson's Hydrometer

Ex. 138.—Find the density of a solid by Nicholson's hydrometer. Only a small piece of the solid must be used.

Place the hydrometer in a tall cylinder containing water that has been recently boiled, and remove air bubbles. To prevent weights falling into the water, close the top of the jar by a piece of cardboard having a slot in it.

(1) Place weights in the upper pan until the instrument just sinks to th mark on the stem (fig. 74*a*). Let the weight be w_1 grm.

(2) Remove the weights; place the substance in the upper pan, and ad weights in the upper pan until the stem mark is again at the surface of the wate (fig. 74*b*). Weights in the pan (w_2).

Remove the weights and the substance.

(3) Place the substance in the lower pan. If it is lighter than water tie i there with a small piece of thin thread (unless there is a cage on the lower pan and put the instrument again into the water.

Place weights in the upper pan until the stem mark is again at the liqui surface of the water (fig. 74*c*). Weights in the pan (w_3).

Then we have—

Weight of the substance in air, i.e. its mass ... $w_1 - w_2$ grm.

Weight of the substance in water ... ... $w_1 - w_3$ grm.

$\therefore$ Loss of weight in water $= (w_1 - w_2) - (w_1 - w_3)$ ... $w_3 - w_2$ grm.

$\therefore$ Volume of substance ... ... $w_3 - w_2$ cub. cm.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{w_1 - w_2}{w_3 - w_2}.$$

Ex. 139.—Find the density of a liquid by Nicholson's hydrometer.

(1) Weigh the hydrometer ... ... w_1 grm.

(2) Place the instrument in water and sink it to the mark on the stem by placing in the upper pan weights amounting to ... ... w_2 grm.

(3) Remove the hydrometer; dry it; place it in the given liquid, and sink it to the mark by weights in the top pan amounting to ... ... w_3 grm.

Then we have—

Mass of water displaced by the hydrometer ... $(w_1 + w_2)$ grm.

Mass of liquid „ „ „ ... $(w_1 + w_3)$ grm.,

and these different masses have the same volume.

$$\therefore \text{ Relative density of the liquid} = \frac{w_1 + w_3}{w_1 + w_2}.$$

Ex. 140.—Miscellaneous Examples.

(1) Coil a piece of copper wire round a piece of lead. Find the density this mixed substance. Obtain the density of each of the metals from the tabl Knowing these three quantities, calculate the proportions of copper and lead p sent. Verify by weighing them separately.

(2) From the above experiment describe a method by which you could asc tain whether a piece of metal supposed to be gold is all gold or contains so other substance mixed with it (Archimedes' experiment).

(3) Three liquids are given you, all having the appearance of water. On water. How would you find out which was the water?

(4) A glass stopper weighs 4·5 grm. in air and 2·7 grm. in water. H much would it weigh in turpentine of relative density ·87?

78. SURFACE TENSION OF LIQUIDS.—It has alrea been noticed that where liquids touch the sides of vessels surface is not horizontal.

Ex. 141.—(1) Place a fine steel needle on a forked piece of wire, and lower it gently in a horizontal position on to the surface of water. With care the needle may be made to float.

(2) Place similarly a piece of fine wire gauze turned up at the edges on the surface of water. This may be made to float.

There is a special cause acting in these cases. The metal is not wetted by the liquid; it lies on the top as on a table. The reason of the flotation is that there is a special condition of the surface layer called **surface tension**, which has the same effect as though the surface were covered by a skin. Unless this surface layer or skin is pierced the solid does not get into the liquid.

Ex. 142.—(1) Take two rectangular glass plates; place a *thin* piece of wood between them at one end leaving the other ends in contact; and bind the plates together.

First immerse them entirely in water; then set them straight up in a vessel containing a little water. The water rises between the plates as shown in fig. 75, reaching its greatest height where the plates are closest.

(2) Dry the plates, and place them in a similar position in mercury. The mercury between the plates takes the form of a curve. Draw the curve.

Fig. 75.—Change of Level of Liquid between Plates

Ex. 143.—Take a glass tube of small bore, such as a piece of thermometer tube. Such tubes are called *capillary* tubes.

Draw water through the tube so as to wet the inside. Fix the tube upright in water. The liquid stands up the tube as shown in fig. 76.

Place a similar tube in mercury contained in a test tube. The position of the mercury may be seen by holding the capillary tube against the side of the test tube. The liquid in the capillary tube is depressed below the general level as shown in fig. 77.

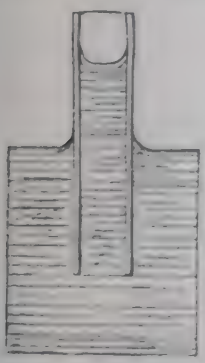

Fig. 76.—Elevation of Water in a Capillary Tube

Fig. 77.—Depression of Mercury in a Capillary Tube

Ex. 144.—Take two or three capillary tubes of different diameters. Determine their diameters by the method of Exercise 113. Draw water through them so as to wet the insides. Place the tubes vertical in a vessel of water (fig. 78). Measure the height at which the water stands in each tube above the level of the water in the vessel.

Compare the heights with the diameters of the tubes.

For each tube multiply together the diameter of the tube and the height to which the water has ascended. The products are approximately the same.

The experiments show that liquids in contact with solids change their levels at the line of contact. When the liquid wets the solid the liquid is elevated, and the surface is concave; when the liquid does not wet the solid it is depressed, and shows a convex surface.

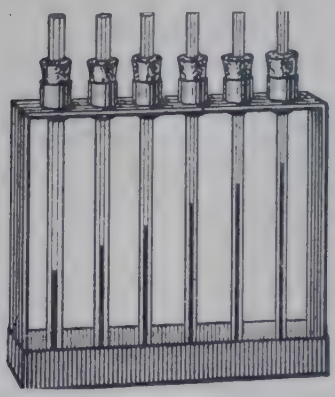

Fig. 78. — Capillary Elevation varies with the Diameter of the Tube

The amount of the elevation or depression of a liquid in a capillary tube depends upon the diameter of the tube, the exact law being that it is inversely proportional to the diameter of the tube (Exercise 144).

The amount of the capillary elevation or depression is different for different liquids in the same size of tube.

The cause of the capillary action is the surface tension of the liquid. The surface layer of the liquid has liquid beneath it and air above it, so that the surface layer is acted on in a different manner on its two sides. This layer is therefore in a different condition to the rest of the liquid.

CHAPTER VIII

Properties of Gases

79. Although air is invisible, and cannot be handled in definite quantities as easily as solids or liquids, we can infer many of its properties from its effects on other things.

Thus we know that air flows from one place to another by feeling the wind, and by seeing it carry along smoke, clouds, and light objects. Air is therefore a **fluid**. Gases and liquids are all fluids.

There are many other gases besides air; oxygen, hydrogen, nitrogen, and carbonic acid being those most commonly met with. The elementary properties of all gases, so far as they are dealt with here, are the same, and the experiments of this chapter may be made with **any gas**.

80. PRESSURE OF A GAS.

Ex. 145.—(1) Push an empty flask, preferably one that has a long narrow neck, mouth downwards into water.

Water rises up the neck, and the farther the flask is lowered into the water the higher the water rises into the flask (fig. 79).

No air has escaped. Hence the air has been compressed into a smaller volume. The compression is produced by the pressure of the water demonstrated in the last chapter.

(2) Slowly blow out a toy balloon or a paper bag with its mouth gathered up into a narrow neck. The pressure of the air inside may be felt by holding it between the finger and thumb. Such a vessel is said to be *inflated* with air.

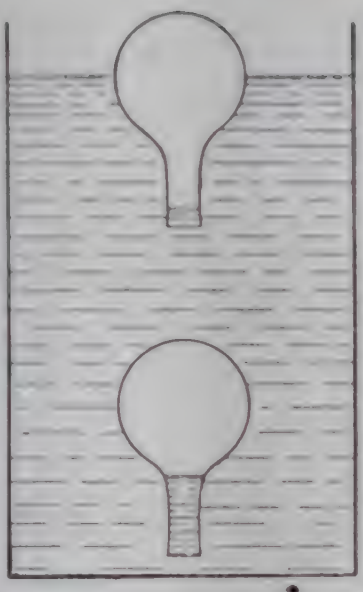

Fig. 79.—Compressibility of Air.

A football or a tennis ball is blown up in the same way till the ball feels quite hard, but it becomes smaller under considerable pressure. Note the resistance that it offers to compression.

A bicycle tyre is similarly blown up, and a cyclist tests the sufficiency of its inflation by pressing it with his fingers. The pneumatic (air-containing) tyres of a motor car are similarly inflated.

If the outlet of an inflated tyre or football be opened, the air that has been forced in rushes out again. A puncture in the tyre of a bicycle is followed by a great outrush of air.

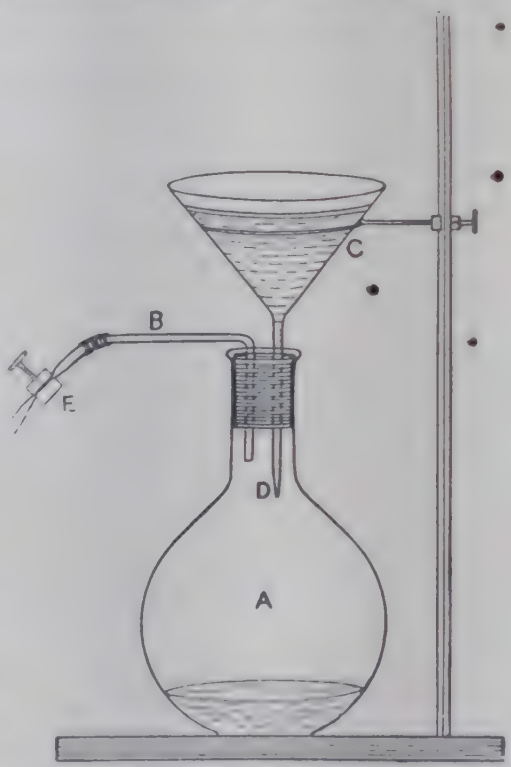

Fig. 80.—Pressure of Air

Ex. 146.—Fit a flask A, fig. 80, with a well-fitting stopper having two holes through it. The stopper must be air-tight. Through one hole pass a piece of glass tubing B, that terminates in a piece of indiarubber tubing on which there is a screw clip or pinch-cock E (fig. 81); and through the other hole pass the stem of a funnel C, the aperture D of which is small. The small aperture may be obtained by fastening to the stem of the funnel a piece of small-bore tubing by means of an indiarubber joint, or as described in Art. 179.

Pour water into C, leaving the tube B open. Screw up the pinch-cock while there is still plenty of water in C.

If the stopper is air-tight, water soon ceases to flow from D. What is keeping it back?

Loosen the screw clip. Does air pass out along the tube B? Why? What is the effect on the flow of water? Why?

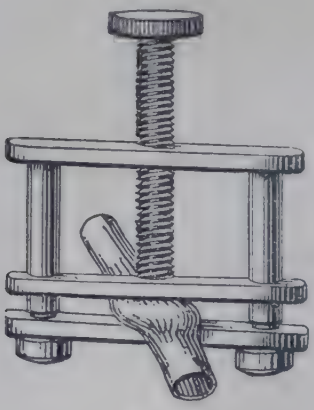

Fig. 81

These experiments show that air can be compressed, that a quantity of air contained in a closed vessel presses against the walls of the vessel in all directions, and that air which is under a greater pressure flows out where it can to a place where the pressure is less. Of course as the air presses against the walls of the vessel, those walls resist or press back. Thus we may speak of the pressure to which the air is subjected (which it is *under*) or of the pressure which it exercises. These are equal.

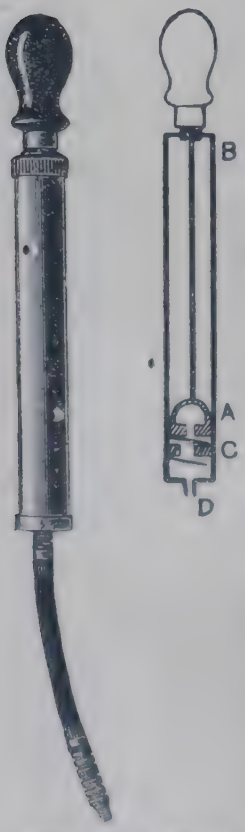

Fig. 82.—A Simple Compression Air Pump

81. THE AIR PUMP.—In Exercise 145 the water moved up the neck of the flask, pushing the air in front of it. A well-fitting solid piston would compress the air in the same way.

A tube containing a piston which when pushed down compresses the air in front of it, is called a **compression pump**. A bicycle pump is a simple compression pump.

Fig. 82 shows such a pump. By means of a handle the piston A can be moved up and down in the barrel BC.

You know that air rushing into a room blows the door open if it is not fastened, and when rushing out of a room blows the door shut. The **valves** shown in section in fig. 82, at C and in the piston, are small trap doors, hinged so that when air presses on one side they open a little way and allow air to pass through; but when air presses on the other side they shut closely and lap over the opening against which they are placed, so that no air can pass through.

Valves are made in many forms, differing in shape from those shown, but always fulfilling the same purpose.

Suppose the piston A of the pump is at the top of the barrel at B. On pushing it down the valve in A closes, and drives the air in front of it. This causes the valve C at the bottom of the barrel to open, and air is driven through the tube D into any vessel that is connected to the tube.

On pulling the piston back the valve in A opens, while that at C is closed by the pressure of the air in the vessel connected with D. Thus at every stroke air is driven out of the barrel along the tube D.

Ex. 147.—Take the apparatus of fig. 65, and put the funnel mouth downwards into a deep vessel full of water, so that water rises up the glass tube. Now attach a small compression pump to the end of the small glass tube, and work the pump moderately. Note the result of each stroke of the piston.

This experiment shows how air is supplied to diving bells, and to other vessels in which men work below water. The pressure of the air in the sunken vessel is equal to that of the water, and the water is thus kept out of the vessel.

82. TENDENCY OF GASES TO EXPAND.

Ex. 148.—Take two football bladders or two indiarubber toy balloons. To one of them, A (fig. 83), connect a glass tube in which there is a tap. Or in

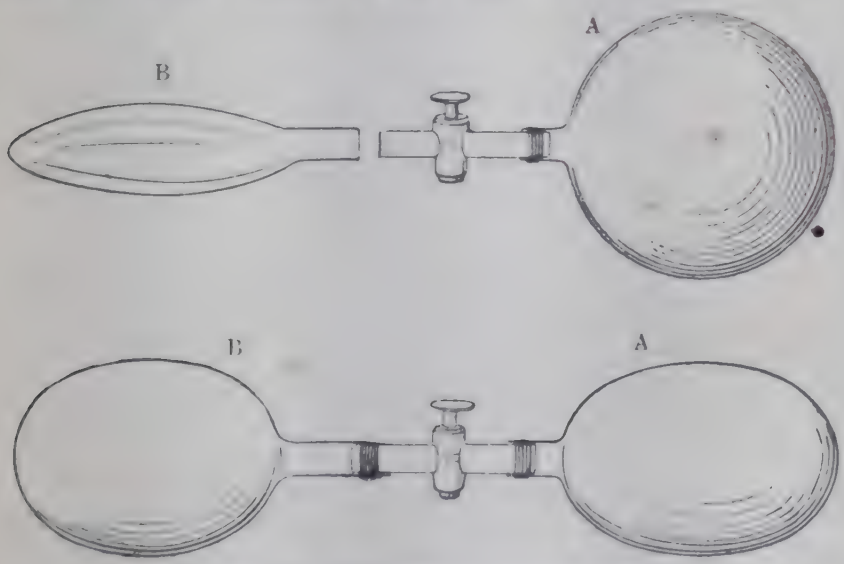

Fig. 83.—Expansive Power of Gases

the absence of a tap, join two short pieces of glass tube by a short piece of indiarubber tube, and put a pinch-cock on the indiarubber tube. See that the joints are air-tight.

Inflate A fully until the membrane is tightly stretched; then close the tap.

Join the other balloon B uninflated to the other end of the glass tube.

Open the tap. The vessel B becomes partly inflated and A less inflated.

This experiment shows a very important property of gases, in which they differ both from liquids and solids. A quantity of gas shut up in a vessel occupies all the space not filled by anything else, and if a second vessel containing no air be connected with the first the air spreads itself equally throughout both vessels, and presses against their walls.

If the small balloon B above were connected with an inflated bicycle tyre instead of with the vessel A, it would probably be inflated to such an extent as to burst, because the air in the tyre is greatly compressed, and the pressure of the air that passes into the small balloon might be greater than it would bear.

83. THE EXHAUST PUMP.—By making use of this fact, that a quantity of air expands so as to occupy all the available space, air can be pumped out of a closed vessel. The vessel that contains the air is connected with an empty one in which there is no air. Immediately some air leaves the full vessel and enters the empty one. Thus some of the air has been got out, and the process is repeated a number of times. The arrangement is effected by an exhaust pump, which is like the compression pump, except that the valves are differently arranged.

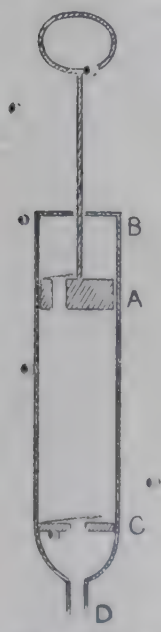

Fig. 84. — A Simple Exhausting Air Pump

Fig. 84 shows a section of a simple exhaust pump.

The nozzle D is fitted on to the mouth of the vessel from which the air is to be pumped. The piston A is then pulled outwards. The pressure of the air in the vessel opens the valve C, and the lower part of the barrel B is occupied by the air that has thus passed. The piston A is then pushed inwards. The pressure of the air in the lower part of the barrel opens the valve in A and closes that at C. Thus some of the air that was in the vessel is now outside. Each complete up-and-down stroke of the piston removes some of the air.

Can the air *all* be got out in this way?

Ex. 149.—Fit up an apparatus similar to that shown in fig. 65, but with a larger vessel, such as a bell jar, in place of the funnel. Lower the bell jar mouth downwards into a vessel containing water. Attach the exhaust pump to the tube, and pump out the air. Be careful that no water comes up into the pump.

Fig. 85 shows a more elaborate air pump, with better-fitting

valves than a cheap pump has. Its construction and action are exactly the same as described above.

The pump is fixed to a stand, and the connecting tube passes from the pump along an arm that ends in a flat plate. On this plate stands a large glass vessel called a **receiver**.

Ex. 150.—Grease the bottom of the receiver. Place on the plate a partly inflated toy balloon, and cover it with the receiver. Work the pump. What is the effect on the balloon? Why does it increase in size?

Using such a pump we may learn something more of the action of air pumps.

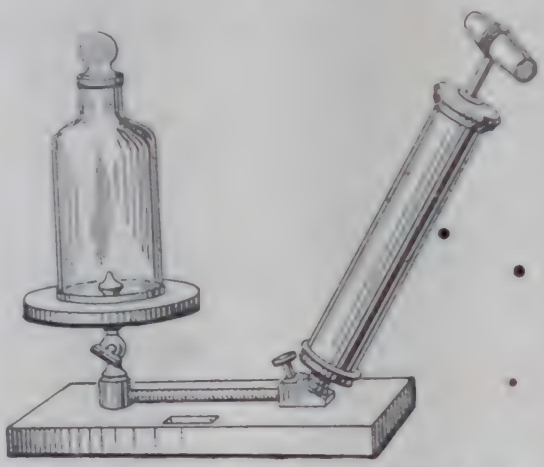

Fig. 85.—A Fixed Air Pump with Receiver

Suppose the volume of the receiver and connecting tube to be 8 times that of the barrel. When the piston is raised these 8 volumes expand and become 9, of which 8 remain in the receiver; when the piston is lowered, then $\frac{1}{9}$th of the air originally in the receiver has passed through the valve and $\frac{8}{9}$ths remain. The next stroke similarly takes away $\frac{1}{9}$ of this $\frac{8}{9}$, i.e. $\frac{8}{81}$, and there remains $\frac{8}{9} - \frac{8}{81}$, i.e. $\frac{64}{81}$ of the original quantity. The next stroke removes $\frac{1}{9}$ of this $\frac{64}{81}$, i.e. $\frac{64}{729}$, and there remains $\frac{64}{81} - \frac{64}{729}$, i.e. $\frac{512}{729}$ of the original quantity; and so on.

On examining the numbers it will be noticed that this law holds.

Quantity left after	1st	stroke	.....	$\frac{8}{9}$	of the	original quantity.
,,	,,	2nd	,,	..... $(\frac{8}{9})^2$	,,	,,
,,	,,	3rd	,,	..... $(\frac{8}{9})^3$	,,	,,

And generally, if a be the volume of the receiver and b of the barrel, the quantity left after the nth stroke of the piston $= \left(\frac{a}{a+b}\right)^n$th of the original quantity. Since the quantity of air removed at each stroke of the piston is only a constant fractional part of what is actually present before the stroke, it follows that all the air can never be completely pumped out of the receiver.

84. AIR HAS WEIGHT.—We know that windmills are turned by the wind, as watermills are by water; that heavy boats and ships

are driven along by the wind; and that sometimes trees and buildings are blown down by it. This suggests that air, which behaves in many respects like water, may also possess weight.

The direct way of ascertaining this is to weigh a large flask or globe, such as is shown in fig. 86, with air in it; to pump out the air, and to weigh the flask again. With such a globe, a good air pump, and a well-made balance, the weight of the air that fills the globe can be ascertained accurately.

Fig. 86

Ex. 151.—In the absence of a specially fitted globe, fit a large flask (fig. 87) with an indiarubber stopper through which there passes a glass tube, either provided with a tap, or lengthened by a piece of indiarubber tube that can be closed by a screw clip. The stopper must be air-tight and tied in position.

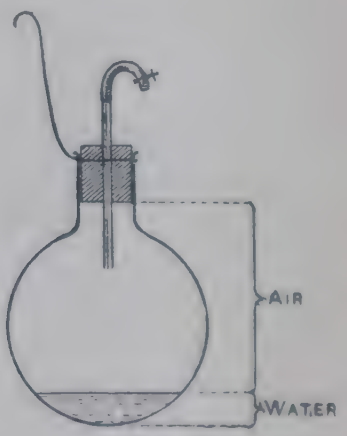

Fig. 87

Globe Fitted for showing Weight of Air

Connect the tube to the exhaust pump and pump out as much air as possible. Screw the clip up tightly, remove the flask and weigh it.

Connect the tube to the compression pump, and force as much air as possible into the flask. Again weigh the flask.

The difference between the two weights is due to the difference in the quantity of air in the flask on the two occasions. The difference will be less than 1 grm., and can only be measured by using a good balance. But the fact that there is a difference shows that air possesses weight.

But all the air was not got out of the flask in the first operation. With such a pump it could not all be got out.

The air can nearly all be got out of the flask by sweeping it out by steam. Steam when cold condenses into water.

Ex. 152.—Take the flask used in the last experiment (fig. 87) and place in it a little water (about 50 cub. cm.). With the exit tube open boil the water. As the steam rushes out it carries air with it. When steam has been issuing for several minutes screw up the clip and quickly remove the heating flame. When the flask is cold weigh it with the accessories on an accurate balance.

Open the screw clip; air rushes in. Again weigh the flask and accessories. The difference between the two weights (m grm.) is the weight of the air that rushed in.

Find the volume (v_1) of the water left in the flask, and the total capacity (v_2) of the flask.

Then a volume of air equal to ($v_2 - v_1$) cub. cm. weighs m grm.

Hence find the weight of 1 cub. cm.

Example.—In an experiment the following values were obtained:—

Difference between the weighings	... ...	·73 grm. (m)
Volume of water left in the flask	... ...	75 cub. cm. (v_1)
Total volume of the flask	... ...	685 cub. cm. (v_2)
∴ 610 cub. cm. of air weighed	... ...	·73 grm.
And 1 cub. cm. weighed approximately	... ...	·0012 grm.

It will be seen later that the mass of 1 cub. cm. of air varies according to its temperature and pressure. The correct value at standard pressure and temperature is ·001293 grm.

The weight of the air enclosed in the vessels in the experiments described in this chapter is thus very small, and it must be clearly understood that the pressure of the air enclosed in the vessels is not caused by its weight. The air enclosed may weigh very little, but the pressure it exercises may be very great, and the smaller the space it is caused to occupy the greater is the opposing pressure that it exerts.

The cause of this pressure is explained in Art. 112.

Ex. 153.—(1) Using the correct value given above, find the weight of a litre of air, and of a cubic metre of air.

(2) What is the weight of the air contained in a room 8 m. long, 5·5 m. wide, and 3·5 m. high?

85. THE CARTESIAN DIVER.—The instrument shown in fig. 88 furnishes an instructive illustration of the pressure of gases and liquids.

Take a glass bulb with a neck 2 or 3 cm. long. Wind fine wire round the neck until the bulb just floats in water with the mouth downward. This represents the diver.

Fig. 88.—The Cartesian Diver

Nearly fill a tall jar with water and place the diver in it. Cover the mouth of the jar with thin sheet indiarubber, so that it is air-tight.

Gently press the indiarubber; the diver descends. Take off the pressure; the diver ascends. Why? What is the effect of your pressure on the height of the water column in the neck of the diver, and therefore of the weight of the diver?

Remember that your pressure on the membrane has caused the air to press more on the water.

These suggestions should enable you to answer the above question—Why?

CHAPTER IX

Pressure of the Atmosphere

86. We have shown that air has weight, is easily compressed, and expands of its own accord when the pressure on it is lessened. Now the earth is surrounded by a layer of air several miles high. Men who have ascended to great heights in balloons, or up very lofty mountains, such as those to the north of India, have found that the air extends upward as far as they have been, although its density there is much less than at the level of the sea.

There are reasons for thinking that it extends upwards for a

distance of about 80 or 100 miles, so that we are living at the bottom of a deep ocean of air, the total weight of which is many millions of tons.

Now we know that in consequence of their weight, solids and liquids press upon any surface on which they rest.

Does the atmosphere, owing to the fact that air has weight, press on the surface of the earth and on objects on that surface?

To answer this question make the following experiment:—

Ex. 154.—Fit a thin metal can (fig. 89) with a good tightly fitting stopper. Instrument makers supply vessels for this purpose, but any ordinary workman's tin can will do if it does not leak. Remove the stopper; put about 80 or 100 cub. cm. of water in the vessel, and boil the water for several minutes.

Push in the stopper tightly, and at the same time remove the heating flame.

Pour cold water on the vessel, or plunge it into cold water. What is the result?

Some glass flasks are thin enough to be smashed in this way.

The collapse of the vessel took place when the air was removed from the inside of it. If the air were removed from the inside of a house, the house would collapse in a similar manner.

Fig. 89.—An Empty Vessel crushed by the Pressure of the Atmosphere

When a book rests on and presses on a table, the table supports the pressure. If you remove the table from under the book, the book falls to the ground. So long as the pressure of the book on the table is balanced by the back pressure of the table on the book, it does not fall; but if you remove this back pressure it does fall.

Hence we see that the reason why the vessel when full of air did not collapse, but when empty of air did collapse, is that in the one case there is a back pressure which supports the sides of the tin, and in the other case this back pressure was removed so that there was nothing to resist the pressure on the other side.

If the tin vessel containing air only had been corked up and lowered in water to a depth of 30 or 40 ft. it would have collapsed similarly, but if full of water it would not collapse.

Thus at the surface of the earth there is a pressure due to the atmosphere similar to the pressure shown by experiments in Chap. VII to exist in water.

Ex. 155.—(1) Fill an ordinary tumbler with water (quite full). On the top of it place a thin card or a piece of thick paper. Press the card to ensure that

it touches the rim of the glass all round. See that no air is enclosed. Raise the tumbler, and gently turn it about—sideways, bottom upwards, &c. (fig. 90). **Why does the water not fall out?**

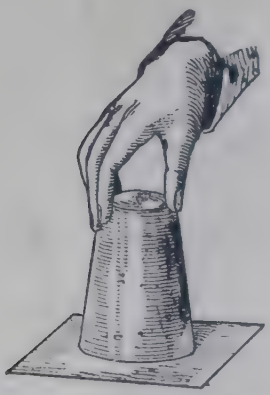

Fig. 90.—A Card supported by Atmospheric Pressure

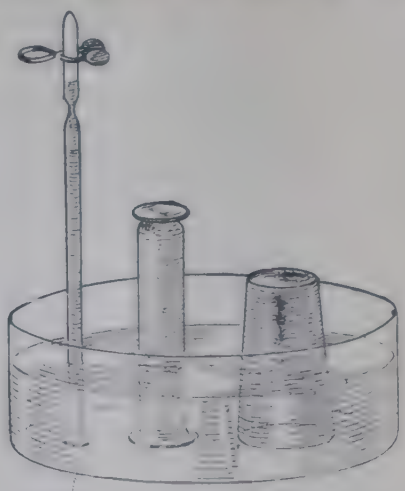

Fig. 91.—Columns of Water supported by Atmospheric Pressure

(2) Fill a tall measuring cylinder with water: close the top by a card or glass plate; holding the plate in position invert the cylinder, and place it mouth downwards in a shallow vessel containing water; remove the plate (fig. 91). What is the behaviour of the water? Raise the cylinder a little from the bottom of the vessel. Why does the water not descend?

Repeat the experiment, using a burette with one end closed by a clip or a tap. Open the clip a little. Why does the water now descend?

(3) Fig. 92 shows a bottle with three apertures closed by corks. Through the two side corks small stopped tubes are passed. Fill such a bottle with water. Open the small tubes. Explain why air passes in at the top hole and water out at the bottom hole.

Fig. 92

Figs. 93 and 94 illustrate two experiments showing effects of the pressure of the atmosphere. They may be made with an air pump such as is shown in fig. 85.

Ex. 156.—Take out the stopper from the receiver, and tie a flexible membrane over the aperture. Put the receiver in position and pump out the air.

Ex. 157.—Screw on to the plate of the air pump the bottom Magdeburg

Fig. 93

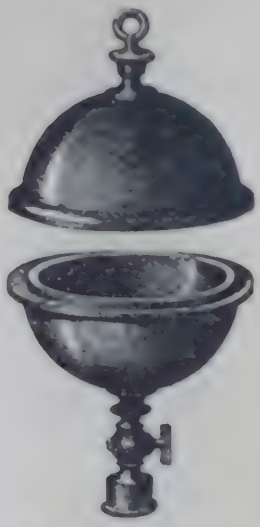

Fig. 94.—Magdeburg Hemispheres

hemisphere, and open its stopcock. Grease the rim of the top hemisphere; press it on the lower one with a turning motion. Pump out the air. Close the bottom tap, and unscrew the sphere from the plate.

Endeavour to pull the hemispheres apart with a straight pull.

87. USE OF THE PIPETTE.—The action illustrated in Exercise 155 is made use of in the pipette, which is a glass tube enlarged in the middle and drawn out to a fine point at the bottom (fig. 95).

The lower end is placed in a liquid, and the air being sucked out at the other end, liquid rises in the tube. The upper end is then closed with the finger, and on the pipette being removed the liquid remains in it.

Pipettes are generally made so that when filled up to a mark on the stem they hold a convenient volume, such as 50 cub. cm. or 100 cub. cm.

Fig. 95.—A Pipette

Ex. 158.—(1) To use a pipette. Draw water into it slightly above the mark. Remove it from the liquid. By slightly lessening the pressure of the finger allow water to fall out drop by drop until the level of the mark is reached.

(2) By means of a pipette verify the graduations on a cylinder. Using the pipette as above, deliver into the cylinder an exact number of cubic centimetres

of water, and read the level on the cylinder. Repeat the operation as far as the graduations extend.

(3) Graduate an ungraduated cylinder. Gum a strip of millimetre paper on its outside; deliver water into it as before, and mark the quantities on the strip of paper.

88. THE BAROMETER.—Torricelli's experiment.

The pressure of the atmosphere and some of its effects having been demonstrated, we have now to show how that pressure is measured.

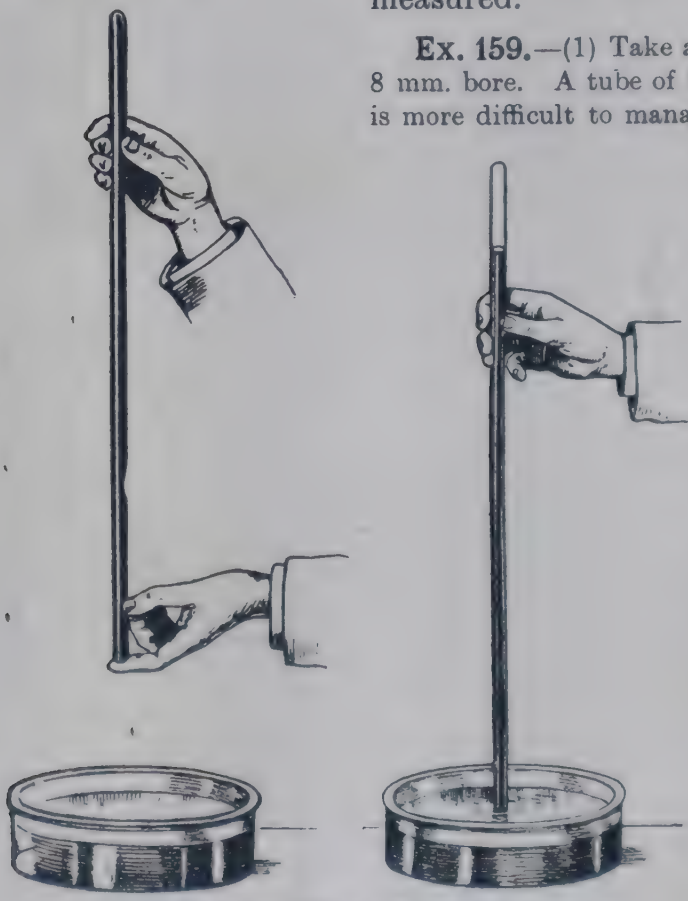

Fig. 96.—Filling a Barometer

Ex. 159.—(1) Take a glass tube of about ¼ in. or 8 mm. bore. A tube of larger bore may be used, but is more difficult to manage. The length of the tube should be about 90 cm. (36 in.); it is useless if less than 76 cm. It must be open at one end and closed at the other. The tube should be quite clean and dry inside. It may be cleaned if necessary by passing down it small pieces of clean cotton wool tied to the end of a wire.

This tube has to be filled with clean dry mercury. Warm a quantity of mercury and also the tube.

Fix a small clean funnel to the open end of the tube by means of a short piece of indiarubber tubing, as illustrated in fig. 203.

Pour some of the mercury into a small strong dish such as is shown in fig. 96, and fix the tube in a vertical position over this dish. Slowly pour mercury into the funnel until the tube is nearly full, within about 1 in. or 3 cm. of the top.

Remove the funnel. Place the thumb firmly over the open end of the tube, and slowly invert it two or three times. The air bubble will thus be made to pass up and down the tube, and will collect any little bubbles of air that may be clinging to the glass.

When these have been removed, complete the filling of the tube with mercury. Close the open end with the finger or thumb, taking care that no air bubble is shut in. Invert the tube (fig. 96), and set it upright in the dish of mercury, keeping the finger in position until it is well under the surface of the mercury in the dish. Gently remove the finger. Note the behaviour of the column of mercury.

Does the tube remain full? Measure the height of the mercury column from the level of the mercury in the dish.

Whatever be the size, shape, height, or form of the tube, the difference in height between the upper and lower mercury surfaces is the same, about 76 cm., or 30 in.

(2) What is there in the space in the tube above the mercury? Incline the tube as in fig. 97. If the inclination be sufficient, the mercury will go quite to the top of the tube. This shows that there is nothing in the space above the mercury. Such a space is called a *vacuum*.

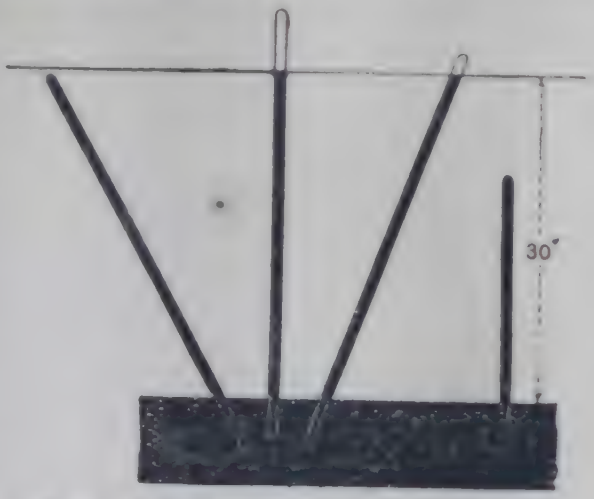

Fig. 97.—Effect of Inclining a Barometer Tube

Fix the tube in various inclined positions, and measure the height between the surfaces of the mercury. Is it the same as when the tube was vertical?

If the tube be in a deep mercury cistern and be pushed straight downward it becomes full when the difference of level is about 76 cm., or 30 in., and if it be pushed lower than that remains full.

89. THE U-TUBE BAROMETER.—Less mercury is required if the preceding experiment be arranged rather differently so as to avoid the use of the dish.

Ex. 160.—To the long 90-cm. tube fix by india-rubber tubing another shorter one, as shown in the left-hand side of fig. 98.

Proceed exactly as previously described to clean, dry, and fill the tube. When this has been done hold the long tube with your left hand, and grip the short tube with your right hand, the thumb or a finger of the right hand closing the end.

Hold the tube with your right hand over a tray or dish, and bend the tube round so that it takes the position shown in the right-hand side of fig. 98.

Loosen the pressure of the thumb a little, and allow mercury to trickle out into a dish underneath it. Only a small quantity will come out, the remainder standing up in the tube as before.

Fix the tube on a stand with both arms vertical, as in the right-hand side of fig. 98.

Gently lower the open arm. What is the effect on the height of the column in the closed arm? Measure the difference of level between the two surfaces.

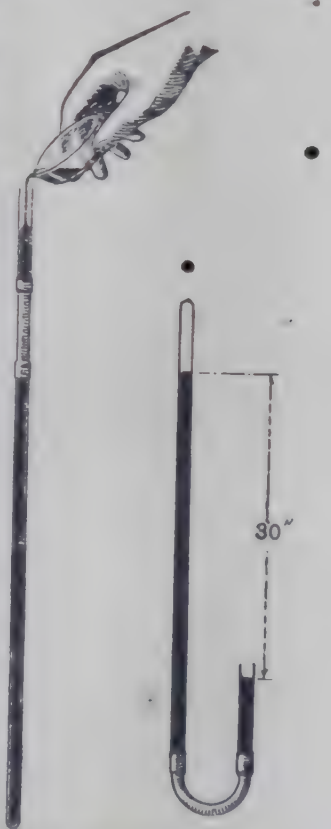

Fig. 98.—Setting up a U-tube Barometer

Raise the open arm and repeat the measurement. Continue to raise it. Note what occurs.

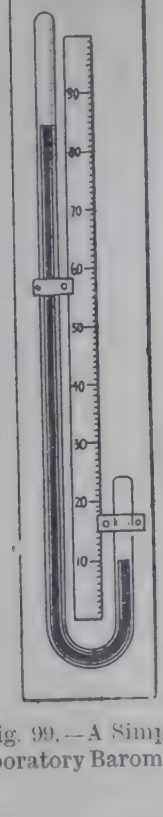

Fig. 99.—A Simple Laboratory Barometer

A barometer attached to a board having a scale attached, as shown in fig. 99, should be kept permanently hung on the wall of the laboratory.

The U-tube or siphon form of barometer is sometimes made entirely of glass, with a small cistern attached to the short arm. Such a tube must be filled in several steps, inclining it each time to allow mercury to enter and air to pass out.

90. HEIGHT OF THE BAROMETER.

A comparison of Exercise 160 with Exercises 119 to 121 suggests that in Exercise 160 we have a light fluid balancing a heavy one. It makes no difference how high the open arm is. It may therefore be imagined to be many miles high, as high as the air extends.

Then we have in Exercise 160 a U tube with the atmosphere and the short column of mercury on one side and the long column of mercury in the other?

Why do we measure the *difference* in level of the two mercury surfaces? What does this difference represent?

This difference is called the **barometric height** or **height of the barometer.**

Ex. 161.—Having set up a siphon barometer, attach to the open end the small compression pump, and pump in a very little air. What is the effect on the barometric height? Using the exhaust pump similarly, pump a very little air out. What is the effect?

It is found that the first of these effects follows if the barometer be taken down a deep mine, and the second effect follows if the barometer be taken up a hill. What explanation can you give of this? These effects are called respectively **a rise and a fall of the barometer.**

On the top of very lofty mountains the barometric height is not more than about half its value at sea level.

As air is very compressible, the bottom layers of the atmosphere pressed by the weight of those above are comparatively dense, and as one ascends each succeeding layer is less dense, because the pressure of the layers that are still above it continues to diminish. (Compare fig. 66.)

Fig. 100 shows graphically the height of a barometer at differe

distances above sea level. Thus at a height of 4000 m. the barometer stands at a height of 46 cm. Compare the other values given in the diagram. Note that the fall of the barometer is not proportional to the height above sea level.

Also it is found that in the same place the barometric height varies from day to day. This effect is generally due to changes in the amount of water vapour present in the air (Chap. XXVI). Water vapour is lighter than air, and

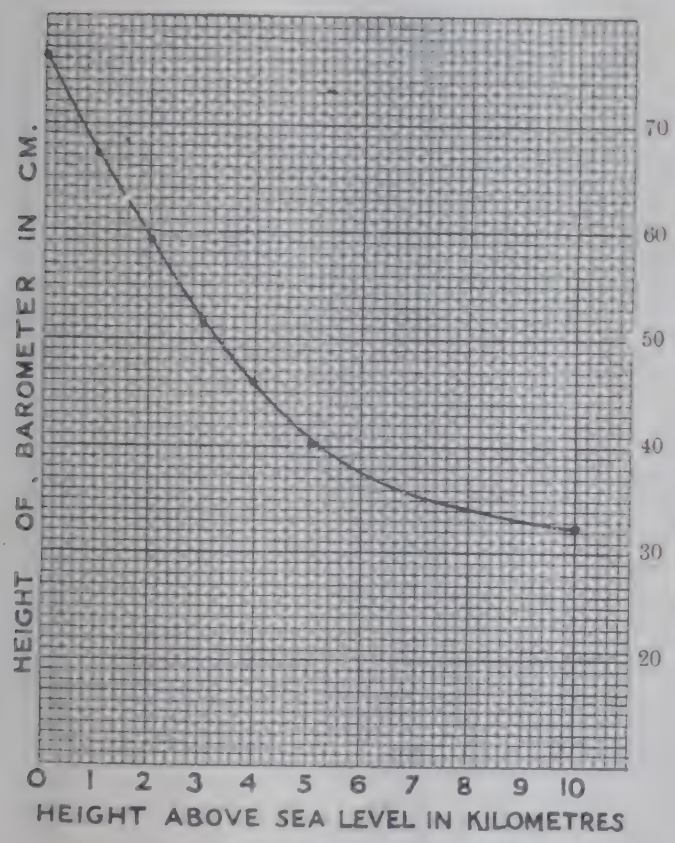

Fig. 100.—Height of a Barometer at Different Distances above Sea Level

Fig. 101.—The Dial Barometer

when a quantity of it comes into the atmosphere over any spot it drives some of the air away and takes its place. Hence the pressure of the atmosphere is lessened and the barometer falls, i.e. the barometric height becomes less. Thus a fall of the barometer at any place generally means that more water vapour is present in the air, and that it may probably rain.

As the height of the barometer thus changes from day to day,

it is convenient to make an arrangement by which these variations may be easily seen. Fig. 101 shows how the rise and fall of the barometer is made to turn a hand round a dial. The mercury supports a float on which there is a rack whose teeth work in those of a pinion wheel.

Since the barometric height varies from day to day and from place to place, one particular value has been selected and called the **standard height**. This is 760 mm. or 29·92 in.

91. VALUE OF THE ATMOSPHERIC PRESSURE.—Suppose the bore of a barometer tube is exactly 1 sq. cm. in area, and the height of the barometer column 76 cm. (the standard height), then the volume of the mercury is 76 cub. cm., which weighs $76 \times 13{\cdot}6 = 1033{\cdot}6$ grm. The standard atmospheric pressure is thus 1033·6 grm. per square centimetre, or 14·7 lb. per square inch. It must be noticed at this stage that *pressures are always expressed as so much weight on every unit of area.*

Ex. 162.—(1) If a piece of leather attached to a string is soaked in water and pressed down upon a flat stone, the stone may be lifted by the string. Explain the reason of this.

(2) The end of a glass tube is closed by a large apple forced on to it, so that the tube cuts into the apple. Air is pumped from the other end of the tube. What happens to the apple? Why?

(3) Explain the action of the common syringe or water squirt. Why does it fill with water?

(4) Calculate the total weight of air that is pressing on 1 sq. Km. of the earth's surface if the mercury barometer stands at 760 mm.

(5) Explain why the size of the tube makes no difference in the height of the barometer. Compare Exercise 118.

(6) Read the height of the barometer at the same hour every day for a week, and make a graph showing the change of height from day to day. (On squared paper write under the successive vertical centimetre lines the names of the days of the week, and mark the height of the barometer each day on the vertical line by a dot. As the dots are added join each to the preceding one by a straight line.)

92. PUMPS.

Ex. 163.—Repeat Exercise 159. with water as the liquid instead of mercury. What is the difference in the result? Why does the tube remain full of water while the mercury fell to 30 in.? Would the experiment succeed with a long tube, supposing that the tube could be practically managed? About what length of tube would you require?

This experiment shows that the pressure of atmosphere will support a tall column of water. A barometer in which water was the liquid must be about 33 ft. high. Hence, if the air is pumped out

of the top of a pipe, the bottom of which is in water, the water is driven up the pipe to a height of about 33 ft. by the pressure of the atmosphere. This fact is made use of in the water pump.

The Common or Lift Pump. — Examine a model of a common lift pump (fig. 102).

Compare the model with the diagram of the exhausting air pump (fig. 84).

The valves in such models are often small movable cones, which when on their seating press against the sides of the tube. Thus they fulfil the same function as those in fig. 84. In fig. 102 the piston has a hole through it, and the valve opens or blocks that passage according to the position of the valve.

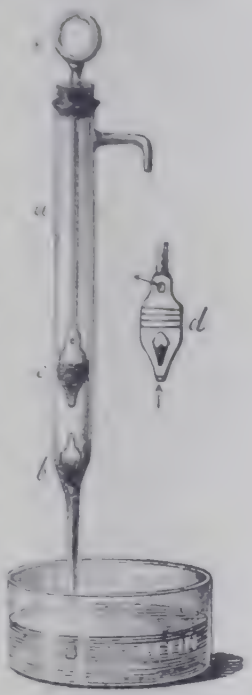

Fig. 102.—Model of a Common Pump

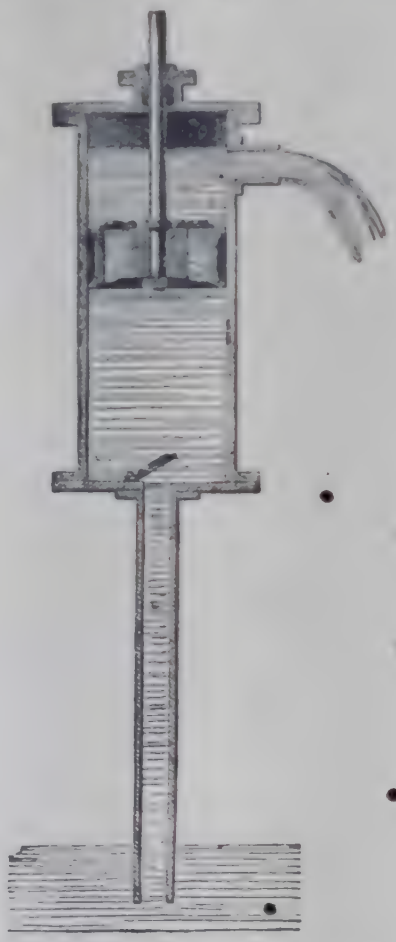

Fig. 103.—A Working Pump

Ex. 164. — Work the pump slowly and watch its action. The water in the vessel represents the water in the well. The first upward stroke of the piston causes water to pass up the tube through the valve *b* (fig. 102) into the barrel.

Why does the water rise?

The first downward stroke causes the water to pass through the valve *c* in the piston while *b* is closed. Why is the valve *b* closed during the downward stroke?

Water is now above the piston, and at the next upward stroke the valve in the piston closes and water is lifted up and out of the spout. At the same time the valve *b* opens and more water comes up from the well.

During the first upward stroke nothing was seen to issue from the spout. But something did issue. What was it?

Fig. 103 shows the barrel and upper parts of a working pump, which acts in exactly the same way as the model. Compare them.

The Force Pump.—Examine a model of a force pump, such as is shown in fig. 104.

The piston g is solid. There is a valve e at the bottom of the barrel as in the lift pump, and just above e a side tube b is inserted, the bottom of which is closed by a valve f. Both valves open upwards.

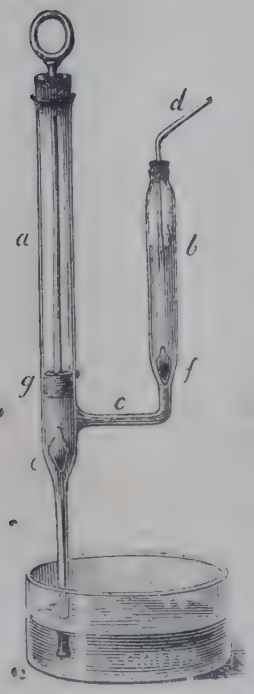

Fig. 104.—Model of a Force Pump

Fig. 105.—A Working Force Pump

Ex. 165.—Work the pump slowly and observe its action.

The first upward stroke gives the same result as in the lift pump. On the first downward stroke the valve e (fig. 104) closes, and the pressure of the piston forces the water through f into b.

Each successive stroke forces more water into b until it reaches the top and flows out into a cistern.

Fig. 105 shows a commercial force pump in which the piston is a solid plunger. By such a pump water is forced up to great heights. When the water in the tube D reaches to a great height, it requires great force to make the plunger descend. Why is this?

CHAPTER X

Fluid Pressure

93. In the previous chapter some acquaintance has been made with the main facts of fluid pressure, both liquid and gaseous. We have now to study one or two quantitative laws, and some further applications. The meaning of the word *law* in physical science is not that of a decree which the substance must obey, but an expression of some regularity in its behaviour.

Liquids

Ex. 166.—To ascertain the exact way in which the pressure of a liquid varies with the depth below the surface lower a mercury barometer into water.

(1) If the barometer is set up in the form of fig. 96, place the cistern at the bottom of a deep vessel (empty), fix the tube upright, and tie a metre scale to the tube, so that the height of the mercury column may be read. Read the height carefully.

Pour some water into the vessel until the mercury has risen about half a centimetre; measure the depth of the water and the rise of the mercury.

Add more water and take the two readings again.

Continue thus in successive steps until the vessel is full of water.

(2) If the barometer is of the form shown in fig. 98, fill the vessel with water first, and lower the barometer by successive steps until it reaches the bottom of the vessel. In this case the level of the mercury changes in both arms, and the measuring scale must be adjusted each time to get the correct height of the mercury column.

With this form of tube, if the short arm be curved, it may be shown, as in Exercise 115, that the pressure measured acts in all directions.

(3) Make a graph showing the relation between the depth of the water and the increase of pressure registered by the barometer.

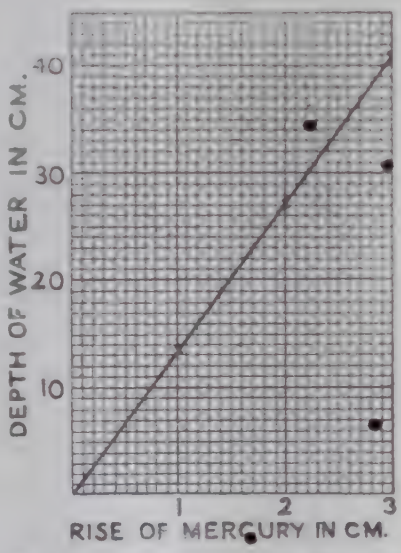

Fig. 106.—The Pressure in Water is Proportional to the Depth

Such a graph is shown in fig. 106. It is a straight line, showing that the pressure is exactly proportional to the depth.

The graph also shows that the depth of the water is 13·6 times as great as the rise of the mercury. Mercury is 13·6 times as dense as water. Hence the height of a barometer in which water is used is 13·6 times that of the mercury barometer.

Compare this experiment with Exercise 116.

A column of mercury or other liquid used in this way to measure pressure is called a manometer.

94. THE MANOMETER OR PRESSURE GAUGE.—A common form of manometer is a U tube containing a suitable liquid, usually mercury or water (fig. 107). If the liquid stands at different heights in the two arms, it is because the pressure on one surface is greater than on the other. The pressure at the bend of the tube must be the same in both directions. Hence the difference in level of the liquids is produced by the difference of pressure on the surfaces,

and this difference of level is proportional to and measures the excess of pressure on one surface above that on the other. It is often expressed as so many inches or centimetres of mercury or water.

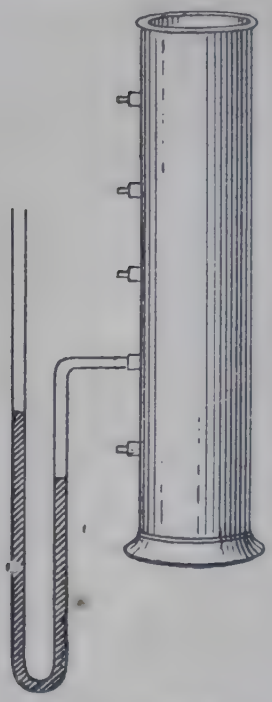

Fig. 107.—Liquid Pressure Measured by a Manometer

Ex. 167.—(1) By means of indiarubber tube attach a manometer containing a considerable quantity of mercury to nozzles in the side of a deep vessel containing water (fig. 107). The difference between the mercury levels shows the pressure due to the depth of water in the vessel down to the nozzle used. Use the different nozzles in turn, and measure as before the pressure and the depth of the water.

A comparison of the results of this experiment with those of the last show that the liquid presses horizontally to exactly the same amount as vertically.

(2) In the same way (*if possible*) measure the pressure of the water in the laboratory pipes. Proceed cautiously, as the pressure may be too great for the manometer. Fix a stopper tightly in the open end of the manometer. Fill the indiarubber tube with water; join it to the nozzle of the water tap. Turn the tap slowly. If the mercury is forced rapidly round, turn the water off, and abandon the experiment. If the mercury does not move far, slightly loosen the stopper, and allow air to escape and the mercury to rise. If it is safe to do so, remove the stopper; and take the difference between the mercury levels.

95. The pressure of the fluid in a vessel acts at any point on the wall of the vessel in a direction perpendicular to the wall, as indicated in fig. 108. And pressure applied to any part of a liquid is passed on instantly throughout the whole body of the liquid.

Fig. 108

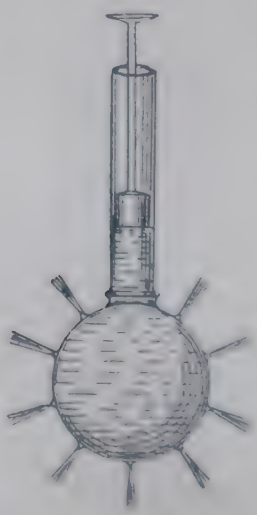

Fig. 109

The Pressure of a Fluid is always Perpendicular to the Walls of the Containing Vessel

Ex. 168.—(1) Fig. 109 shows an apparatus to illustrate these facts. Fill the vessel with water and drive in the piston. The water squirts out from every nozzle along a radius of the spherical vessel.

(2) Fill an ordinary elastic syringe with water, and press with one finger in any position. Wherever the pressure is applied, water is forced out

from the nozzle. Make a few small pinholes in different places. On squeezing the ball, the water spurts out of every hole.

This transmission of pressure in all directions is a distinguishing character of all fluids—both liquids and gases.

96. THE HYDRAULIC PRESS.—Fig. 110 shows a U tube with the diameter of one arm 4 times as great as that of the other, and therefore with the areas in the ratio of 16 to 1. If the small arm contains 1 litre of water, the larger contains 16 litres. If watertight pistons be fitted in the tubes weights may be placed on them, and 1 Kg. thus balances 16 Kg., 1000 Kg. supports 16000 Kg. Such an arrangement is called a hydraulic press, and is used to obtain very great pressures or to raise very heavy weights.

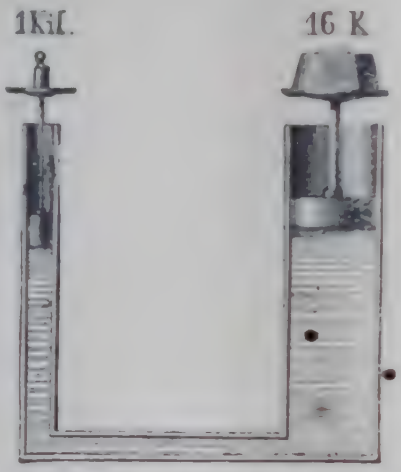

Fig. 110.—Principle of the Hydraulic Press

Instead of putting a weight on the smaller arm, water may be poured in, and the weight of this water acts in the same way.

It produces a pressure along the bottom tube of x grm. per square centimetre; this pressure is transmitted all through the liquid, and produces an upward pressure of x grm. on every square centimetre of the larger piston. Hence, if this piston is large, the total upward pressure is very great.

97. THE SIPHON.—An inverted ∩ tube, with one arm longer than the other, is often used to remove liquids from vessels. It is then called a **siphon** (sometimes spelled syphon).

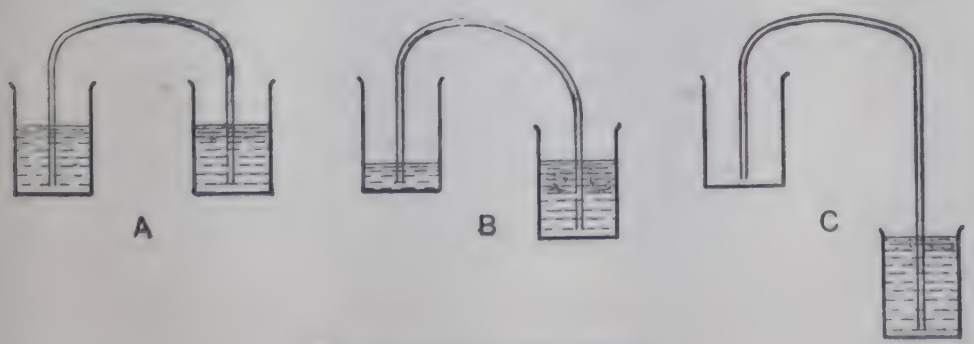

Fig. 111.—Action of the Siphon

Ex. 169.—Take two beakers containing water and in them place the two ends of a siphon (fig. 111), which may be a tube of indiarubber or glass.

(1) With the siphon full of air place the beakers successively at different levels, as indicated at A, B, C in the figure. Is there any flow of water?

(2) Fill the siphon with water and repeat the operations. Cause the water to flow into and out of either beaker. What determines the direction of the flow? Under what conditions (fig. B) does the flow stop?

(3) Take a glass siphon with unequal arms, and study its action. Since the water moves over the top bend there must be more pressure on a layer of liquid there in one direction than in the other. In which direction? Why? On each side of this layer there is a column of water—one longer than the other. The downward pressure of the longer column is greater than that of the shorter, and the two columns do not balance. Hence liquid passes out at the bottom of the taller column. Why does other water pass up the shorter arm to take the place of that which flows out?

How high could a water siphon be from the bend to the higher liquid surface? How high a mercury siphon?

98. EXPLANATION OF UPTHRUST.—Consider a prism ACDB in the middle of a body of liquid (fig. 112). The liquid presses on the solid all over. The horizontal pressures counteract each other.

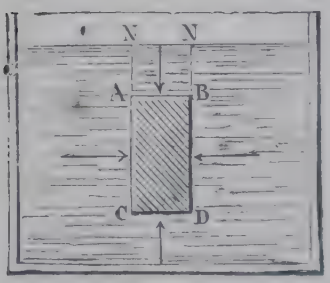

Fig. 112. Explanation of the Upthrust of a Liquid

On the top surface AB there is a downward pressure equal to the weight of the column of liquid NABN. On the bottom surface CD there is an upward pressure equal to the weight of the column of liquid NCDN. The result is an upward pressure equal to the weight of a column of liquid ACDB, which is the quantity of liquid displaced by the solid.

Gases

99. Previous experiments have shown that a quantity of air enclosed in a vessel exercises a pressure on the walls of the vessel. It must be fully realized that this pressure is not due to the weight of the air. It can rather be compared to the pressure of a spring. Indeed the term "spring of the air" used to be applied to this quality that gases possess.

We have now to see how this elastic pressure may be measured, and how it changes when the volume of the air is caused to change.

100. TRANSMISSION OF PRESSURE IN A GAS.—Everyone knows that if a hollow indiarubber ball be squeezed in any direction the air rushes out of any hole in it, wherever the hole may be. The air inside it transmits the pressure in all directions

Ex. 170.—The fact that it does so *equally* in all directions may be demonstrated by the apparatus shown in fig. 113. A little mercury is placed in each of the manometers attached to the bulb. On pushing in the piston the mercury rises to the same height in all the manometers.

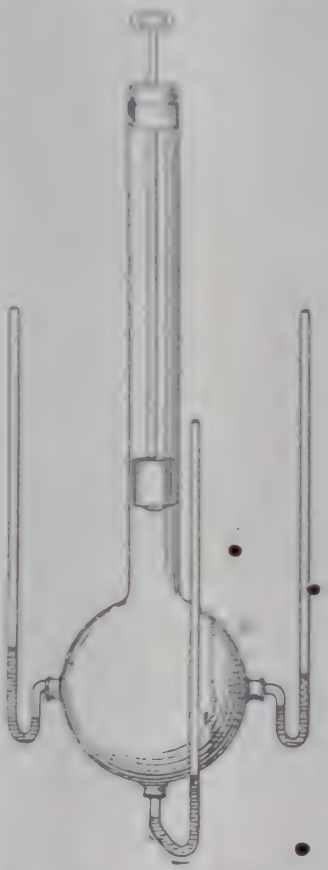

Fig. 113.—Transmission of Pressure in a Gas

Ex. 171.—Measure the pressure of the gas supply. Put some water in a manometer tube. To one end attach a piece of indiarubber tube. Attach the other end of the rubber tube to a nozzle of the gas supply. Measure the difference of level produced.

The extra pressure of gas in town pipes over the atmospheric pressure is generally equivalent to the pressure of an inch or two of water. Why is this extra pressure necessary? How is it produced?

If there be no gas supply imitate it by forcing air along a long glass tube by means of a small compression pump.

101. BOYLE'S LAW.

Ex. 172.—(1) Take a burette or graduated tube, and if both ends are open, close the larger end with a stopper. Attach a piece of rubber tube about a metre long to the smaller end, and into the other end of the rubber tube insert a length of straight glass tube (fig. 114). Clamp each glass tube vertically on a retort stand with the rubber connection lying on the table. You now have a U tube with one arm closed.

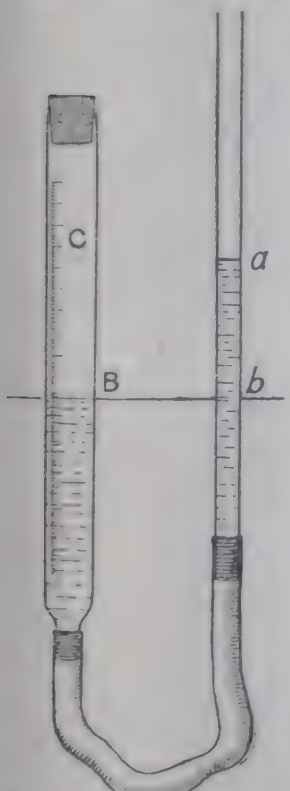

Fig. 114.—A Simple Boyle's Tube

Pour water into the open tube *ab*, loosening the stopper at the top of the burette if necessary to allow water to enter the burette until it stands a few inches up each arm, and then close the stopper tightly.

Some air is now confined within the burette C.

Raise the open tube, and if necessary pour more water into it. Note the decrease in the volume of the enclosed air as the movable arm of the U tube is raised, and its increase when this arm is lowered.

What produces these changes of volume?

Take two or three measurements of these quantities:—Volume of air; difference between water levels.

The readings show that the volume of air grows less as the level of the liquid in the open arm is higher, and greater when that level is lower.

Fig. 115.—Boyle's Law Apparatus

Remember that the atmosphere is pressing on the liquid surface in the open arm.

The pressure exercised by the enclosed air is equal to that of the atmosphere when the two liquid surfaces are level; greater than that of the atmosphere when the liquid in the closed arm is lower than that in the open arm; and less than that of the atmosphere when the liquid in the closed arm is higher than that in the open arm.

(2) Close the stopper tightly and again raise the open arm of the U tube, so that the liquid stands as in fig. 114. On the surface of the open liquid at *a* there is the pressure of the atmosphere; at *b*, in addition to the pressure of the atmosphere, there is the pressure of the liquid column *ab*. Measure the height *ab*. With what force must the enclosed air now be pressing on the surface B, which is level with *b*? The pressure of the air in C is equal to the pressure of a water column of what height?

Ex. 173.—(1) We now proceed to a more accurate form of apparatus (fig. 115) which is the same in all essential points as that of fig. 114. The glass tube that contains the gas (the volume tube) is an accurately graduated burette. the liquid used is mercury, the pressure is varied by sliding the movable arm of the U tube up and down.

To take an observation, arrange the movable arm so that the mercury is visible in both arms of the tube. From the graduations on the volume tube read the volume in cubic centimetres of the enclosed air *v*. On the millimetre scale read the levels of the two mercury surfaces in centimetres; subtract one reading from the other and thus obtain the height of the mercury column *h*.

Read the height H of the barometer. Call the pressure of the enclosed air *p*.

Record thus:—

Height of Barometer, H.	Difference of Levels, *h*.	Pressure, H + or − *h* = *p*.	Volume of Air, *v*.	Product, *p* × *v*.
75·5 cm.	25·2 cm.	100·7 cm.	24·6 cub. cm.	2477
...	...	...	...	...

Commence with the movable arm in its lowest position, and take several observations as above, raising the movable arm a few centimetres each step, until it is as high as it will go.

In some cases, to obtain the pressure *p*, *h* must be added to H; in some cases *h* must be subtracted from H. Write down the rule showing when to add and when to subtract.

(2) Make a graph showing the relation of *p* and *v*.

The following is a set of observations, the volumes being measured in cubic centimetres and the pressures in centimetres of mercury, through a range above and below atmospheric pressure:—

p	54	58·5	63·4	69	76	84·5	95
v	70	65	60	55	50	45	40
$v \times p$	3780	3802	3804	3795	3800	3803	3800

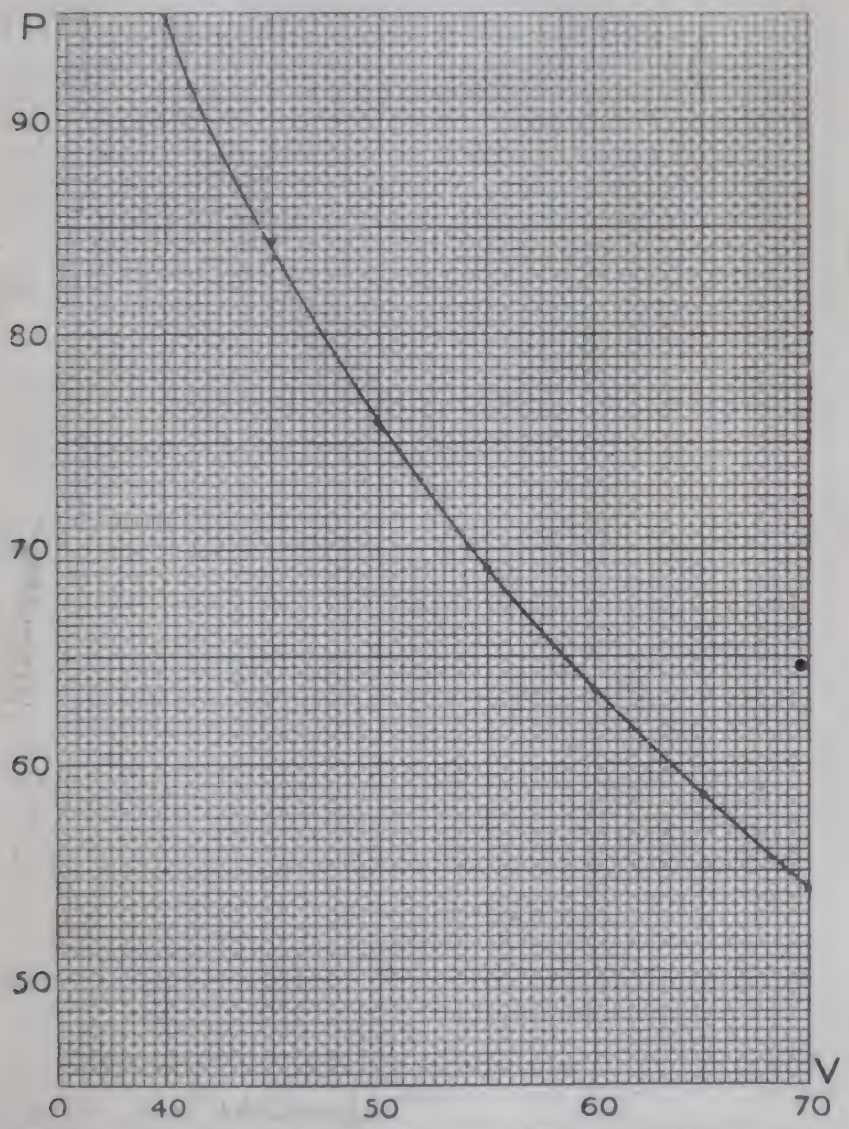

Fig. 116.—The Relation between Volume and Pressure of a Mass of Gas

They are set out in the graph above (fig. 116). It is observed that the products of P by V are approximately the same. In a

perfect experiment they are exactly the same. The exercise should be repeated until the agreement is fairly close.

The fact that the pressure and volume multiplied together gives always the same result is expressed by saying that **the volume of a mass of gas varies inversely as the pressure to which it is subjected**. This is Boyle's law.

102. DENSITY OF A GAS. — In Art. 63 we found the density of solids and liquids by finding the mass of a cubic centimetre of the substances.

In Exercise 152 was found the mass of a cubic centimetre of air.

Now we see that a cubic centimetre of air does not always contain the same quantity of air. When air is compressed, 1 cub. cm. contains more air than when it is rarefied.

Also it will be shown later that the mass of 1 cub. cm. of air differs according to whether the air is hot or cold.

Hence we cannot call the result of Exercise 152 simply the density of air. The correct statement is that the density of dry air at 760 mm. pressure at 0° C. temperature is ·001293 grm. per cubic centimetre.

Solids and liquids are not capable of being compressed as air is. By very great force some of them may be compressed to a small extent, but in all elementary operations they may be regarded as incompressible, so that 1 cub. cm. of a solid or liquid contains the same amount of material under the conditions of our present experiments. Solids, liquids, and gases all expand when heated, and their density then changes (Chap. XXVII).

Hence, neglecting for the present changes due to heating, we see that gases present a great contrast to liquids and solids. The density of a gas increases or diminishes according to the pressure to which it is subjected; and Boyle's law may be expressed thus: **the density of a gas is proportional to its pressure**.

103. UPTHRUST OF AIR. — We are unable to make simple experiments, such as those in Chap. VII, to see whether the air exercises an upthrust on bodies immersed in it. But several commonly known facts show that it does so.

Balloons are made of thin material, filled with a gas that is lighter than air. They ascend, and can even carry weights. The air does then exercise an upthrust. Knowing the volume of a balloon and the mass of a cubic centimetre of air, we can calculate the weight of the air it displaces; and it is found that in order that

a balloon may ascend, the weight of the balloon and its load must not be greater than the weight of the air it displaces, which is what was found to be the case with water.

A light balloon goes up high, but does not float on the top of the atmosphere, as wood floats on water. The atmosphere has no definite upper surface; it merely gets less and less dense at greater and greater heights. The increase of density downwards is produced by the pressure of the upper layers on the lower layers. Air being very easily compressible, this pressure has a great effect on it, so that the air near the surface of the earth is much denser than that a few thousand feet up. Fig. 66 gives an idea of the effect. A balloon floats like the egg in that picture at a level where the air is just so dense that the mass of the balloon is equal to the mass of the air it displaces.

Soap bubbles blown with hydrogen ascend like balloons.

The smoke of chimneys ascends for the same reason. Smoke is hot soot-laden air, which being less dense than cold air is forced upward by the upthrust of the heavier cold air around it.

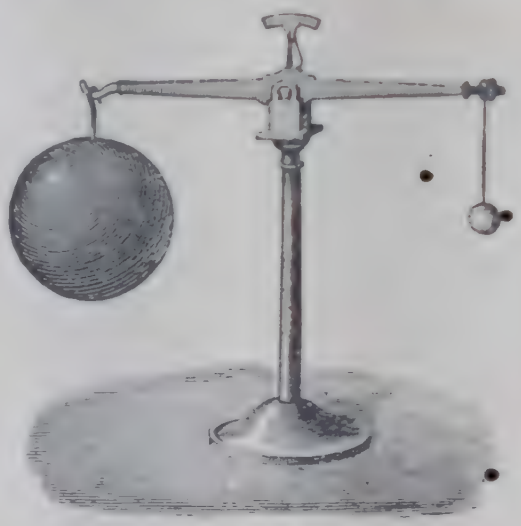

Fig. 117 —The Baroscope

All bodies weigh less in air than in a vacuum owing to this upthrust. This may be proved by means of the apparatus shown in fig. 117, called a baroscope. A small balance has a large globe made of light material on one arm and a small volume of heavy material on the other arm. In air the masses balance.

The baroscope is put under the receiver of an air pump and some air pumped out. The larger body at once begins to sink, and as the pumping proceeds sinks gradually lower.

If the large sphere exactly balanced the small weight when in a vacuum, then in air the larger body would weigh less, exactly as a glass stopper was found to weigh less in water than in air.

104. DENSITIES OF LIQUIDS.—On the above principles are based methods of determining densities of liquids.

Ex. 174.—Fix a U tube upright on a stand. Such a tube may be made by

joining two straight glass tubes by indiarubber tube. It is convenient to have a centimetre scale between the vertical arms. Pour in water until it stands not quite halfway up the tubes. With a pipette or thistle funnel gently pour some oil into one arm. Then the liquids stand as A and B in fig. 118, where C is the surface of the liquids in contact.

Mark the horizontal line CD. Measure the heights DE, CF.

Then if A is water the pressure of the column of water of height DE is equal to the pressure of the column of liquid of height CF.

$$\therefore \text{ Density of liquid } = \frac{DE}{CF}.$$

Add more liquid and make a second determination, and a third.

Ex. 175.—If the two liquids are such as mix with each other, first put mercury in the bend DGC, then put the liquids one in each arm in such quantities that the two mercury surfaces are on the horizontal line CD. The measurement and calculation are then the same as above. By this means find the density of a solution of salt or copper sulphate in water. Make three determinations with the columns of different heights and take the mean of the values obtained for the density of the solution.

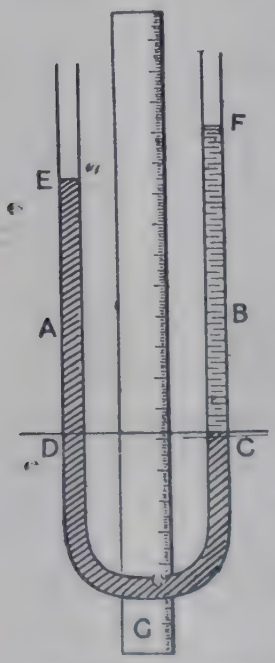

Fig. 118.—Determination of the Density of a Liquid by the U Tube

Ex. 176.—Proceeding in the same way, find the density of the foregoing liquids by a tube in the shape of a V. Not the slanting lengths but the vertical heights of the liquid columns must be measured. Why?

Ex. 177.—Sometimes it is convenient to use a tube in the form of a W.

Join together two U tubes with unequal arms, as in fig. 119, by a piece of indiarubber tube G. Pour in the water A and the liquid B very gently, so that none goes up into the bend G. They take up levels, as shown in fig. 119. Then the upward pressure at D is that produced by the atmosphere + the liquid column D′E: the upward pressure at C is that produced by the atmosphere + the liquid column C′F: and these upward pressures are equal to each other because they are each equal to the pressure of the air confined in the bend. Thus the relative density of liquid B (if A is water) is as before $\frac{D'E}{C'F}$.

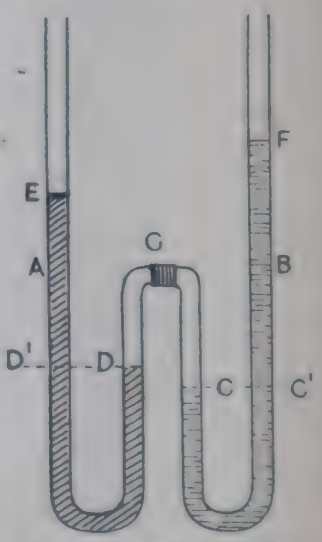

Fig. 119.—The W Tube

The apparatus shown in fig. 120 may be made from two straight glass tubes and a ⊥ tube connected by pieces of indiarubber tube. It is an inverted ∩ tube, and when used to determine densities is sometimes called Hare's apparatus.

Ex. 178.—Fix the two vertical tubes upright, standing one in water the other in the liquid whose density is to be determined.

Suck out some air through the ⊥ piece and then close this tube by a clip.

Why do the columns of liquid rise in the tubes? What is the pressure at the top of the liquid columns? What at the bottom?

The same considerations as those pointed out above show that the densities of the liquids are inversely proportional to the heights of the liquid columns. Note that the height of each column is to be measured from the surface of the liquid in its vessel.

Fig. 120.—Hare's Apparatus

Ex. 179.—Miscellaneous Exercises.

(1) From the work of previous exercises write down a list of all the methods of determining the density of (*a*) solids, (*b*) liquids, (*c*) gases.

(2) With the help of the table of densities on p. 408 calculate the height of a standard barometer in which the liquid was (*a*) water, (*b*) glycerine, (*c*) petroleum.

(3) Calculate the density of each of the following mixtures:—

10 cub. cm. of alcohol and 20 cub. cm. of water.
15 cub. cm. of turpentine and 25 cub. cm. of water.
10 grm. of alcohol and 20 grm. of water.
15 grm. of turpentine and 25 grm. of water.
20 grm. of silver and 20 grm. of gold.

(4) How far must a diving bell be sunk in water in order that the air inside it may be doubled in density?

(5) What will be the change in the volume of 100 cub. cm. of air when the pressure on it is changed from 1030 to 900 grm. per square centimetre?

A quantity of air occupying 120 cub. cm. at ordinary atmospheric pressure is compressed into a space of 50 cub. cm. What is its pressure?

(6) On a day when the mercury in a barometer tube 80 cm. high stands at 760 mm., some air is admitted above the mercury, which falls to 700 mm. What would be the true barometric height on another day when this barometer stands at 650 mm.?

(7) Calculate the weight of 200 cub. cm. of air which is under a pressure of 200 mm. of mercury.

PART III.—MECHANICS

CHAPTER XI

Matter and Force

105. All the different kinds of stuff that we meet with are included in one general term **matter**. Everything that can be seen or touched or tasted or smelled or weighed or measured is matter. So far as we have yet seen, the one property common to all forms of matter is *extension*; matter in any form occupies space. But this fact does not furnish a convenient definition.

We have seen that matter is met with in three different physical states—solid, liquid, gas. Water is commonly known in all three states—as ice, water, and steam, which are all the same in material but differ in state. Air is known to most of us only in the form of gas, but air has been liquefied, and solidified. Iron is known to most of us only as a solid, but it has been liquefied, and vaporized (turned into a gas).

In fact, most of the substances we know can be changed from one state to another by suitable means, usually heat or cold, combined with increase or diminution of pressure.

There are other properties of matter which demand attention.

106. Hardness is a property possessed by solids only, and by them in different degrees. The diamond, glass, and some kinds of steel are very hard; gold and lead are comparatively soft. The test for hardness is the resistance which a body offers to being scratched. Hardness cannot be definitely measured, but a scale has been formed consisting of ten minerals of different degrees of hardness by means of which the relative hardness of solids may be compared. The ten minerals are arranged in a numbered series from (1) the softest (talc) to (10) the hardest (diamond). Each one

scratches all that come before it on the list, and can be scratched by all that follow it.

Hence the degree of hardness of any substance is determined by finding out the highest numbered mineral that it will scratch and the lowest numbered mineral by which it can be scratched.

Ex. 180.—By scratching one with the other, arrange the following substances in order of hardness:—Tin, lead, zinc, the blade of a penknife, copper, pieces of glazed and unglazed crockery, flint, a piece of common window glass, any available minerals.

107. MALLEABILITY.

Ex. 181.—Hammer out a piece of copper wire and a small piece of lead into as thin a sheet as possible by even blows on a hard surface. Which substance can be worked into the thinnest plate?

Malleability is the property by virtue of which some cold solids can be hammered or rolled out into thin sheets. Such solids yield to the action of pressure by changing their form, at the same time retaining their continuity. The following is the order of malleability for some of the metals: gold, silver, copper, tin, platinum, lead, zinc, iron. The gold leaf used in gilding is extremely thin (·00001 cm.); copper-foil, tinfoil, platinum-foil are much used. Lead is beaten out by plumbers into very varied shapes. Iron when cold is only slightly malleable.

The malleability of substances is greatly affected by heat, e.g. hot iron is malleable. Metals hammered out in this way generally become more dense; their particles besides being forced over one another are forced closer together. They become more brittle in the process, but they recover their malleability by annealing (Art. 179).

Some solids possess no malleability. If a piece of glass or brick be struck it breaks up rather than changes its form. Such substances are **friable** or **brittle**.

108. PLASTICITY.—A plastic body is one which is capable of being moulded. Clay and wax are common examples of plastic substances. Under pressure a plastic body does not break but yields, at the same time preserving its continuity. Plasticity involves both malleability and ductility.

Most solids have a certain amount of plasticity. Thus many metals when taken in the form of a rod may be bent into various shapes and do not spring back. The yielding of some cold bodies occupies time—a steady pressure acting for a long time produces change of shape in some bodies which would be smashed by a blow

Many metals become very plastic when red hot, as also does glass. Two pieces can then be pressed together into one.

109. DUCTILITY is the property by virtue of which substances can be drawn into fine wires; it involves both plasticity and tenacity, i.e. the power of resisting a pull (Art. 119).

Wires are made by drawing rods of metal through holes in a steel plate. The process consists of several stages, the wire being drawn successively through smaller and smaller holes.

Silver, gold, and platinum are the most ductile metals. Warm wax and hot glass are also ductile.

110. POROSITY.

Ex. 182.—(1) Weigh a piece of brick; soak it in water; dry the outer surface; weigh again. What is the result?

(2) Make a cone of filter paper and place it in a funnel; put some muddy water in the cone. What passes through the paper?

(3) Place an empty dry pot of unglazed earthenware and a glass cylinder mouth upwards, in water for several minutes. Note the difference in the behaviour of these substances.

In these experiments water has passed through or into a solid substance, which appeared to be continuous. But through glass water will not pass.

These results are not due to some special property of water.

Fig. 121 shows a cup of cane or wood fitted on to the top of the receiver of an air pump. The cup is filled with mercury. On exhausting the receiver the mercury falls in a fine shower into the beaker placed to receive it. What forces it through?

Fig. 121.—Mercury Percolating through Cane

Marble allows oil to pass into or through it, and some glazed pottery is penetrated by a strong solution of bichromate of potash.

The passage of one substance into another indicates that there are small channels between the particles of most solids. They are called **pores**. Some pores are large enough to be seen, and are called **sensible** pores. But by pressure water has been forced through lead and iron in which no pores are visible. Such very minute channels are called **physical** pores.

Liquids are also porous, as may be shown by the following experiment.

Ex. 183.—Place 10 or 20 cub. cm. of water in a burette; add an equal measured volume of alcohol and stir thoroughly. Read the volume of the mixture. Is it equal to the sum of the two separate volumes?

The diminution of volume shows that there has been penetration of the particles of one liquid into spaces between the particles of the other.

These experiments show that minute spaces exist between the parts of bodies which appear to us to be quite continuous.

III. DIVISIBILITY OF MATTER. — Ordinary experience shows that every piece of matter we meet with can be divided into smaller pieces. We generally deal with masses of considerable size. Very small pieces are called **particles**.

Ex. 184.—Pound a piece of brick to fragments. Examine the fragments: some are fairly large, some are dust. Continue pounding: the longer you keep on the smaller will the pieces be.

Shoot the fragments into a beaker almost full of water, and watch the result. The largest pieces fall quickest; smaller ones slower. But in time if the vessel be not shaken all the fragments will reach the bottom. The particles that remained suspended for some time are very small.

Ex. 185.—Examine with a magnifying glass the dust on a table. Some of the pieces are too small to be seen with the naked eye. Powerful microscopes show still smaller particles.

Ex. 186.—Drop a crystal of permanganate of potash into a test tube containing water. The crystal dissolves, that is divides up into very small particles which travel through the water and colour the whole mass. No microscope will show these particles. Heat the tube until the water has evaporated. The solid substance remains deposited on the glass.

A piece of sugar by a similar process sweetens a large quantity of water. There are substances which, if 1 grm. be dissolved in 70000 grm. of water, can be detected by tasting a very small fragment of a gramme.

When one scents an odour it is by means of very small particles that reach the nose. A grain of musk will scent a very large volume of air, showing that the grain must have divided itself up into an immense number of particles.

The above and similar experiences show that matter is extremely divisible. But there are other experiences which lead to the conclusion that there is for every substance a certain size of particle which is the smallest that can exist. These smallest possible particles are called **molecules**. The different kinds of substances—iron, water, &c.—are different in density, because the molecules are

different. The molecules of different substances are built up of still smaller particles called **atoms**. If the molecules of a substance are broken up into atoms the substance ceases to exist: other substances are formed. This takes place in electrolysis (Art. 374).

112 MOTION OF MOLECULES.—Experiments with liquids and gases such as those described above and in Art. 82, and many others of a like character, also show that the molecules of liquids and gases are always moving about. In gases this movement is very rapid. A flask of gas must be thought of as consisting of millions of molecules flying rapidly about in all directions. They hit the sides of the flask and rebound. It is this motion which produces the pressure of a gas.

Force

113 It is not easy to grasp the full scientific meaning of the term force. The following illustrations, and similar suitable experiments, if thought over, will help to form a working idea.

(1) If two persons pull at the ends of a rope in opposition to each other they are each exercising force. One person may pull the other along, or, as sometimes happens in such a "tug of war", for a time no motion may take place. But the exercise of force takes place in either case, and there is a characteristic sensation aroused by its exercise.

The force in action is **muscular force**, which most animals possess the capacity of exercising.

(2) Lift up a heavy mass from the floor and hold it suspended. You have the same sensation of exercising force, and of force being exercised against you. The muscular force is opposed by a non-personal force called the **force of gravitation**, which the earth exerts on the mass of matter.

(3) Pull off the keeper from a horseshoe permanent magnet (Art. 306), or an excited electromagnet (Art. 307). In this case muscular force is acting against an opposing force called **magnetic force**. Pieces of iron or steel, when in the special condition which is described by saying that they are magnetized, attract or pull each other.

(4) Pull or push a large mass along a table. In this case the opposing force is caused by **friction** between the body and the table due to the roughness of the surfaces. When an engine pulls a train along at a steady speed, or a horse draws a cart, the opposing force is of this nature.

(5) Pull steadily a piece of thread until it breaks; pull a string fixed to a peg; push against a solid wall. In these cases the opposing force is that due to **cohesion** between the particles of the substance against which the muscular force is exercised. It is the force of cohesion that holds solids and to some extent liquids together. The particles of gases show no cohesion under ordinary conditions.

(6) Suspend a heavy mass by a long string. Push the mass horizontally with

the finger. Again resistance is felt. The opposing force is that due to the **inertia** of the body moved.

(7) Pull or push against a spring; press a ball inflated with air. The opposing force is that due to the **elasticity** of the substance.

These illustrations of the different agencies that manifest force are those with which we are here mainly concerned, and from them some idea may be formed of the meaning of the term *force* in physical science.

(*a*) Force is not a *thing*. The muscles of animals can exercise it. But there are many other means by which it is exercised—gravitation, cohesion, friction, magnetic action, &c. These are sometimes called **natural forces** or **forces of nature.**

(*b*) In each of the foregoing experiments, when you were exercising muscular force on a mass of matter, another force was being exercised against you. Forces always act in pairs. When their directions are away from each other, thus ←——→, they produce a **tension**, as in the case when a string is pulled at the two ends. When their directions are towards each other, thus —→ ←—, a **pressure** is produced, as when a book lies on a table, or as was the case in many of the experiments in hydrostatics.

(*c*) In some of the above experiments motion resulted from the action of the forces.

In other experiments there was no visible motion of the whole body, but in those cases the effect of the action of the forces was to produce a change in the shape or volume of the body acted on. Such a change is called a **strain** or **deformation**, and the special term **stress** is often applied to forces that produce a strain.

In some of the experiments there was during part of the time a strain, and during part of the time motion.

Thus we see that there are two effects—motion and strain—that may be brought about by force, and *nothing else but force can produce these two effects.* Hence force has been defined as "muscular exertion, and whatever else is capable of producing the same effects".[1] It is more usually defined as *that which produces or tends to produce motion of matter.* It might also be defined as that which by its action on bodies produces a strain.

That part of Mechanics which deals with motion as the result of force is called **Dynamics**: that part which deals with forces which produce strains only is called **Statics**.

[1] Lodge, *Mechanics.*

114. UNIT OF FORCE.—Just as the measurement of volume, mass, &c., consists in choosing a certain volume, mass, &c., as a unit, and comparing other volumes, masses, &c., with that unit, so we have to choose some amount of force, call it the **unit of force**, and measure other forces by comparison with that unit.

Now the commonest force that we are concerned with is the force of gravitation, and it has been agreed that for practical purposes the unit of force shall be the pull which the earth exercises on a cubic centimetre of water; that is, the **weight of one gramme**. This is called the **gravitation unit of force**.

Ex. 187.—Hold up a gramme weight by a thread, and feel the pull of the earth on it. That pull is the unit of force. Hold up a 100-grm. weight in a similar way. The force which you are now exerting is 100 units, 100 grm. weight. In whatever way the force is produced it is of the same character, and is measured in the same way.

115. MEASUREMENT OF FORCE.—An instrument used to measure forces is called a **dynamometer**. Such an instrument—the spring balance—was employed in Chap. III to compare masses, because the only simple way open to us to compare masses was to compare their weights, i.e. the force of attraction between them and the earth.

Ex. 188.—To graduate a dynamometer.

A useful form for preliminary work may be made from a piece of indiarubber cord. Fig. 122 shows one that is easily made. The cord is fixed at one end, either to a board or by a loop on a hook. A scale pan is hung to a loop on the other end of the cord. Behind and parallel to the cord is fixed a millimetre scale. Two points are marked, one on each end of the cord, either by tying a thread loosely round it or by sticking a fine needle into it at those points.

The distance between these marks changes when weights are placed in the pan.

(1) Ensure that the cord is quite straight. If the weight of the pan is insufficient to straighten it, place some shot or dry sand in the pan.

Record the positions of both marked points on the millimetre scale. The distance between these marks is the piece of the cord that we are concerned with.

Weigh the pan and its contents on a balance. Record the weight. In using the dynamometer afterwards the pan must always weigh this amount.

Fig. 122.—A Cord Dynamometer

Place weights on the pan sufficient to stretch it a few millimetres. Record the weights thus added and the length of the cord between the points.

Add more weights, and again record the weight and the length.

Continue thus to add weights and to measure length step by step. It is better to add the same weight (5, 10, or 20 grammes, according to the thickness of the cord) each time. Do not proceed far enough to injure the cord.

Take off the weights in successive steps as before, and take the readings backward. To get the extension that corresponds to any weight take the mean of the ascending and corresponding descending reading. If there is much difference you have strained the indiarubber, and rendered it for the time at any rate useless.

Set out the readings of the weights, and the amounts by which the cord was stretched, as in the following table, which is a set of observations:—

Tension of cord, in grammes	12·4	22·4	32·4	42·4	52·4	62·4	72·4
Extension of cord, in millimetres...	9·5	19	29	40	51·5	64	79·5

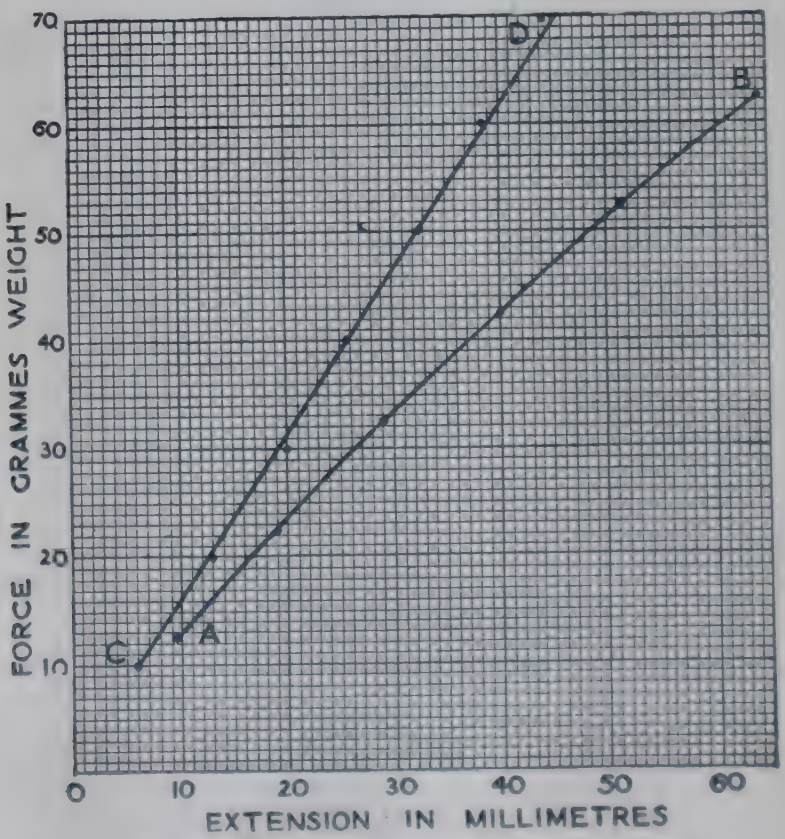

Fig. 123.—Calibration Curves of Dynamometers

These are set out in a graph (fig. 123) and give the line AB.

The fact that the line is not quite straight shows that the extension of the cord was not exactly proportional to the tension of the cord.

(2) Set your results out on a similar graph.

Such a curve is called a **calibration curve**, and when it is once

made it may be used to measure any force (within the limits of the curve) that may be applied to the dynamometer. Thus, referring to the graph AB above, suppose the cord to be pulled until its length has increased by 60 mm. The curve crosses the 60-mm. vertical line at the point corresponding to 59·5 grm., which is therefore the force applied.

What force would extend this cord by 35 mm.? by 45 mm.?

116. THE SPRING BALANCE.—A more reliable dynamometer may be made from a steel spring, mounted, as shown in figs. 124 and 125.

Ex. 189.—Make a set of observations on a spring. The process of graduating it and the method of using it are precisely the same as with the cord dynamometer in the preceding exercise.

Fig. 124 Fig. 125

Spring Balances (Rintoul's form)

The following is a set of observations taken on a spring:—

Tension on spring, in grammes weight	10	20	30	40	50	60	70	90
Extension of spring, in millimetres	6	13	20	25·5	32·5	38·5	44·2	57·5

These are also set out in fig. 123, and give the line CD which is practically straight, showing that the extension of the spring was proportional to the force applied.

What is the force that would extend the above spring by 60 mm.?

Forces that extend the spring of the dynamometer by an equal amount are equal, whatever the agency that produces them. The reading of the dynamometer tells us to how many grammes weight the force is equivalent.

We proceed to a further acquaintance with the forces distinguished in Art. 113

117. THE FORCE OF GRAVITATION.—The earth pulls all objects towards itself, just as the magnet pulled the soft iron. It is this pull which causes bodies to have weight, which keeps bodies down on or close to the earth, which holds the sea in its bed and the rivers in theirs, which holds the atmosphere in its place, which brings down the rain, which holds the moon in its course.

The force of gravitation, the cause of which is unknown, is not only exercised by the earth, but also by all the ordinary objects which we daily meet with. Every piece of matter whether large or small attracts and is attracted by every other piece of matter. There is a pull between them of precisely the same kind as that between the earth and a lump of iron. The lump of iron pulls the earth just as the earth pulls it, and pulls every other object round it in the same way. The strength of the pull depends on the quantity of matter that the objects contain and on their distance apart. The pull between the earth and the moon is very strong, because the earth and moon are both very massive bodies. The pull between the earth and a cricket ball is much less, because the mass of the ball is less than the mass of the moon. The pull between two lumps of iron standing side by side is very much smaller still, because both the bodies are of small mass. On the other hand, the pull between the sun and the earth is very great in spite of their long distance apart, because the sun is a body of immense size and mass, and the earth is large.

The pull exerted by the earth on a mass of matter is not quite the same all over its surface. A 1000-grm. weight is a little lighter at the equator than it is in London. If a spring balance were loaded in London with such a mass, and a mark made on the scale; and if the same spring balance and the same mass were carried to the equator, it would require the addition of 3·15 grm. to bring the spring down to the same mark. The quantity of stuff—the mass—contained in the lump of metal has not changed. But the pull of the earth is not the same, and the dynamometer registers this pull. Hence it is quite clear that the mass of a body is not the same as its weight. The reason for this difference is that at the equator the surface of the earth is farther away from its centre than the surface at London, the earth being not quite a sphere.

In all previous experiments in which a spring balance was used for weighing, the gravitation force between the earth and the body "weighed" was being measured.

118. FORCE OF MAGNETIC ATTRACTION

Ex. 190.—Measure in grammes weight the force of attraction between a magnet and its keeper.

Attach a dynamometer to the keeper of a magnet (fig. 126). Fix the magnet firmly, and see that the line of pull of the dynamometer is at right angles to the end faces of the magnet. Pull steadily until the keeper leaves the magnet. Take several readings of the dynamometer, and find their mean.

Place a piece of paper between the magnet and the keeper, and again find the mean attractive force.

Increase the thickness of the paper layer, and repeat the observations.

Fig. 126.—Measurement of the Force of Magnetic Attraction

The readings show that the force of attraction between a magnet and its keeper diminishes as the distance between them increases.

The gravitation force of attraction between the earth and pieces of matter diminishes in the same way when the distance between them increases. This is the reason why a body does not weigh the same at places on the earth's surface which are at different distances from its centre.

119. FORCE OF COHESION.—It is the force of cohesion that holds bodies together When the molecules are very close to each other they attract one another. When a solid is ground into smaller pieces, the force of cohesion between these pieces has been overcome and the pieces separated. It still holds together the particles that make up the separate pieces. On the other hand, some substances in a state of fine powder, such as graphite, when compressed with great force become a solid mass that can be sawn up. If a flat clean surface be scraped on two pieces of lead, and the two faces be pressed hard together with a screwing motion, the two pieces cohere and become one.

The force of cohesion is different in amount in different bodies. A **tenacious** body is one in which it is very great.

Ex. 191.—To compare the tenacities of wires.

(1) Pass a piece of fine iron wire through a hole in a fixed peg, and wind the wire several times round the peg, or clamp the wire in such a way as to grip it tightly. Attach a spring balance to the other end of the wire, and gently increase the pull on the wire, taking readings of the dynamometer as the process proceeds. At a certain value of the stress, the force of cohesion is overcome, and the wire breaks. Note the force just before breakage occurs.

(2) The experiment may be done by utilizing the force of gravitation to overcome the force of cohesion. Hang the wire from a support, as in fig. 127; put a scale pan on the end of it; add weights in the pan until the wire breaks. The scale pan should be close to the floor, or a table. Note the breaking weight.

(3) The tenacities of different metals may be compared by finding the breaking stress of wires of the same diameter. Repeat the experiment with a piece of wire of a different material, such as copper, of the same diameter as the iron wire. Then the tenacities of the two wires are in the ratio of the breaking forces.

(4) The *measure* of the tenacity of a wire is the value of the breaking force divided by the sectional area of the wire. Find the diameters of the wires used, calculate their sectional area, and determine the value of the tenacity in grammes per square centimetre.

The cohesion of solids is greatly affected by heat.

Ex. 192.—Suspend a piece of wire of the same kind as that used in the last exercise. Stretch it by a weight less than the breaking weight. Heat the wire by a flame. It breaks, showing that the tenacity has been diminished.

Liquids also show cohesion.

Ex. 193.—Dip a glass rod into water, and on withdrawing it examine the formation of drops on the end of the rod. Repeat with a test-tube, in place of the rod. Draw the shapes of the drops at different stages of their formation.

120. ELASTICITY.—When force is applied to a body in such a way as to produce a strain (Art. 113), the body exercises an opposing force that resists change. Moreover, on the removal of the stress, many substances return to their original condition. This power of recovery is called **elasticity**.

The elasticity of air has been demonstrated in Chap. VIII, and that of indiarubber and of a steel spring in Arts. 115 and 116.

All bodies resist an effort to change their volume, i.e. possess *elasticity of volume*. *Elasticity of shape or figure* is possessed by solids only, many of which return completely or partially to their original shape when the stress that deforms them is removed.

If there were a solid which did not change its shape at all when force was applied to it, this body would be *perfectly rigid*. No solid is perfectly rigid: all are more or less so; comparatively, the less change they undergo under the same force, the more rigid they are.

Bodies which on the removal of the stress which is deforming them return completely to their original volume or shape are called *perfectly elastic*. With regard to elasticity of volume all gases and liquids are perfectly elastic. Many solids are perfectly elastic within certain limits, i.e. if the force applied to them be not too great the removal of the stress is followed by complete disappearance of the strain. Glass and steel are such bodies. But if the

force applied be greater than a certain amount called the **elastic limit** they do not recover.

Some solids, on the other hand, such as putty, clay, and dough do not recover their shape even when the strain is small: they are called **inelastic**.

Ex. 194. (1) Drop a glass or steel ball on a stone slab. Note the many rebounds, gradually diminishing in height.

Consider what took place at the moment of contact. What is the effect on a tennis ball during its contact with a wall against which it is thrown? A similar effect is produced on the glass ball at the moment of contact with the slab.

Drop a lead ball on the slab: examine the place of contact on the ball. Is it slightly flattened?

(2) Smear a very thin layer of paste on the slab: drop the glass ball from different heights, and examine the places of contact on the slab. Are the spots of different sizes?

These experiments show that the glass ball is flattened at the place of contact by the force of the blow, and that the rebound is produced by the instantaneous return of the deformed ball to its spherical shape.

This is the ordinary behaviour of an elastic solid when subjected to force. It is deformed by the force, but returns to its original form when the force is removed.

Ex. 195.—To compare the elasticity and ductility of wires. Make the following experiment with each of two wires of the same length and diameter, but of different materials. Copper and German silver are suitable. The length should be about 2 m.; a suitable diameter is ·3 to ·4 mm.

Fix one end of the wire securely to a firm hook or nail in the ceiling or wall of the room. Mark a point near the bottom of the wire. Attach a scale pan to the bottom of the wire. Fix a millimetre scale on or close beside the wire (fig. 127). Measure the length of wire down to the marked point.

Place weights (m grm.) in the pan sufficient to stretch the wire by about 1 mm.

Read very carefully the elongation and the weight. (The elongation is small, and may be measured more accurately if a vernier is fastened to the wire.)

Continue to add weights, m grm. at a time, and to measure the elongation at each stage until the wire breaks.

Make a graph showing the relation between the elongation and the stretching force.

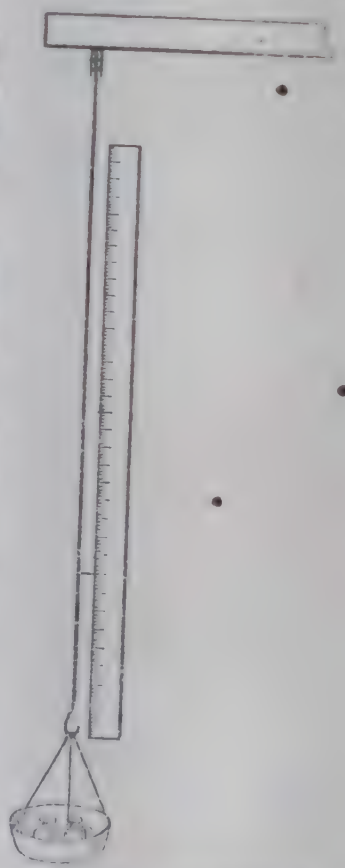

Fig. 127.—Comparison of the Elasticity and Ductility of Wires

Such a graph is shown in fig. 128. The graph is divided into two parts, OA and AB.

The part OA is practically straight, and the elongation is small. During this part of the experiment the stretching was within the elastic limits, and the graph shows that the elongation l was proportional to the stress F. This rule is called Hooke's Law. From the readings taken in this part of the curve, find the mean value of $F \div l$ for each wire. These numbers are the comparative measures of the elasticity.

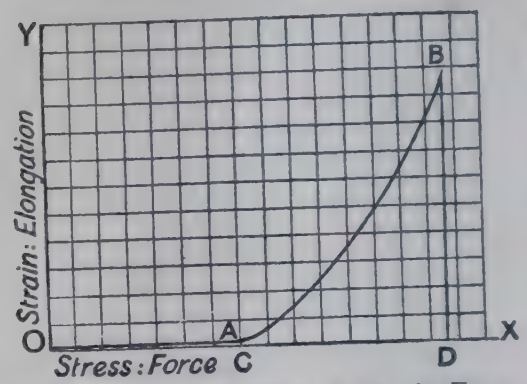

Fig. 128.—The Extension of a Wire by Force

The part AB of the curve rises rapidly. During this period of the experiment the elastic limit was exceeded; the wire was being drawn out, and ductility comes into play. The measure of the **ductility** is the ratio between OD, the stress required to break the wire, and OC, the stress when the wire began to draw. Find the value of OD ÷ OC.

The *brittleness* of the wire is measured by the ratio OC ÷ OD. Find the values of each of the above ratios for the two wires, and thus compare their elasticity, ductility, and brittleness.

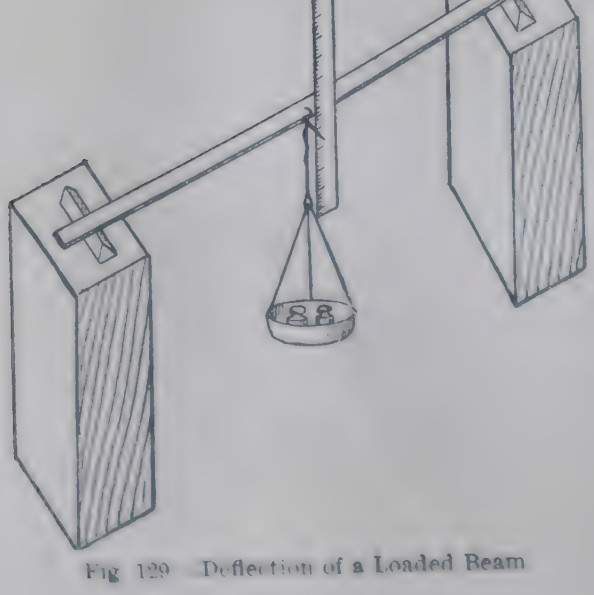
Fig. 129.—Deflection of a Loaded Beam

121. ELASTICITY OF BEAMS.

Ex. 196.—(1) Support a thin flexible lath, such as a metre scale, near each end on a knife edge (fig. 129). From its centre suspend a scale pan, and at the centre fix a needle (by wax), so that the needle point is near the graduation marks of a vertical scale.

Read the position (a) of the needle on the vertical scale.

Place a weight w in the pan sufficient to bend the lath, and read the position (b) of the needle.

Then the *deflection* of the beam produced by a weight w is the difference between a and b.

Take several observations

the same kind with different weights, and record weight and deflection in two columns. Do not load the bar too heavily.

(2) Make a graph showing how the deflection varies with the load. The graph is a straight line, showing that the deflection is directly proportional to the load. Compare with Exercise 195, on the elasticity of a wire.

Ex. 197.—Using the same apparatus, make another set of observations, keeping the load the same throughout. Observe the length of the beam, i.e. the distance between the knife-edges, and the deflection produced by the load.

Take a series of readings with different lengths of beam, recording the deflection and length of beam, and make a graph showing how the deflection varies with the length of span.

The result shows that a small difference in the span corresponds to a large difference in the deflection. No simple law can be obtained from these observations.

The amount of bending of a lath loaded at one end may be obtained by clamping one end and placing the load and scale at the other end.

The investigation of the elasticity of beams is of great value in connection with the building of houses and bridges, the beams and girders in which must never be strained beyond the limits of elasticity.

122. FRICTION.

Ex. 198.—(1) Upon an accurately planed and level board place a smooth block of wood having a dynamometer attached to it, as shown in fig. 130. Its mass may be adjusted to any desired value by placing a weight on the block.

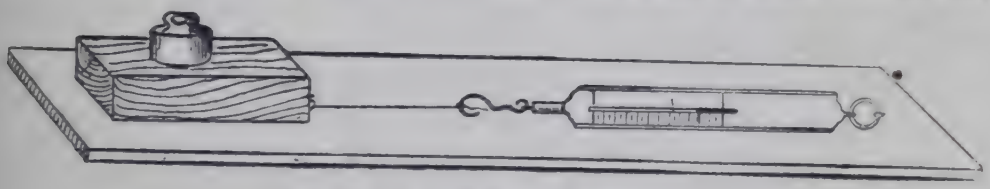

Fig. 130.—Measurement of Friction

Pull the dynamometer slowly until the block begins to move, and draw it along the board slowly and steadily. Note the dynamometer during the process. It shows that the force increases to a certain amount, and then diminishes. Record (*a*) the highest reading just as the mass begins to move, and (*b*) the reading when the block is in steady motion. Repeat several times, using the same face of the block and the same part of the board. Take the mean of the (*a*) readings and also of the (*b*) readings. The results show that the force of friction is greater before the body moves than when it is in motion.

The mean value of the (*a*) readings is the *limiting force of friction* for the substances used: it is the highest value that the friction can attain for those bodies. The mean value of the (*b*) readings is the value of the *sliding friction*.

(2) Repeat the observations on a sheet of smooth glazed paper and on a plate

of glass, finding for these substances the limiting force of friction and the value of the sliding friction.

These experiments show that friction is a surface effect varying with the nature of the surfaces in contact; that it gives rise to a force parallel to the surfaces in contact which opposes any motion of one surface along the other; that for any two surfaces it cannot exceed a certain value; and that it is greater when the surfaces are at rest than when one is moving over the other.

Ex. 199.—To find the effect of the area of the rubbing surfaces. Place a load upon the block, and by drawing it along the board several times, find the value of the sliding friction when its whole surface is rubbing.

Using the same load, allow the block to overlap the edge of the board, so that only part of its surface is rubbing. Find as before the mean value of the sliding friction.

A comparison of the values shows that the area of contact does not affect the amount of the friction.

Ex. 200.—To find how the force of friction depends on the pressure between the surfaces. Weigh the sliding block (W grm.). Taking the mean of several observations, find the value of the force of sliding friction (F grm.).

Find the ratio $\frac{F}{W} = \frac{\text{friction}}{\text{pressure}}$.

Place a weight upon the block so as to increase W, the total pressure between the surfaces. Again find the value of F, the sliding friction, as before, and the ratio $F \div W$.

Place in this way various weights on the block, and for each obtain the value of $F \div W$.

Make a table of the values and a graph.

The results may not be very concordant unless the work has been carefully done. But a straight line best represents the results, showing that the friction is proportional to the pressure. Find the mean value of $\frac{F}{W}$ given by the graph

This is the **coefficient of friction** for the two surfaces concerned.

Ex. 201.—To find the coefficient of friction by an inclined plane (fig. 162). Prop up one end of the board, until at a certain height the mass just begins to slide down the plane. Find the tangent of the angle which the plane makes with the horizontal when sliding begins (Art. 26).

Make several determinations of this tangent, and take their mean. The mean value is the coefficient of friction, and should be the same as that obtained in the last exercise, if the same board and block of wood have been used.

123. INTERNAL FRICTION OF A LIQUID.—It is sometimes difficult to decide whether a substance is a solid or a liquid

A tallow candle resting horizontally on two supports remains horizontal, but in time a stick of sealing wax bends. Pitch, which is very hard, flows under pressure. However hard a substance may be, if the application of a small force steadily for a long time causes it to flow, the substance must be classed as a fluid.

The different layers of a fluid can slide over one another just as one solid slides over another. In so doing they experience an opposing force due to friction. In some liquids, such as treacle, pitch, honey, this friction is great. When the surface of such liquids is disturbed from the horizontal, it is some time before horizontality is again attained. Such liquids are specially called **viscous**. But all fluids exhibit the property of **viscosity** more or less.

Ex. 202.—By means of a glass tube blow a small bubble of air to the bottom of a vessel containing water, and a vessel containing glycerine. Compare the rates at which the bubbles ascend. Glycerine is much more viscous than water.

The suspension of mud in water, and of the clouds in the air, is due to the viscosity of the water and the air. The small particles in suspension fall, but fall very slowly.

Ex. 203.—To compare the viscosities of liquids. By indiarubber tube attach to a pipette a long capillary tube. Place a measured quantity of water (20 or 50 cub. cm.) in the pipette, and put the capillary tube horizontal. Measure the time taken by the water to flow through the capillary tube.

Repeat the experiment in exactly the same manner with the same volume of alcohol, or a mixture of glycerine and water. Then the viscosities of the two liquids are proportional to the times of flow.

124. We are now in a position to take a wider view of the properties which are common to and the properties which distinguish the three states of matter. Matter may be defined as that which is capable of being moved by force. All the forms of matter possess mass, weight, extension, and divisibility. Solids do not flow, they are more or less rigid: liquids and gases flow, they are not rigid. In solids the force of cohesion is great, in liquids small, and gases have practically none. Solids are almost incompressible, liquids are only slightly compressible, gases are very easily compressible. Such qualities as hardness and brittleness belong to solids only.

CHAPTER XII

Relations between Forces in Equilibrium

125. TRANSMISSION OF FORCE.—When we pull a piece of thread until it breaks, the breakage often occurs at some point distant from the ends where the force is applied.

When we move a mass by pushing at it with a long stick, the force is applied at one end of the stick and its result appears at the other end.

This process is called the transmission of force. When a string is transmitting a force it is in tension, and the amount of tension means the pull along it measured in units of force. A rod may be in tension in the same way.

A rod can also transmit a pressure; a string cannot do so.

In Chap. X it was shown that one of the principal properties of fluids was the transmission of pressure in all directions equally, so that pressure applied to any part of it may produce motion or strain at any other part of the vessel or machine that contains it.

The place at which a force is considered as acting is called its **point of application**.

126. EQUILIBRIUM OF FORCES.—A body on which forces are acting, but which nevertheless does not move, is sometimes said to be in **equilibrium**. The term is also applied to forces. A body cannot remain at rest under the action of one force, but may do so under the action of two or more forces. If two or more forces act on the same body at the same time and keep the body at rest **the forces** are said to be **in equilibrium**.

127. DESCRIPTION OF A FORCE.—In order to avoid complications at this stage, the forces about to be considered are disposed so as to act on a very small body: they are considered then as acting at **a point**.

It is obvious that every pull or push must be in some definite direction. Thus to fully describe a force there must be stated **its amount, its direction, and its point of application.**

Forces may be represented on paper by straight lines, for a line can always be drawn in any given direction, starting from any given point, and its length may contain as many units as the force contained units of force.

This graphic method of dealing with forces is of great use.

128. FORCES ACTING IN ONE LINE.

Ex. 204.—Tie three strings together at a point (D in fig. 131), and attach the other ends of the strings to three spring balances.

Fix one spring balance C, and pull the other two so that the strings DA and DB are in the same straight line as CD. Hold A and B steadily or fix them, and read all the dynamometers.

Call the readings of A, B, and C respectively a, b, and c. Compare $a + b$ with c for several pulls of different values.

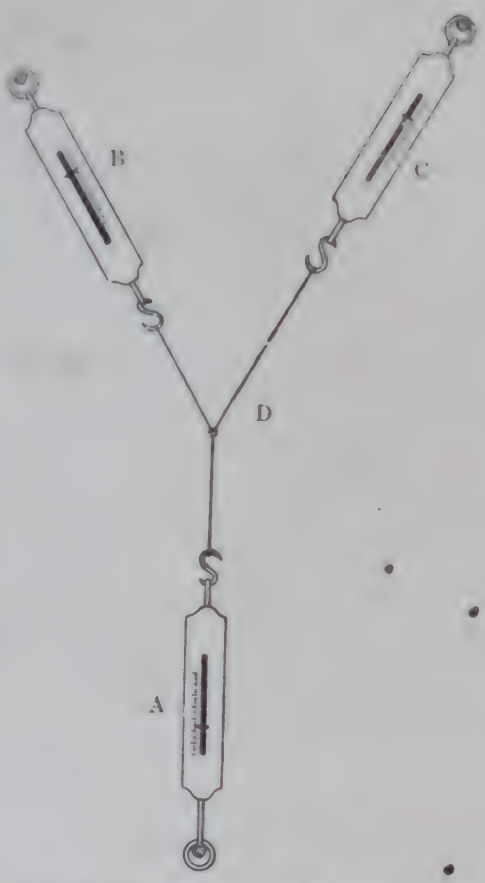

Fig. 131.—Three Forces in Equilibrium

The experiment shows that forces may be added or subtracted, and that when a number of forces acting in one straight line keep a point in equilibrium the sum of the forces acting in one direction is equal to the sum of those acting in the opposite direction.

129. THE PARALLELOGRAM OF FORCES.

Ex. 205.—Using the same apparatus as in the last exercise, make a small angle between the strings AD and BD. Take the readings, and compare $a + b$ with c.

Increase the angle ADB, and again compare $a + b$ with c.

Make the angle ADB = 2 right angles, and ADC = 1 right angle.

What is the reading c?

From this experiment we see that when three forces are acting on a point and maintain that point in equilibrium—

(a) If no two of the forces are in a straight line the sum of any two of them is greater than the third. (What geometrical proposition does this suggest?)

(b) The amounts of the forces depend on their inclination to each other.

Ex. 206.—(1) Using the same apparatus, fix a sheet of paper under the strings. Make the angle ADB (fig. 131) about 30°. Fix the spring balances; mark accurately on the paper the line of the three strings, and read the forces a, b, c.

Remove the paper, and complete the drawing as in fig. 132. From D along the line DA mark off a distance DE whose length is a units. Similarly, along DB

a length DF = b units; and along DC a length DH = c units. On the lines DE, DF thus obtained complete the parallelogram DFGE. Measure the diagonal DG, and compare the number of units of length it contains with the force c, represented by the line DH.

(2) Repeat the experiment for two or three different values of the angle EDF, for each one making a parallelogram as above, and comparing the length of the diagonal through D with the length of the line representing the third force.

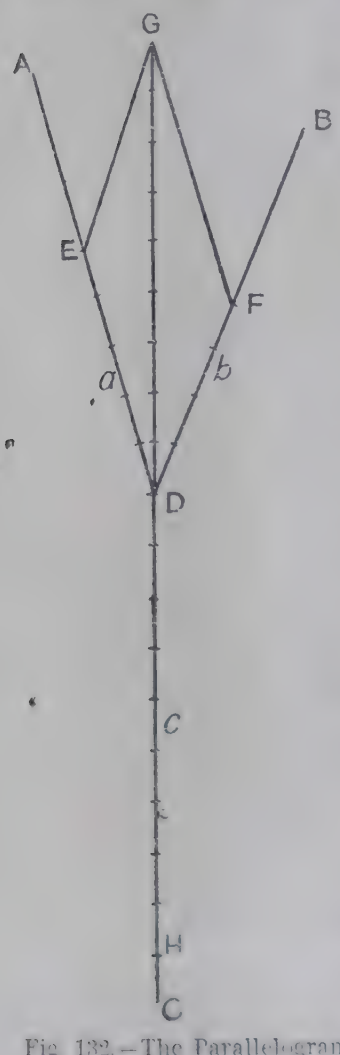

Fig. 132.—The Parallelogram of Forces

The results show that the diagonal of the parallelogram drawn as indicated is a continuation of the line of action of the third force; and contains the same number of units of length as the third force contains units of force.

Now, since the point D was kept in equilibrium by the three forces, a, b, and c, the combined effect of a and b must have been equal and opposite to c. This combined effect is called their **resultant**, the two forces a and b themselves being called the **components**. The resultant of a and b must therefore be equal and opposite to c. But the diagonal DG comes out in the same straight line as D and equal in length to DH. Therefore the diagonal DG represents the resultant of the two forces a and b.

This principle is called the **parallelogram of forces**, and may be stated in general language thus:

If two forces acting at a point be represented in magnitude and direction by two straight lines which form two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point.

This important experiment should be done in various ways. Fig. 133 shows three weights connected to strings, two of which pass over pulleys on a board. The three forces thus arranged are the three weights which are known. The board is placed vertically, and the line taken by the strings is marked as before. The point corresponding to D in the previous figure is the point where the lines of the strings meet.

Ex. 207.—Demonstrate the parallelogram of forces by means of the apparatus shown in fig. 133. Tap the board to ensure that the pulleys do not stick. If scale pans are used to hold the weights, the weight of the pans must be included in the forces.

Mark the lines of the strings or measure the angles between them by a protractor, draw a large figure like fig. 132, and measure the lengths.

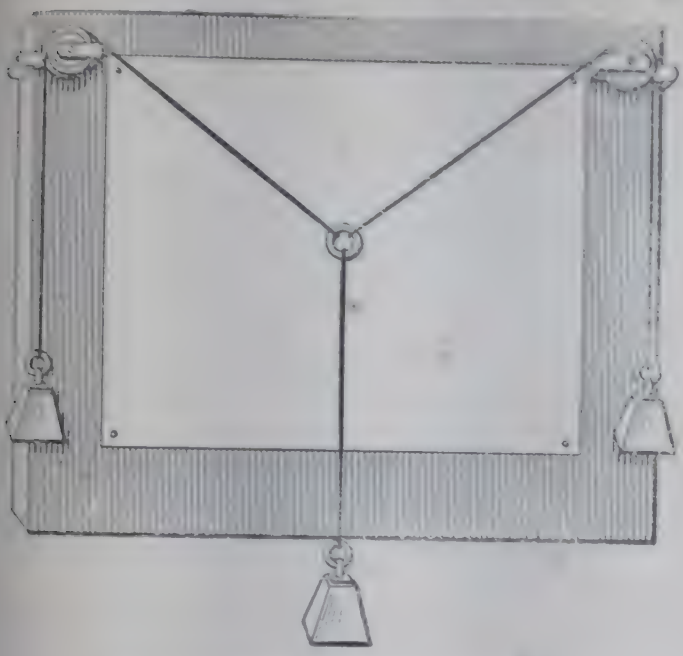

Fig. 133.—Three Forces in Equilibrium

Ex. 208.—(1) Find the resultant of the following pairs of forces:—

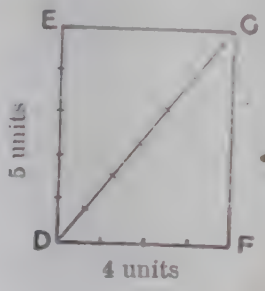

Fig. 134.

(i) 4 units and 5 units acting at an angle of 90° (fig. 134).

The answer may also be obtained by calculation—

$$(DG)^2 = 4^2 + 5^2 = 41. \quad \therefore DG = 6{\cdot}4.$$

(ii) 45 grm. and 90 grm. at right angles.

(iii) 4 units and 5 units acting at an angle of 60°. Measure the angles accurately with a protractor. Make a large figure.

(iv) 90 grm. and 90 grm. acting at an angle of 120° with each other.

(2) Is it possible for a point to remain at rest under the action of—

(*a*) Three forces of 8 grm., 13 grm., 5 grm.?

(*b*) Three forces of 8 grm., 13 grm., 4 grm.?

(3) A picture weighing 3 Kg. is hung by a single string passing over a nail. The two parts of the string make an angle of 60° with each other. Find the tension in the string.

[The resultant is 3000 grm.: the two equal components act along the strings.]

(4) The resultant of two forces is 6 lb., and the sum of the two forces is 9 lb. The resultant is at right angles to the smaller force. Find the components.

[Draw a figure. If x and y are the two components, then $x + y = 9$ and $x^2 + 6^2 = y^2$.]

130. TRIANGLE OF FORCES

Ex. 209.—(1) Set up the apparatus shown in fig. 131 or fig. 133. Obtain

equilibrium of the three forces, and record their values. Read the three angles, or mark the direction of the strings accurately on the paper.

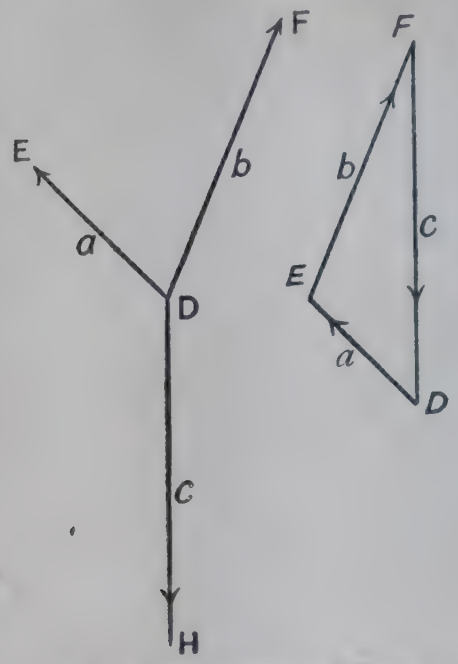

Fig. 135.—The Triangle of Forces

Set out the observations on paper, obtaining the three lines DE, DF, DH on the left-hand side of fig. 135, each in the right direction and of the right length. Mark the directions in which the forces were acting by arrows.

At the side of this figure draw a triangle (fig. 135), proceeding in this way:—Fix a point *D*. Draw *DE* parallel to DE, and in the direction in which the force *a* was acting, as shown by the arrow. Cut off *DE* of length *a* units. This fixes the point *E*. From *E* similarly draw *EF* parallel to DF, of length representing *b*, and in the direction in which *b* was acting. Thus the point *F* is determined. From *F* draw a line *FD* parallel to DH in the direction in which the force *c* was acting, and containing *c* units of length. Then if the whole of the work has been accurately done, this line closes the triangle DEF.

(2) Repeat the experiment with different angles and forces. For each experiment make the above construction. It is an excellent exercise in accurate measurement.

Ex. 210.—On paper draw a triangle *FED* of any shape. Measure the lengths of the sides *a*, *b*, *c*. Arrange three spring balances as in Exercise 205, to give three pulls of respective values *a*, *b*, *c*. Mark the directions of the strings, and make a triangle as in the last exercise.

Measure the angles of this second triangle, and compare them with those of the triangle *FED* with which you started.

Exercises 209 and 210 demonstrate the principle of the **triangle of forces**, which may be thus stated:—

When three forces acting at a point are in equilibrium they can be represented in magnitude and direction by the sides of a triangle taken in order; or, When three forces acting at a point can be represented by the sides of a triangle taken in order the forces are in equilibrium.

Ex. 211.—(1) Suspend a mass of 100 grm. from a dynamometer attached to a vertical board.

Attach another dynamometer to the mass and pull it horizontally along the board.

Read the dynamometers, and mark the angles between the forces. From the readings verify the triangle of forces.

(2) If a mass of 40 grm. is suspended by a string, what horizontal force applied to the mass will cause the string to make an angle of 30° with the vertical?

(3) Find the resultant of two forces of 45 units and 25 units acting at an angle of 60° with each other.

131. POLYGON OF FORCES.—The methods and principles enunciated above may be applied to any number of forces.

Ex. 212.—By means of strings passing over pulleys, as in fig. 133, or by spring balances, as in fig. 131, find the relation between five forces which act at a point and keep that point in equilibrium. Having recorded the forces and the angles between the springs proceed as follows:—

Example. — Forces of 120 grm., 160 grm., 200 grm., 100 grm., and an unknown x grm., acting at angles shown in the upper part of fig. 136, kept the point A in equilibrium. Find the resultant x.

The line AF is produced to Q, and in it a point a is taken.

The line ab, making an angle of 105° with AQ, is parallel to AB; the length ab is taken = 120 units.

From b similarly bc is drawn parallel to AC, and of length = 160 units.

In the same way from c, the line cd is drawn, representing the force D; and from d the line de, representing the force E.

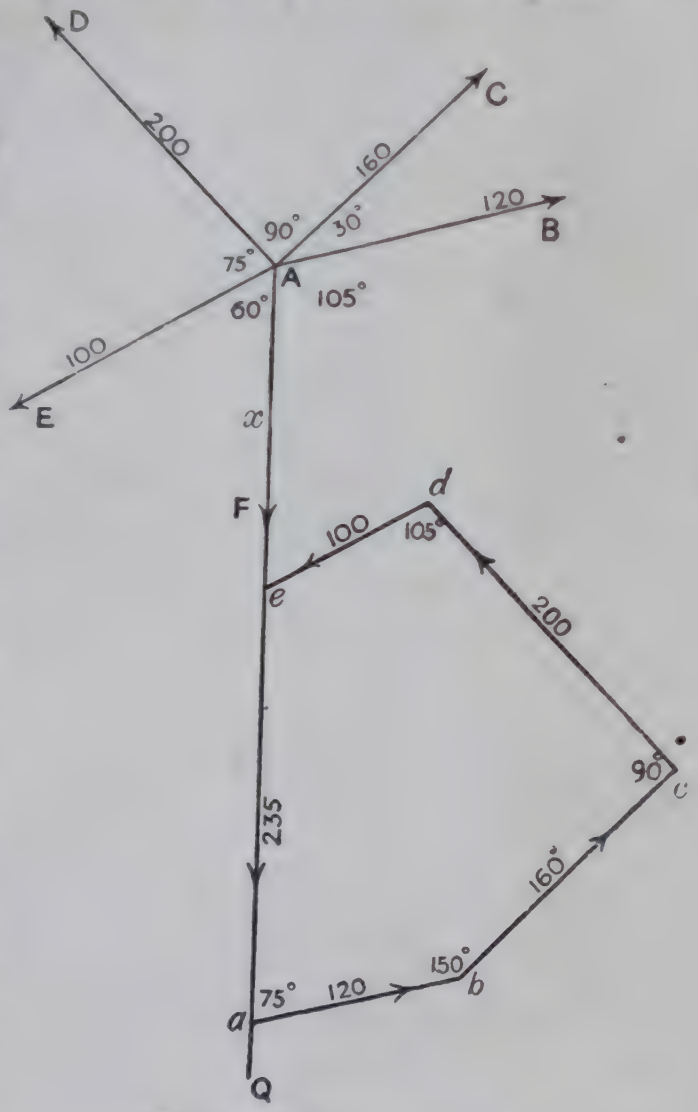

Fig. 136.—The Polygon of Forces

The point e comes on the line AQ, thus making a closed figure.

The length ea corresponds to 235 units, which is the value of the resultant force x.

The principle of the polygon of forces is thus stated:—

If any number of forces act at a point so as to keep that point in equilibrium, they may be represented in magnitude and direction by the sides of a polygon taken in order.

Ex. 213.—Forces of 3, 4, and 5 Kg. act on a point: the angles between them lines of direction are each 120°, and all the forces act away from the point.

Find the direction and magnitude of the fourth force, which is necessary to keep the point at rest.

132. PARALLEL FORCES.

Ex. 214.—(1) By means of a thread passing over a pulley suspend a uniform bar, such as a metre scale, from its central point. Counterpoise the weight of the bar as shown in fig. 137, and adjust it so that it stands horizontally.

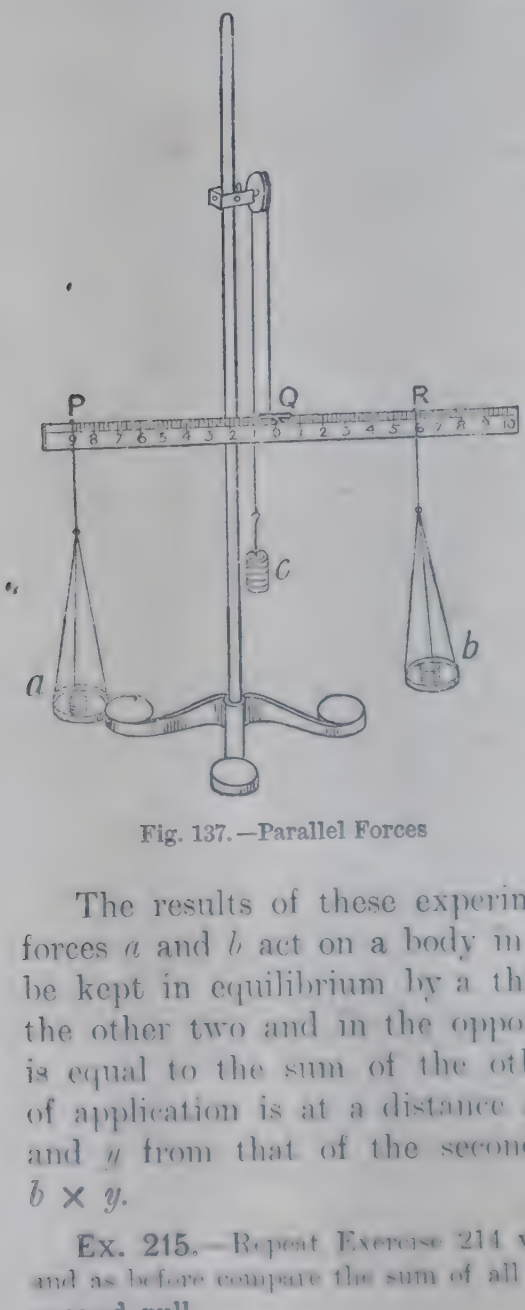

Fig. 137.—Parallel Forces

On the bar hang two weights a, b, and add a weight c to the counterpoise sufficient to keep the bar in equilibrium. When it is accurately adjusted read the values of a, b, and c.

Compare c with $a + b$.

The points P, Q, R being the points of application of the forces, compare the ratios $\frac{a}{b}$ and $\frac{RQ}{PQ}$.

(2) Repeat the experiment with different values of the forces a, b, c, keeping the arms PQ, RQ unchanged.

For each experiment compare c with $a + b$, and find the value of the above ratios.

(3) Alter the lengths of the arms PQ and RQ; take a similar set of observations, and make the same comparisons.

(4) Make a similar set of experiments with the rod lying on a table (no counterpoise being therefore required), the forces a, b, c being applied by dynamometers. Stops must be placed to keep the bar from rotating entirely out of position.

The results of these experiments show that when two parallel forces a and b act on a body in the same direction, the body may be kept in equilibrium by a third force c, which acts parallel to the other two and in the opposite direction, that the third force is equal to the sum of the other two forces, and that its point of application is at a distance x from that of the first component and y from that of the second component, such that $a \times x = b \times y$.

Ex. 215.—Repeat Exercise 214 with three weights hanging from the bar, and as before compare the sum of all the downward pulls with the value of the upward pull.

Ex. 216.—(1) Support a heavy bar horizontally by means of two spring balances. Take their readings in different positions along the bar.

The sum of the readings is always equal to the weight of the bar.

(2) Hang weights in various positions, and record the downward pulls in one column and the upward pulls in another, remembering the weight of the bar. Compare the readings in the two columns.

(3) By the method of the previous exercise calculate the position and magnitude of a single force d, which would be equivalent to the pulls of the dynamometers; and the same for a force e, which would be equivalent to the weights. Then $d = e$.

132*a*. COUPLE.

Ex 217.—On a board pivot a bar AB (fig. 138) by a pin C through its centre. Arrange four stops, m, n, p, q, so that the bar can only rotate through a small angle.

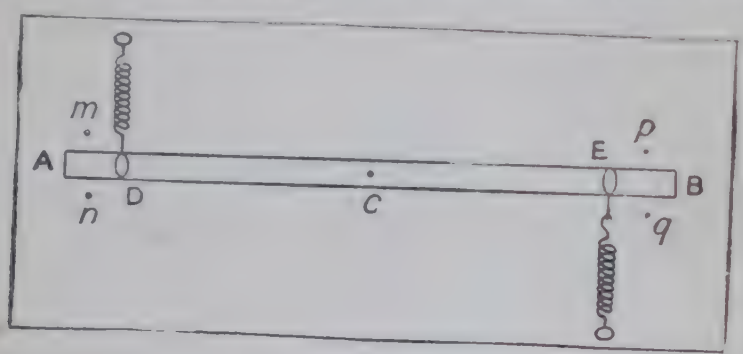

Fig. 138.—A Couple

Parallel forces are to be applied to the bar either by spring balances, or by weights passing over pulleys, as in the previous exercises.

If spring balances are used, place two at points D, E, as shown in the figure, at equal distances from C, and apply force to each.

The bar is pulled round against the stops. Make the pulls of the dynamometers equal, and fix them.

Two equal parallel forces are being applied to the bar, both tending to turn it in the same direction. Such a pair of forces is called a **couple**.

By means of a third spring balance endeavour to bring the bar away from the stops. Try various positions. Can the bar be brought back into the central line between the stops?

Use a fourth spring balance to assist the third. Positions for the dynamometers can now be found, which will enable the bar to be brought back to the central line.

When that is done read the values of the forces and the distances between the pairs of dynamometers. Record thus:—

	Pull.	Length of Arm.	Product.
1st dynamometer ..	a	x	$a \times x$
3rd dynamometer ...	b	y	$b \times y$

The experiment shows that a couple can only be counteracted by another couple.

Ex. 218.—(1) Find the value and position of the resultant of two forces of 50 grm. and 40 grm. applied at the ends of, and at right angles to, a rod 90 cm. long.

(2) The resultant of two parallel forces that are acting at the ends of a bar 5 ft. long is 20 lb., and it acts at a point 1 ft. from one end of the bar. Find the component forces.

(3) On a rod, whose weight may be neglected, two men carry a weight of 80 Kg. The weight is 1·5 m. from the shoulder of one man and 2·5 m. from the other. Find the weight borne by each man.

(4) A couple, each force of which is 150 grm., acts on a rod at points 2 m. apart. Find another couple which will keep the rod in equilibrium.

CHAPTER XIII

Centre of Gravity

133. The weight of a body is in all cases equal to the sum of the weights of its parts, whether the parts into which it be divided are few or many.

Fig. 139.—Centre of Gravity

This fact shows that the pull of the earth is exerted on every particle of a body, whether it is whole or ground to powder. Hence we can think of this pull as consisting of a multitude of parallel forces, directed towards the earth's centre (fig. 139).

But we saw (Art. 132) that a number of parallel forces are equivalent to one force acting at a certain point. Hence any object may be supported by a force acting at a certain point

Ex. 219.—Balance common objects on the top of one finger, or on the head of a screw fixed into a block of wood.

(i) A uniform rod. Where is the support?

(ii) A rod heavier at one end than the other. To which end is the support nearer?

(iii) Thin plates. If made of glass, the position of the point of support may be seen. Where is the support required for a circular plate? a square plate? a rectangular plate?

Each of these objects should be thought of as a collection of many particles of which each has the same mass. Each particle

is pulled vertically downwards by the earth, but they are held together by the force of cohesion. The finger supplied an upward force of the correct amount and in the correct position to counteract all these downward forces. The position of the finger thus indicates the position of their resultant (fig. 139), which is called the **centre of gravity** or **mass centre** of the body. When the centre of gravity of a rigid body is supported the whole body is supported.

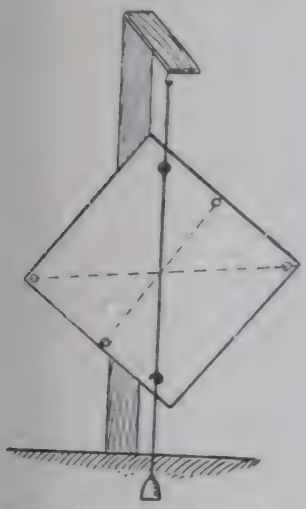

Fig. 140.—Determination of Centre of Gravity

Ex. 220.—To find the position of the centre of gravity of a thin uniform plate.

(1) Take a piece of cardboard having three holes near its edge. Hang it by a thread from a hook, and on the same hook hang a plumb bob (fig. 140). When both are hanging freely, mark by some dots the line of the plumb bob on the plate. Hang the plate up from each hole in turn, and mark the vertical lines similarly.

Remove the plate; draw the lines accurately. Do they pass through one point? Support the card on the head of a nail, and test whether this point is the centre of gravity.

Fig. 141.—Position of the Centre of Gravity of a Triangle

(2) Do this for a circle, a parallelogram, a triangle, a hexagon, a semicircle.

In the case of the rectilineal figures draw lines through the centre of gravity to each of the angles of the figure, and endeavour to find the geometrical position of the centre of gravity.

The triangle needs special notice. Fig. 141 shows the result. Measure how far the centre of gravity is up each of the bisecting lines of the triangle.

134. If the bodies are not uniform the centre of gravity is not at the centre of figure.

Ex. 221.—(1) Find the centre of gravity of a rod of which one end is weighted so as to be heavier than the other.

(2) Weigh a plate whose centre of gravity has been found (mass M grm.). Mark the position of its centre of gravity (A in fig. 142).

Weigh also a coin or metal disk (mass *m* grm.). Gum the disk on to the plate, marking the position (B) of the centre of gravity of the disk. Find the centre of gravity (C) of this weighted plate.

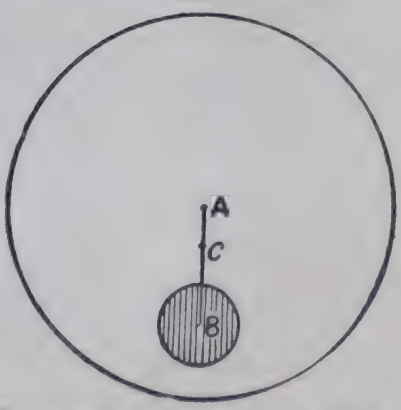

Fig. 142.—Mass-centre of a Loaded Plate

Is C on the line AB?

Measure the distances BC and AC, and find their ratio. Compare this ratio with the ratio of M to m.

(3) Place unequal weights at the angles of a triangular plate supported on a point, and find the position of the centre of gravity of the body thus formed.

135. In the previous experiments the plates were supposed to be so thin that their thickness was neglected.

The centre of gravity of a thick plate is midway between the two faces; of a rod, on its central axis; of a sphere, at its centre; of a cube, at the point of intersection of its diagonals.

The position of the centre of gravity of a triangular pyramid is illustrated in fig. 143. The point D is the centre of gravity of one

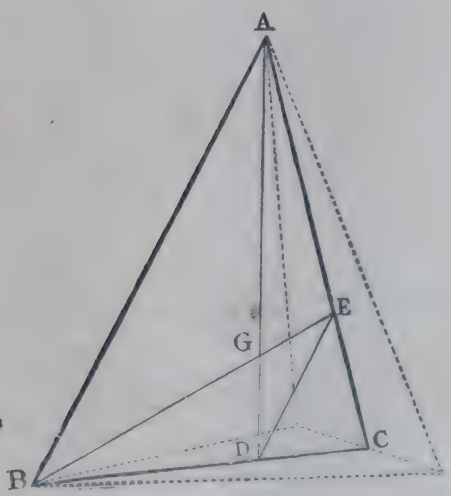

Fig. 143.—Mass-centre of a Pyramid

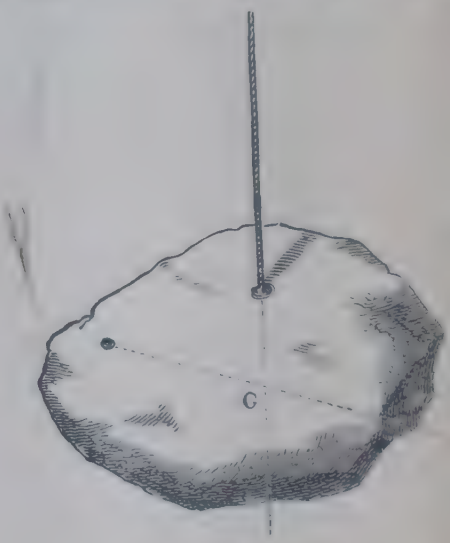

Fig. 144

of its faces, and if the pyramid were cut into slices parallel to thi face, the centre of gravity of all the slices would be in the line D Similarly E being the centre of gravity of another face, the centr of gravity of the whole solid is in the line EB. Therefore th centre of gravity of the pyramid is at G, where BE and AD intersec The geometry of the figure shows that DG $= \frac{1}{4}$DA.

Any pyramid may be divided into triangular pyramids, so th the result applies to all pyramids. Also a cone is a pyramid wi many sides. Thus the centre of gravity of a cone and of a pyram is one-fourth of the distance from the centre of gravity of the ba to the apex of the solid.

The practical determination of the centre of gravity of an irreg lar solid might be found in the same way, as illustrated in fig. 1 but is too complicated an operation for a laboratory exercise.

136. In the case of a solid, such as a chair, which is not continuous, but has some empty space within its contour, the position of the centre of gravity may be within that empty space. Fig 145 illustrates this. The horseshoe-shaped body has its centre of gravity at the point supported on the knife-edge. Of course, to support the body experimentally in the position shown, a rod has to be fastened to the horseshoe

Fig. 145. — Mass-centre outside the Mass Itself

Ex. 222.—(1) Where is the centre of gravity of a wire bent into the following shapes—a circle? a square? a rectangle? a triangle?

(2) Where is the centre of gravity of a hollow sphere? a hollow cylinder?

137. STABILITY.

Ex. 223.—(1) Take a circular disk of cardboard and mark the position G of the centre of gravity.

Pass a needle through this point so that it fits loosely in the hole. The needle being horizontal and the card vertical, turn the card into various positions. It remains where it is placed. The card is in **neutral equilibrium.**

Pass the needle through some other point A in the card. Try to make the card stand with G vertically above A. What is the effect of a slight movement? The card in this position is said to be in **unstable equilibrium.**

There is one position in which the card will rest, and if disturbed it returns to this position. For this condition what are the relative positions of G and A? This position is that of **stable equilibrium.**

(2) Repeat the experiment with plates of other shapes.

These experiments show that when a body is suspended from a point, in order that it may be in stable equilibrium its centre of gravity must be vertically beneath the point of suspension.

It has been seen that the centre of gravity when it can do so tends to descend and to occupy the lowest point possible.

Fig. 146 illustrates this principle, when the body is not suspended but is supported from below.

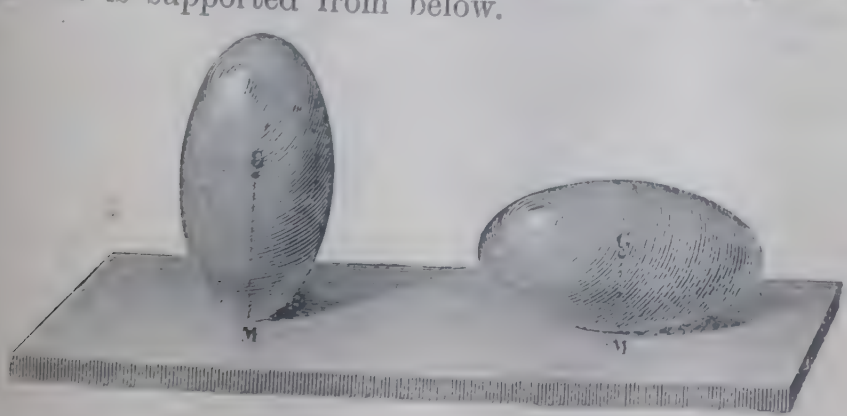

Fig. 146.—Unstable and Stable Equilibrium

Fig. 147 shows how a pin may be made to stand upon its point. Why does it stand thus?

Ex. 224.—Build a leaning tower of cubes symmetrically, as indicated in fig. 148. At each step note the position of the centre of gravity of the tower

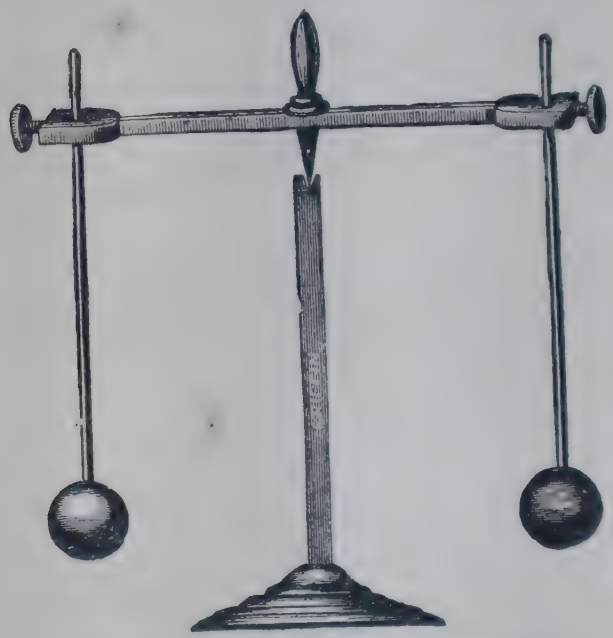

Fig. 147.—Centre of Gravity below the Point of Suspension

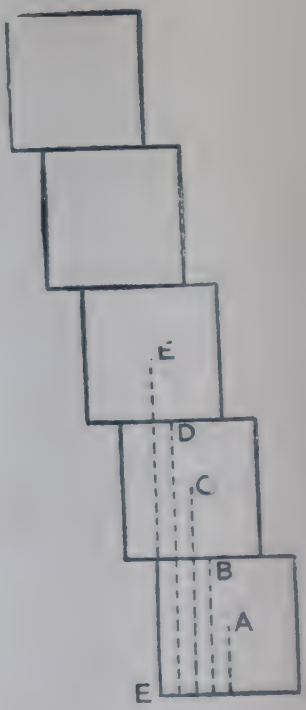

Fig. 148.—The Vertica through the Centre Gravity must fall withi the Base

(1 cube at A, 2 cubes at B, 3 cubes at C, &c.), and where the vertical line throug the centre of gravity falls.

The tower will stand (is stable) so long as the vertical throug the centre of gravity falls within its base. It cannot stand if th vertical through the centre of gravity does not fall within the bas

Ex. 225.—(1) A cylinder or tub is rolled along a flat surface. Does t motion change the height of its centre of gravity? What is its condition of eq librium? Make sketches of a cylinder in positions of stable and unstable eq librium.

(2) Make a sketch of a cone in the positions of the three states of equilibriu

(3) Why cannot a man who is carrying a heavy mass by one hand rem upright?

(4) Why cannot you pick an object off the floor in front of you when stand with your heels in contact with a wall?

CHAPTER XIV

Elementary Machines in Equilibrium

138. A machine is a contrivance by means of which force exerted at one point is made to overcome an opposing force exerted at another point, and the arrangement of its parts is generally such that a small force exerted by a man or an engine overcomes a larger opposing force. When this is the case the machine is said to give a mechanical advantage.

The force that is applied to a machine is called the **effort** or force: it used to be called the power. The force that is overcome is called the **resistance** or **load**: it was formerly called the weight

The **mechanical advantage** of a machine is the ratio

$$\frac{W}{P} = \frac{\text{Resistance}}{\text{Effort}} \text{ or } \frac{\text{Load}}{\text{Force}}.$$

Complicated machines are combinations of certain simple machines often called the mechanical powers.

The mechanical powers are the lever, the inclined plane, the wedge, the wheel and axle, the pulley, and the screw.

In actual working machines the mechanical advantage is diminished by friction and by the weight of the parts. In a first simple treatment the complications produced by these agencies must be neglected. Hence in the following experiments care should be taken to eliminate as far as possible the effect of friction, and to use light apparatus.

139. THE LEVER: THE PRINCIPLE OF MOMENTS.—The lever with equal arms was briefly studied in the exercises on the balance. We now have to examine the lever further. It is an apparatus that is in common use in various forms. In its simplest form it consists of a straight bar which turns upon a fixed point called the **fulcrum**.

The effort is applied at some point of the bar and the resistance at some other point.

There are *three orders* of lever, as illustrated in figs. 149, 150, 151. In a lever of the **first order** (fig. 149) the fulcrum is placed between the effort P and the resistance W

In a lever of the **second order** (fig. 150) the resistance is between the effort and the fulcrum.

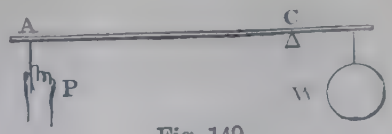

Fig. 149

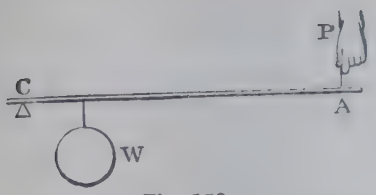

Fig. 150

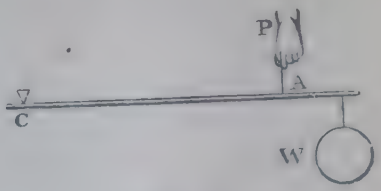

Fig. 151

The Three Orders of Levers

In a lever of the **third order** (fig. 151) the effort is applied between the fulcrum and the resistance.

A lever has two **arms**, viz.: the *perpendicular* distance of the

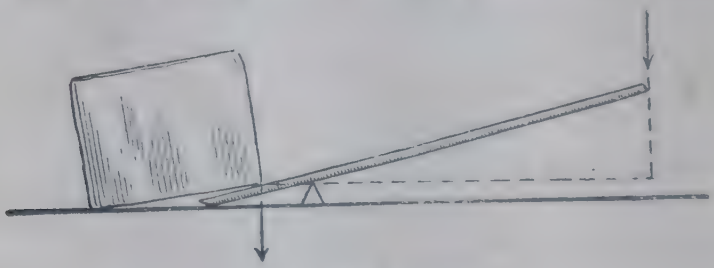

Fig. 152.—Use of a Fulcrum

line of action of the effort from the fulcrum, and the *perpendicular* distance of the line of action of the resistance from the fulcrum, as indicated in fig. 152, which illustrates the action of a lever of the first order in the common operation of raising a mass of stone.

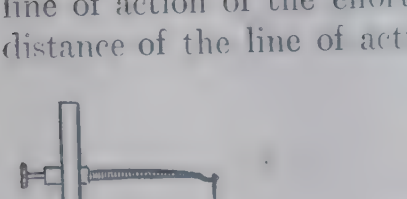

Fig. 153.—Principle of Moments applied to the Lever

Ex. 226.—Support a uniform rod at its centre of gravity as in fig. 28, or as in fig. 1[illegible]

If the rod does not stand quite horizontal, weight the lighter end.

Take two unequal but known masses, e[illegible] numbers of shot, as in Exercise 51, or [illegible] weights, such as a 50-grm. and a 20-g[illegible] weight, and suspend them by threads from [illegible] arms of the lever. Put the lighter wei[illegible] nearly at the end of its arm, and move [illegible] beyond it, and the lever stands [illegible] zontal. Measure the two arms accurately.

Move the lighter weight one or two centimetres towards the fulcrum, and take another similar reading.

Do this for several positions, and record thus:—

Larger Weight, W.	Length of Arm, D	Product, W × D.	Smaller Weight, w.	Length of Arm, d.	Product, $w \times d$
50 grm.	...	...	20 grm.	40 cm	800
50 „	...	...	20 „	35 „	700
50 „	...	...	20 „	...	...

Compare the product W × D with the product $w \times d$ line by line. Then add up each set (to lessen probable error), and compare the totals.

The products in columns 3 and 6—of force by distance of force from fulcrum—are called the **turning moments**, or simply **moments** of the forces round the fulcrum.

The **moment** of a force round a point is the product of the force into the perpendicular distance of its line of action from the point.

Calling the smaller force w the effort, and the larger force W the resistance or load, what is the relation between the moment of the effort and the moment of the resistance when a lever is in equilibrium?

The moments that tend to turn the lever round in the same direction as that in which the hands of a clock travel are described as clockwise or negative moments; those that turn it in the opposite direction as anti-clockwise or positive moments.

Ex. 227.—To find the mechanical advantage of the lever.

Take the readings obtained in the previous exercise for each pair, D and d being the arms of the lever. Calculate the ratio $\frac{\text{load}}{\text{effort}} = \frac{W}{w}$, and compare it with the ratio $\frac{d}{D}$ in each experiment.

The mechanical advantage of a lever of this class is the ratio $\frac{\text{length of the arm to which the effort is applied}}{\text{length of the arm to which the load is applied}}$

Ex. 228.—By means of the lever set up for Exercise 226, find the mass of a stone or lump of metal.

Suspend a known mass as before on one arm of the lever, and the unknown mass on the other arm. Take the same set of observations as before.

From the principle of moments calculate from each line the value of the unknown mass x thus:—

$$x \times 8 = 20 \times 13$$
$$\therefore 8x = 260$$
$$x = 32{\cdot}5 \text{ grm.}$$

Ex. 229.—(1) On one side of the fulcrum place two weights A and B, and on the other a single weight C (fig. 154).

Obtain equilibrium, and read the arms a, b, c. Record thus:—

Negative Moments.	Positive Moments.
C × c	A × a B × b

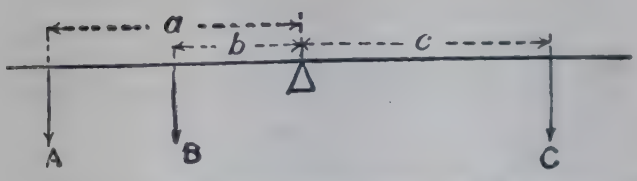

Fig. 154.—Equilibrium of Positive and Negative Moments

Add up the negative moments, and compare the sum with the positive moment. Are they equal?

(2) Repeat the experiment with three negative moments working against two positive moments.

(3) Write out the conclusion to be drawn from the experiment.

Ex. 230.—Take any bar, not necessarily a uniform bar, as the lever. Weigh the bar. Find the position of its centre of gravity, and mark it.

Support the lever at a point not its centre of gravity.

Place weights on the arms as before, and obtain equilibrium.

The weight of the bar is acting at its centre of gravity, and constitutes one of the turning forces.

Remembering this fact, this case is like that of the last experiment. Find the sum of the anti-clockwise and of the clockwise moments as before, and compare them.

140. The **steelyard** (fig. 155) is a bar of this kind. It is supported on a hook above the fulcrum C. The weight of the **bar** m

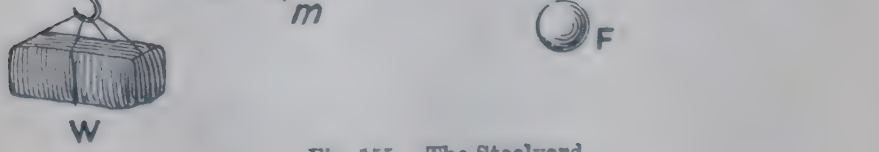

Fig. 155.—The Steelyard

acts downward at its centre of gravity G. The load W is hung on to the end of the shorter arm, and the effort F is applied by a weight that can be moved along the longer arm. When the bar or yard hangs horizontal W × AC = (m × GC) + (F × BC).

Ex. 231.—Support a bar with the fulcrum not at the centre of gravity. Using a constant sliding weight for F, and gradually increasing weights for the load W, graduate the bar as a steelyard.

Why are the graduation marks at unequal distances for equal changes in the load?

Ex. 232.—Find the weight of a bar by the principle of moments.

Find the position of its centre of gravity and mark it. Support the bar at some point not its centre of gravity, as in fig. 155.

Place weights on the arms and obtain a balance. Then taking m as the weight of the bar, equate the moments on each side of the fulcrum, and solving the equation given above find the value of m.

Ex. 233.—To find the correct weight of a body by means of a false balance, i.e. a balance having unequal arms.

Set up a lever as in fig. 153.

Choose the lengths a, b of the arms to be used, making one about 1 cm. longer than the other.

In one pan place an unknown mass x. In the other place weights w_1, and obtain correct balance; then, $x \times a = w_1 \times b$.

Place the mass x in the other pan, and again obtain balance by placing the required weights w_2 in the first pan; then, $x \times b = w_2 \times a$.

Multiply the above two equations together;

$$\text{then } x^2 \,.\, ab = w_1 w_2 \,.\, ab$$
$$\text{and } x = \sqrt{w_1 w_2}.$$

Thus x is correctly found without knowing the lengths of the balance arms.

Ex. 234.—Draw a diagram showing that a pair of scissors is a double lever of the first class. (The central pin is the fulcrum; the resistance is that offered by the object between the cutting edges.)

Ex. 235.—Bend a piece of stiff wire near the middle, and also bend small loops at each end. Hang the wire with the middle bend over a small peg, from which also hangs a plumb bob (fig. 156).

Hang weights on the end loops.

The arms of the forces are the perpendiculars on to the plumb-bob line.

Verify the principle of moments.

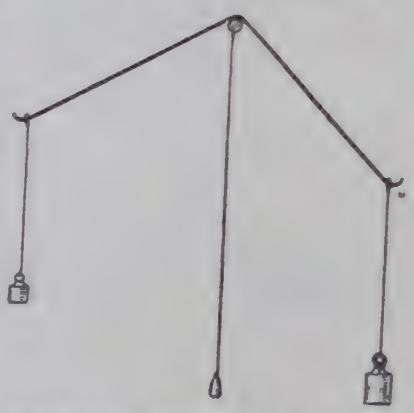

Fig. 156.—Equilibrium of a Bent Lever

141. LEVERS OF THE SECOND ORDER.

Ex. 236.—Set up a bar as shown in fig. 150, the effort (P in that diagram) being supplied by a dynamometer hanging from a support. The fulcrum may be a stiff pin passed through the bar.

Verify the principle of moments for this system.

Find also the mechanical advantage.

Ex. 237.—Show by a diagram—

(i) That a wheelbarrow supported with its legs off the ground is a lever of the second class. (The axle of the wheel is the fulcrum.)

(ii) That a pair of nutcrackers is a double lever of this class. (The nut offers the resistance.)

142. LEVERS OF THE THIRD ORDER

Ex. 238.—Drive a stiff pin through one end of a bar into a block of wood and thus make a lever of the third class as shown in fig. 151. Apply the effort by a spring balance hanging from a support, and put a load on the bar. Taking different positions for the point of applications of the effort and weight, verify the principle of moments for this case. Find also the mechanical advantage. Note that it is less than unity. Thus this order of lever is only used as a matter of convenience.

Ex. 239.—Show by a diagram that the action of a pair of sugar tongs or coal tongs is that of a double lever of the third order. (The resistance is offered by the object gripped: the fulcrum is at the bend of the tongs.)

143. THE TANGENT LAW.—There are several examples occurring in physics in which a body is caused to take up a position under the action of two forces or couples at right angles to each other. This case should be studied.

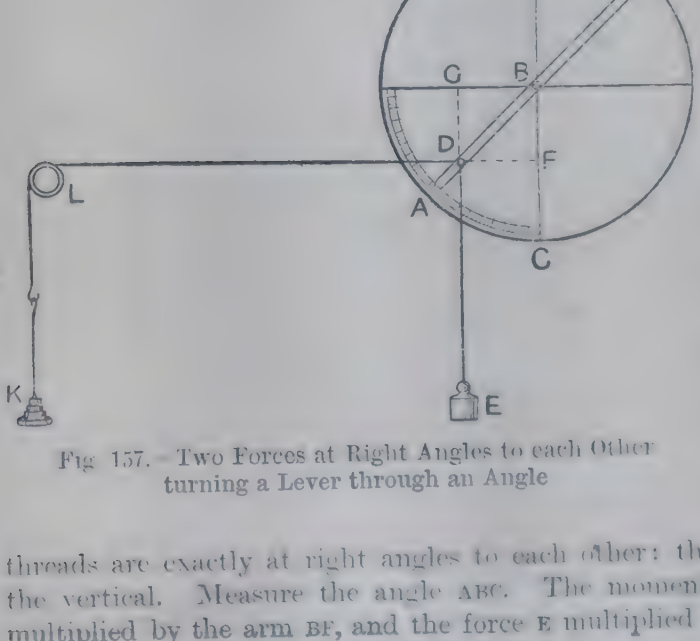

Fig. 157.—Two Forces at Right Angles to each Other turning a Lever through an Angle

Ex. 240.—(1) At the centre of a circle fixed with its plane vertical, and having one quadrant divided into degrees (fig. 157), mount a strip of wood by a pin B so that the strip or "needle" can rotate easily.

At a point D suspend from a pin a weight E.

To the same pin fasten another thread which passes over a pulley L and supports a weight K.

Place a small weight at K. See that the two threads are exactly at right angles to each other: the arm AB is deflected from the vertical. Measure the angle ABC. The moments round B are the force K multiplied by the arm BF, and the force E multiplied by the arm DF. These are equal. Record thus:—

Horizontal Force.	Vertical Force.	Deflection of Needle.	Ratio of Forces.	Tangent of Angle ABC.
K	E	Angle ABC	$\frac{K}{E}$	

(2) Take several observations of this character with different values of K.

as to obtain different values of the deflection ABC, and for each observation compare $\frac{K}{E}$ with the tangent of ABC, which is $\frac{DF}{BF}$.

The observations show that—

$$\frac{K}{E} = \tan ABC; \text{ or } K = E \times \tan ABC.$$

This result is the basis of measurements made by the tangent galvanometer (Art. 360).

144. THE WHEEL AND AXLE.—A lever can only be used for raising heavy masses a short distance. When it is required to raise such masses through a considerable distance an arrangement is made by which the effort is applied to turn a wheel or cylinder having a greater radius, and the load is applied to the radius of a smaller wheel or cylinder which is fixed on the same central axis. Thus a continuous motion is obtained.

Such an arrangement constitutes a **wheel and axle** (fig. 158).

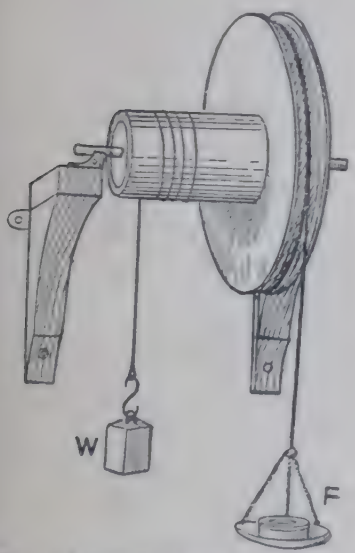

Fig. 158.—The Wheel and Axle

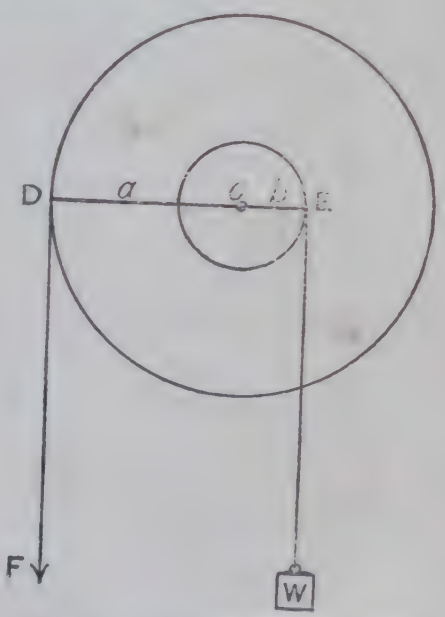

Fig. 159.—Turning Moments acting on the Wheel and Axle

The axis or spindle rests on suitable bearings so that the whole apparatus can rotate. A cord is attached to the circumference of the wheel, another to the circumference of the axle. These cords are wrapped round their cylinders in opposite directions, so that the effort produces a turning moment in one direction and the load in the opposite direction.

Fig. 159 shows how the principle of moments applies. The

machine is like a lever whose fulcrum is at C. and whose arms are CD the radius of the wheel, and CE the radius of the axle.

In our experiments the cords should be thin, flexible, and of small mass, and the spindle where it turns in its bearings should be oiled.

Ex. 241.—Find the mechanical advantage of the wheel and axle.

Measure the circumference of the wheel and of the axle, and calculate the radius of each. Suppose the radius of the wheel is a, and of the axle b.

Affix a weight to represent the load on to the cord which surrounds the smaller cylinder (the axle), and a scale pan (whose weight is known) to the cord which surrounds the larger cylinder (the wheel). Weights placed in this pan constitute the effort.

Put weights in the pan until the load just rises from the ground. Pull the force pan down a little, and thus raise the load. Take off weights until the apparatus is in equilibrium.

Vary the force to a small extent. It will probably be found that equilibrium will still be maintained. Find the force which is just enough to raise the load slowly, and the force which allows the load to descend slowly, and take the mean of these values.

Take a series of readings in this way with different values of the effort F and the load W. Remember that F includes the weight of the scale pan.

Value of F.	Radius of Wheel a.	Turning Moment F × a.	Value of W.	Radius of Axle b.	Turning Moment W × b.
...	...	...	...	...	...
...	...	...	...	...	...

If the effect of friction has been eliminated in the manner indicated above the figures in columns 3 and 6 will be nearly the same. If the cords are thick the radii a and b should include the thickness of the cord.

The mechanical advantage $\frac{W}{F}$ is thus shown to be equal to $\frac{a}{b}$, i.e.

$$\frac{\text{radius of wheel}}{\text{radius of axle}}.$$

Fig. 160.—The Windlass

145. The wheel and axle is used in many common appliances, for example, the ordinary roller blind, in which the blind is the weight attached to the axle, and the force is applied to an endless cord passing round the wheel.

Another common example is the windlass (fig. 160), by which water is drawn from a well. The

wheel in this case is replaced by a crank handle. A comparison of figs. 158 and 160 will show that their action is the same.

The capstan (fig. 161) is another example. It is used largely in ships. Men apply force to the spokes of a wheel, and the rope along which the weight or resistance acts is wound round the hub or axle of the wheel.

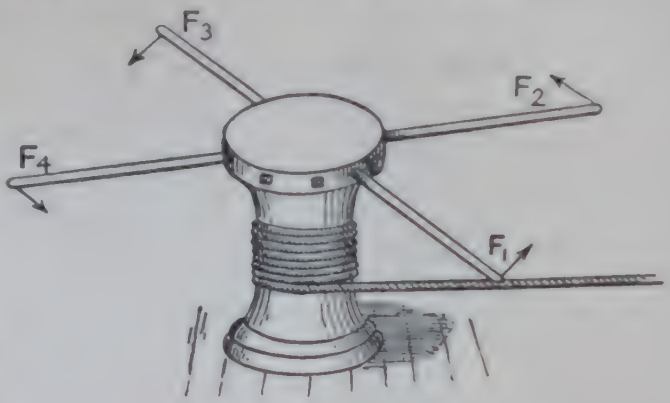

Fig. 161.—The Capstan

Ex. 242.—(1) The axle of a windlass is 15 cm. in diameter, and carries a bucket whose weight is 5 Kg. and capacity 10 litres. The arm (a in fig. 160) of the windlass is 45 cm. long. What force must be applied to the handle to just move the bucket when full of water?

(2) The barrel of a capstan is 2 ft. in diameter, and is worked by four men, each of whom exerts a force equal to the weight of 50 lb. at the end of an arm 5 ft. long.

Find the pull on the capstan rope.

146. THE INCLINED PLANE.—When it is required to raise heavy bodies through comparatively small heights, it is often convenient to roll or push them up a slope. If the friction is not great a man can by this means raise bodies which he could not possibly lift straight up to the required height. He exercises a smaller force along a greater distance.

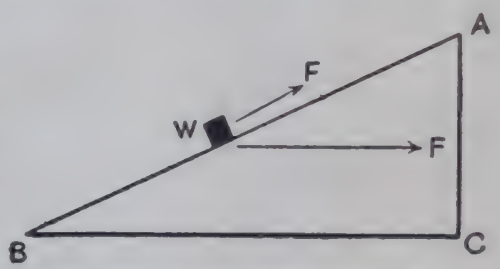

Fig. 162.—A Body supported on an Inclined Plane

Fig. 162 shows a skeleton inclined plane.

AB is called the length of the plane (l).
AC ,, ,, height ,, ,, (h).
BC ,, ,, base ,, ,, (b).

The base BC is taken to be horizontal.

In experiments on the inclined plane the load W is placed on the plane, and is prevented from slipping or rolling down the slope by an effort F, which may be exerted either along the direction BA,

when it is said to be parallel to the plane; or in a horizontal direction, when it is said to be parallel to the base.

The object of the succeeding experiments is to find the ratio between W and F when they are in equilibrium.

To diminish friction the plane should be of polished wood or of wood faced with a sheet of glass. The load may consist of a heavy metal roller, as in fig. 164, or a small trolley on which weights are placed, as in fig. 163.

The effort may be applied by means of weights in a scale pan attached to a string that passes over a pulley, as in fig. 163, or by means of a dynamometer, as in fig. 164. In order that the plane may be inclined at any angle it is hinged to a base board.

Ex. 243.—To find the mechanical advantage of the inclined plane when the effort is parallel to the plane.

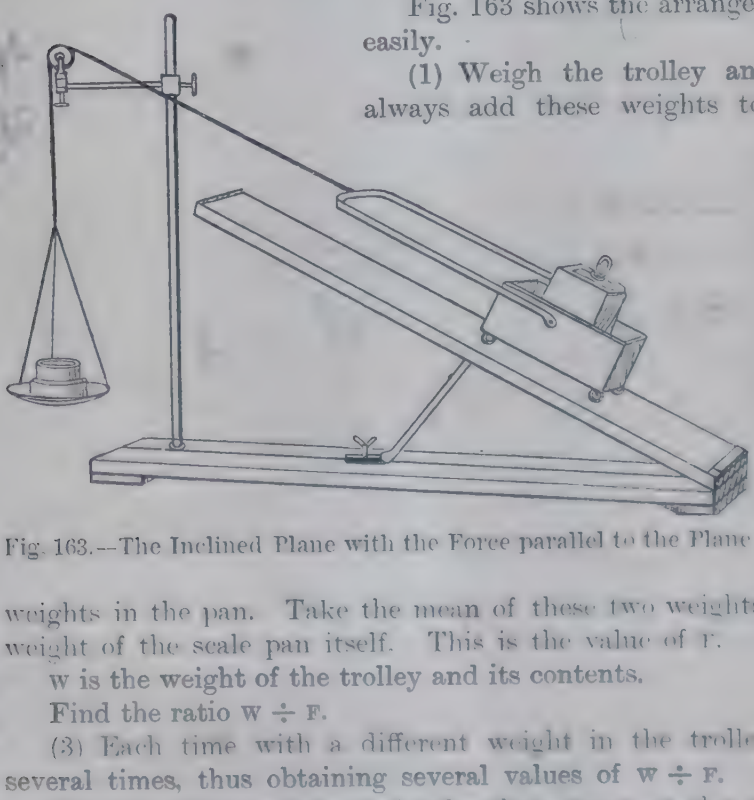

Fig. 163.—The Inclined Plane with the Force parallel to the Plane

Fig. 163 shows the arrangement. See that it runs easily.

(1) Weigh the trolley and the scale pan, and always add these weights to those placed in the trolley and in the pan respectively.

(2) Place a weight in the trolley, and put weights in the scale pan until the trolley just moves up the plane. Note the weights in the pan.

Remove weights from the scale pan until the trolley just moves down the plane. Note the weights in the pan. Take the mean of these two weights, and add to them the weight of the scale pan itself. This is the value of R.

W is the weight of the trolley and its contents.

Find the ratio W ÷ F.

(3) Each time with a different weight in the trolley repeat the operation several times, thus obtaining several values of W ÷ F.

Compare these values, and take their mean as the value of the mechanical advantage. If the weight is a simple roller only one value of W ÷ R can be obtained.

(4) Measure the length (l) of the plane down to the hinge and the height (h) of the plane.

Compare the mean value of W ÷ R with the value of $l \div h$.

Ex. 244. Keeping the weight the same, incline the plane at a different angle and repeat the whole set of observations. Do this for several angles. The

operation alters the height of the plane, but not its length. For each position find as before the ratio W ÷ F and the ratio $l \div h$ and compare these.

The whole set of observations shows that when the effort acts parallel to the plane this relation holds good:

$$\frac{\text{Resistance}}{\text{Effort}} = \frac{\text{length of plane}}{\text{height of plane}} = \frac{l}{h}$$

Ex. 245.—To find the mechanical advantage of the inclined plane when the force acts parallel to the base. Fig. 164 shows an arrangement essentially the

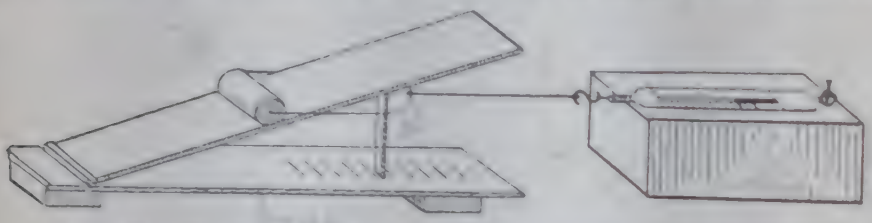

Fig. 164.—The Inclined Plane with the Force parallel to the Base

same as that of fig. 163. The load is a heavy roller. The effort is the reaction of the spring of the dynamometer. The dynamometer is fixed on a support, and the string connecting it with the weight is horizontal, parallel to the base of the plane.

The roller has a pair of shafts connected with its axle, by which it is moved. **Weigh the roller and its accessories** (W).

(1) Place the roller on the plane so that its shafts are horizontal, and connect them by a string to the dynamometer. Roll the load a little way up and down the plane, and adjust the position of the dynamometer or load until the connecting string is horizontal. See that the roller lies straight across the plane, so that the pull on it is really at right angles to its axle. Slight movements and tapping of the plane will enable this to be done.

When the adjustments have been carefully made read F on the dynamometer. **Find the ratio W ÷ F.**

The base of the plane is the distance from the hinge to a point vertically beneath the top of the plane. Measure this base b and the height of the plane h. **Find the ratio $b \div h$. Compare it with the ratio W ÷ F.**

(2) Alter the inclination of the plane. Again find the ratio W ÷ F and $b \div h$.

The mechanical advantage when the effort acts parallel to the base is the ratio of the base of the plane to the height of the plane.

147. PULLEYS.—A pulley is a small wheel, sometimes called a **sheaf**, having a groove round its edge and an axis or spindle through its centre (fig. 165). The ends of the axis are supported in a framework called a **block**. Such pulleys are often mounted with more than one sheaf in a block. Fig. 167 shows two three-sheaved blocks.

A cord is passed over the pulley, and the effort and load are applied at the ends of the cords.

Window sashes are generally hung in this way, most of the weight of the sash being balanced by a weight on the other end of the sash cord.

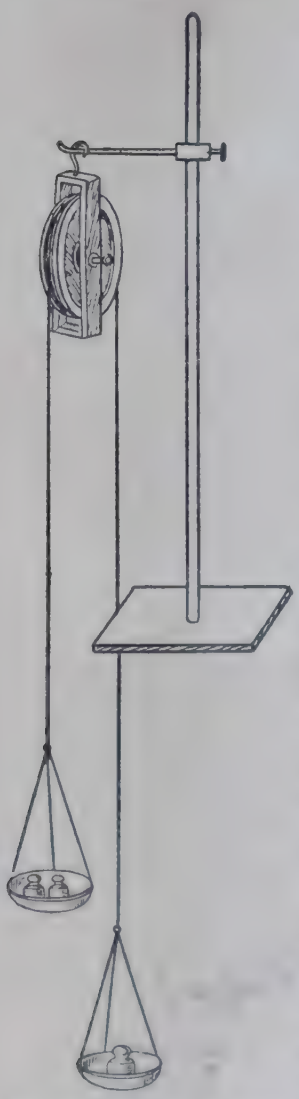

Fig. 165.—A Single Fixed Pulley

Ex. 246.—The Single Fixed Pulley.—Support a single pulley as in fig. 165. See that the pulley runs easily and the string is thin and flexible.

Weigh two scale pans and attach them to the string.

(1) Put a weight in each scale pan and adjust them until with a slight pull of either string the pulley will rotate in either direction. Add the weight of each pan to the weights which it carries, thus obtaining W and F. What is the ratio between them?

Repeat the experiment for several values of F and W. Take the mean of the ratios obtained.

(2) Replace the effort pan by a dynamometer, keeping the strings parallel, and repeat the observations. What is the ratio $W \div F$?

(3) Alter the position of the dynamometer so that the strings are not parallel, and take some observations at different angles. What is now the ratio $W \div F$?

The observations show that W is very nearly equal to F. Any difference is due to friction. Find the value of $F - W$, which measures the friction. There is no mechanical advantage in this arrangement, but it is often used to pull bodies up to the top of a building instead of carrying them up. Moreover, it has been shown that the pulley allows the effort to be applied in any convenient direction.

Ex. 247.—The Single Movable Pulley.—(1) Weigh the pulley that is to move, and set up the apparatus as shown in fig. 166. A single long string having one end fixed to a support passes round the movable pulley and over a fixed pulley. At the other end of the string the effort F is applied. The load is attached to the block of the movable pulley. See that the three parts of the string are vertical and in the same plane, and that the pulleys run freely.

Attach the load, and find the least effort that will keep it raised from the table.

To obtain W, the weight of the movable pulley must be added to the weight it carries. If scale pans are used their respective weights must be included in F and W.

What is the mechanical advantage $\frac{W}{F}$? What is the value of $2F - W$?

Take several measurements with different values of F and W; and for each find the value of $W \div F$ and of $2F - W$. Since two pulleys are used, the disturbing effect due to friction is twice as great as in the last experiment.

(2) Instead of using a weight and a fixed pulley, apply the effort by a dynamometer pulling upwards. You now have two parallel strings. Take a series of readings of the effort F for different values of W as before. There is now only the friction of one pulley, and the result will show 2F more nearly equal to W.

(3) Keeping the load the same change the position of the dynamometer so that the cords are not parallel. Record F and W. Increase the angle between the two parts of the cord, and again read F and W. Take readings for several angles. Note the great effect of increasing the angle between the two parts of the string. Compare with the result of Exercise 205.

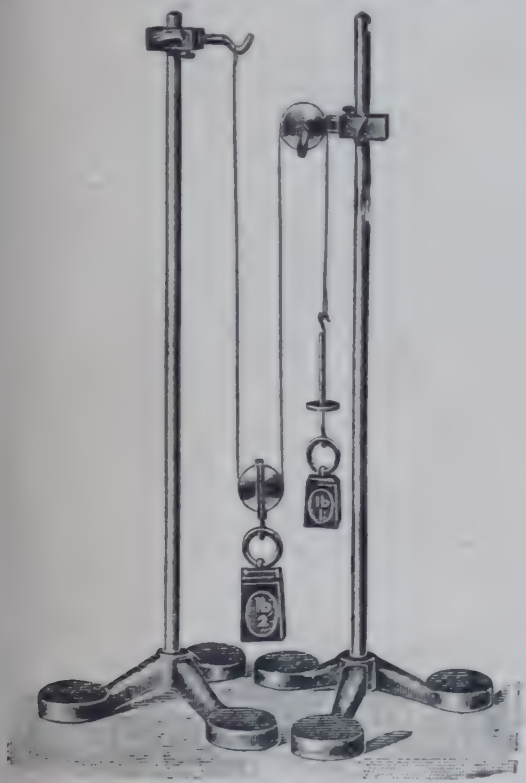

Fig. 166.—A Single Movable Pulley

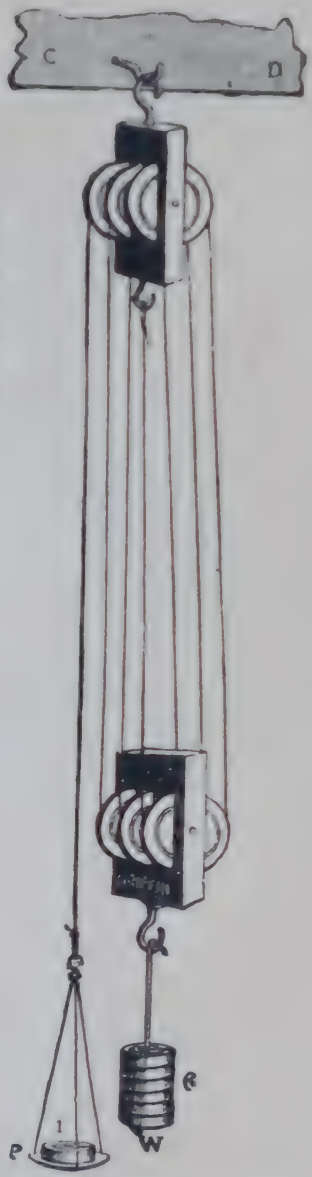

Fig. 167.—Two 3-sheaved Pulley Blocks

Omitting friction the observations show that when a single movable pulley is used—(*a*) If the strings are parallel, the mechanical advantage is 2. An effort F will raise a weight of 2F. (*b*) If the strings are not parallel, the mechanical advantage becomes less and less as the angle between the strings is made greater.

Combination of Pulleys.—Various combinations of pulleys are used. Fig. 167 shows an arrangement much used in practical business operations.

It consists of two blocks, each of which may consist of two,

three, or more sheaves. One of the blocks is fixed, and one movable. The weight to be moved is attached to the movable block. There is only one cord, which has one end fixed to the fixed block, and is threaded through first one of the sheaves of the movable block, then one of the sheaves of the fixed block, and so on through all the sheaves. The effort is applied to the other end of the cord. When effort is applied and the load is supported, there are in the case shown in fig. 167 six strings between the two blocks. Each string carries one-sixth of the load. The effort applied has to be sufficient to keep these strings tight; that is, the effort is equal to the tension in the string, which is one-sixth of the load.

Thus in this case P = $\frac{1}{6}$ W. If there are four cords P = $\frac{1}{4}$ W, and if eight P = $\frac{1}{8}$ W, and so on.

148. THE SCREW.

Fig. 168.—The Screw as an Inclined Plane

Ex. 248.—From a piece of paper cut out a right-angled triangle having one side much longer than the other.

Roll the paper round a small cylinder, such as a pencil, with the shorter side of the triangle parallel to the axis of the cylinder (fig. 168).

Then the hypotenuse of the triangle forms a spiral line travelling up the cylinder, and represents the thread of a screw.

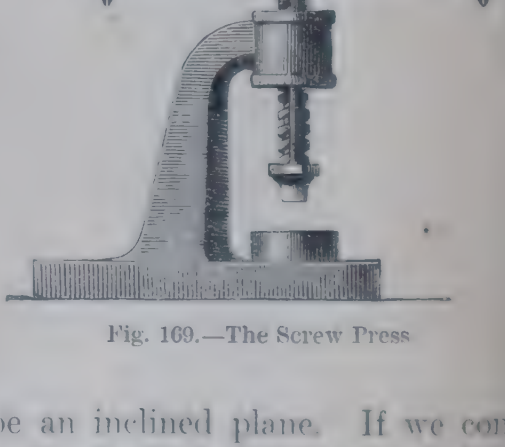

Fig. 169.—The Screw Press

Thus the screw is shown to be an inclined plane. If we consider only one turn of the screw the base of the inclined plane is the circumference of the screw and height the pitch of the screw.

When the head of the screw is turned through one revolution the screw advances one step, i.e. a distance equal to the pitch.

In the screw press (fig. 169) the effort is applied at the end of an arm attached to the screw, and the mechanical advantage is $\frac{\text{circumference of the circle described by the effort}}{\text{the step of the screw}}$. By such a machine great pressure may be applied. The friction in the screw is so great that it cannot be neglected, and no value of the mechanical advantage can be obtained from a statical experiment.

Ex. 249.—Miscellaneous Examples.

(1) The arms of a lever are 20 cm. and 55 cm., and a weight of 80 Kg. is attached to the shorter arm. What is the effort?

(2) Two men carry a load of 60 Kg. suspended from a uniform horizontal pole 4 m. long and weighing 10 Kg. If the pressure on one man's shoulder is 40 Kg., from what point is the weight suspended?

(3) A wheel and axle is used to raise a bucket weighing 15 Kg. The radius of the wheel is 50 cm., and while it makes 6 revolutions the bucket rises 3 m. What is the smallest force that will raise the bucket?

(4) Describe how by means of a wheel and axle a man could pull himself up to the top of a building by exerting only a small force.

(5) The arm of a screw press is 50 cm. long, and the screw descends 5 cm. when it is turned 8 times, what force must be applied to the arm to give a total pressure of 200 Kg.?

(3) A nut is placed in a pair of nutcrackers 1·5 cm. from the fulcrum or hinge. A force of 2 Kg. applied to the arms of the crackers at a distance of 12 cm. from the fulcrum just crushes the nut. Find the crushing force on the nut.

CHAPTER XV

Inertia. The First Law of Motion

149. REST AND MOTION.—These are primitive ideas which everyone realizes from experience, and which can be made no clearer by explanation.

But it must be pointed out that all motion is relative. In walking or running we pass stationary objects. If we did not pass them, but they all moved with us, we could perceive no motion. The earth and its atmosphere are moving very fast through space, but we do not immediately perceive this motion. We only conclude that the earth does move from observations on the sun and the seasons of the year.

To persons riding in a train which is running parallel to another train that moves at the same speed, the second train appears to be at rest, if they cannot at the same time see stationary objects. In a balloon drifting with the wind no motion can be detected by a person looking upwards.

The objects that we regard as at rest in our movements are the earth and things that are attached to the earth.

Our test of motion is whether a body occupies different positions among surrounding objects at different times.

150. INERTIA OF BODIES AT REST.—It is a matter of universal experience that bodies which are at rest but are not fixed remain at rest unless removed by force. But in most of these cases although the body may not be fixed there is a force acting on the body which resists any attempt to move it. Thus great opposition is offered to the movement of a block of stone by the forces of gravitation and friction. These forces, however, are due to agencies outside the body itself. We have now to consider whether it is a property of the body itself to resist being set in motion quite independently of the opposition due to forces acting on it from outside. To do this, make some experiments in which the force of gravitation does not interfere.

Ex. 250.—On a small sheet of paper lying on a table place a mass such as a 100-grm. weight. Sharply slide away the paper along the table. Does the mass move with the paper?

Place a smooth card on a glass tumbler, and on it place a coin. Quickly snatch away the card horizontally. Does the coin go with the card?

In these cases gravitation was counteracted by the support. A horizontal force was applied to the body, but only a very small horizontal motion took place. The body at rest showed a tendency to remain at rest.

The next experiment is more, although not entirely, free from the disturbing effect of friction.

Ex. 251.—On a smoothly running trolley, standing on a smooth board, place a heavy mass (fig. 170). Pulling a dynamometer attached to the trolley, start the mass moving. Read the force required to start it, and the force required to maintain it in motion. Which is the greater?

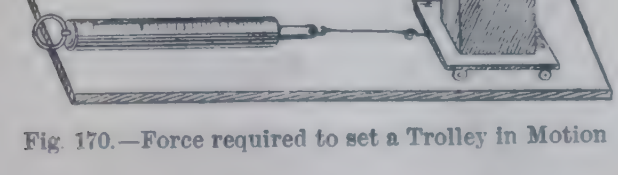

Fig. 170.—Force required to set a Trolley in Motion

Repeat the experiment at different speeds and with different loads.

The readings show that—

(*a*) It requires a greater force to set a body in motion than to keep it in motion.

(*b*) It requires a greater force to produce a rapid motion than to produce a slow motion.

(*c*) It requires a greater force to set in motion a large mass than a small mass.

This experiment carries more conviction when made on a large

scale with a railway truck. The whole strength of two or three men is required to start it, but one man can keep it moving. Similarly, a horse has much more difficulty in starting a loaded wagon than in keeping it moving.

In the next experiment gravitation is counteracted and friction is also very small.

Ex. 252.—Suspend a heavy mass by a long string. The string is counteracting the pull of the earth, while the friction of the air, and at the support, is extremely small.

Does the body offer any resistance to a horizontal push or pull?

Attach a dynamometer to the mass (fig. 171), and set the mass in motion by a horizontal pull. Note the force registered by the dynamometer (*a*) at the moment of starting; (*b*) immediately afterwards. Vary the strength of the pull and the mass moved, and make the same observations as in the last experiment.

These experiments indicate that there is some property in matter which

(*a*) Causes a body at rest to remain at rest;

(*b*) Opposes a resistance to any force that tends to move the body.

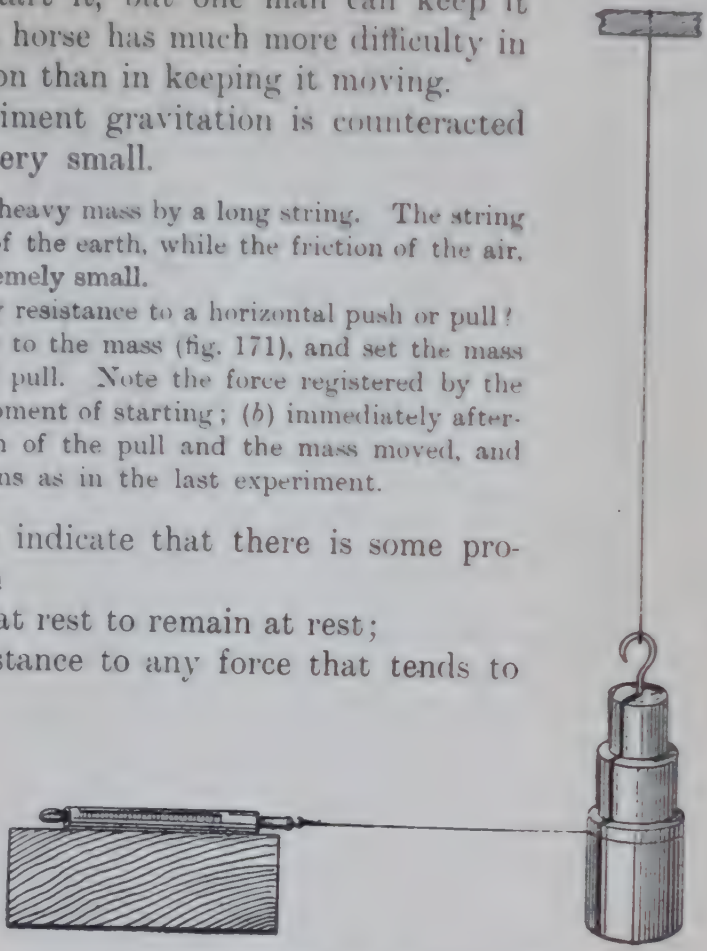

Fig. 171.—Inertia of a Suspended Mass

151. INERTIA OF BODIES IN MOTION.—We now have to consider bodies that are in motion. Everyday experience is that bodies in motion sooner or later come to rest. Are such moving bodies stopped by some external force or not? Do they possess a property which would (if no external force stops them) keep them for ever in motion?

Consider the following experiences:—

If a horse that is carrying a rider suddenly stops, the rider, unless he forcibly prevents it, continues to move forward, and is thrown over the horse's head.

Persons in a swiftly moving carriage are thrown forward in a similar manner if the carriage suddenly stops.

A person alighting from a carriage in motion is thrown forward.

A man running swiftly has some difficulty in stopping quickly.

If water be running through a tap in a pipe and the tap be suddenly turned off, a bang is often heard in the pipe. The moving

water knocks hard against the closed end of the pipe. Lead pipes are often distended and in time burst by this practice.

A cricket ball travels considerably further on smooth turf than on rough ground. On a polished surface, such as ice, it travels much further still. This is because friction has been diminished.

A carefully suspended pendulum continues to oscillate much longer in a vacuum than in air.

If a metal top be set spinning on a glass plate, and then be placed under the receiver of an air pump, and the air pumped out of the receiver, it continues spinning for a very long time.

Ex. 253.—Raise one of the wheels of a bicycle off the ground, and by hand give the wheel a rapid motion of rotation.

Note what a long time, especially if it be on ball bearings, where the friction is small, the wheel continues to revolve.

With a more massive wheel the effort to increase its speed is greater, and the time during which it continues to run is longer, and it is more difficult to stop.

Ex. 254.—Attach a dynamometer to the heavy mass supported as in fig. 171, and set the mass in motion by a horizontal pull on the dynamometer.

When the mass is swinging back through its central position stop it suddenly by a pull on the dynamometer; record the pull. Note that the mass does not stop instantly, but travels on a small distance.

The reading of the dynamometer is greater when the mass is greater.

Ex. 255.—Set up a simple pulley, as in fig. 165, with the string nearly twice as long as the height of the pulley from the ground. Attach a lighter mass to one end of the string, and a heavier mass to the other end. Let the heavier mass rest on the ground; take hold of the string and pull the lighter mass up nearly to the pulley. Release the string and allow the smaller mass to fall. If the larger mass be not too large, it will be jerked from the ground when the string tightens. Thus a large force is required to stop a quickly moving mass.

These experiments indicate that there is some property of matter by virtue of which—

(*a*) When a body is in motion in a straight line it shows a tendency to move on in that same straight line; and

(*b*) That a moving body can only be stopped by the exercise of force.

This is impressed still further upon us when we consider how difficult it often is to stop a moving object, such as a cricket ball, a swinging door, a swiftly revolving wheel, a heavy truck.

152. FORCE REQUIRED TO CHANGE THE DIRECTION OF MOTION OF A BODY.—The same considerations occur when a moving body is not stopped, but has its motion changed in direction or in speed. When a train travels round a curve, or a swiftly moving car turns a corner, the passengers are thrown

to one side. Which side? It is always away from the new direction that the vehicle is taking and towards the direction in which it was previously travelling. The passengers have to exercise force on themselves to keep their positions.

Ex. 256.—(1) Tie a body to one end of a string, and, holding the other end of the string, whirl the body round in a circle. A pull is felt along the string, and if the string be elastic it is stretched. A swift motion will cause the string to break. The explanation is this. The force along the string is always causing the direction of motion of the body to change. That direction at any instant is the straight line in which it happens to be moving at that instant. To pull it out of that straight line, and cause it to change its direction into a circle, requires the exercise of force.

(2) If you twirl a wet mop rapidly round, the drops of water fly off "at a tangent", as is correctly said.

Each drop moves off in a straight line in the direction in which it was moving at the moment when it left the mop.

Thus we see that if a body is moving it requires the exercise of force to change its direction of motion from the line along which it is travelling at any moment into any other direction.

All the experiments that we can make on this subject are more or less disturbed by the presence of forces which we cannot get rid of, and which complicate it and interfere with the result. But it has been shown that the more completely we remove these disturbing forces the nearer we get to the condition of perpetual maintenance of the motion of a moving body.

There are bodies in motion over which we have no control, but whose motions have been carefully studied. These are the earth, sun, moon, and stars. The earth moves round the sun, and we know that it has been so moving for a very long period—millions of years. And we know that during the period within which accurate measurements have been made it shows no slackening of speed. A similar statement holds good with regard to the moon in its motion round the earth.

All the large astronomical bodies maintain their motions continuously with perfect regularity.

153. From all the considerations and experiments such as those mentioned in Arts. 150 to 152 the conclusion is drawn that all matter has an incapacity to change its state of rest into a state of motion, or a state of motion into a state of rest, or to change the direction of its motion To effect such a change the body must be acted on by an external force. This property of matter, of maintaining the condition in which it is, is called **inertia**. The possession

of inertia is regarded in Dynamics as the one characteristic property of matter. Bodies differ in size, mass, density, &c., but they all possess inertia, and whatever does not possess inertia is not matter.

Force is intimately connected with it. Force is that which overcomes inertia either by moving matter or by stopping its motion, or by changing its direction. All changes of motion are the result of force.

Thus we arrive at the **First Law of Motion**:—

Every body perseveres in its state of rest, or of uniform motion in a straight line, except in so far as it is made to change that state by external force. (For uniform motion, see Art. 160.)

Ex. 257.—(1) Why is it that it requires the united efforts of several men to set a railway truck in motion, but one man can keep it going when it has once been started?

(2) Why is it a greater strain on a horse to start a cart moving than to keep it moving at a slow speed?

(3) Why can a nail be driven in by a blow from a hammer, but not by placing the hammer at rest on the nail?

(4) Give some instances in which the inertia of a moving body is used to overcome the inertia of a body at rest.

CHAPTER XVI

Time. The Pendulum

154. One of the most obvious and universal of our impressions is that of the succession of day and night in regular alternation. From this we form an idea of an interval of time, and as we are only concerned with the comparison of intervals of time we compare other intervals with the interval marked out by a day and a night.

The present arrangements for measuring time are the result of centuries of trial and error and comparison.

155. LENGTH OF THE DAY.—If we set up a thin straight rod vertical on a large sheet of paper, and mark the length of the shadow of the rod thrown by the sun on the paper at frequent intervals during the day, we find that the shadow is of different lengths at different times of the day. Let the position of the shortest shadow be carefully marked.

Let the process be repeated the next day.

Then the interval of time that has elapsed between the formation of the two shortest shadows is a **day**. Since it is determined from the sun, it is called a **solar day.**

Day and night are brought about by the rotation of the earth on its axis. Tests continued for many years in the most accurate manner lead to the conclusion that this rotation takes place with the greatest possible regularity. The period of time taken by the earth to perform one revolution on its axis is always the same. This period is ascertained by astronomers by observing the interval between two successive passages of a star across the centre of their telescopes; it is called the **sidereal day**. Its length is not quite the same as that of a solar day.

The "day" which is used as the basis for the measurement of time in all civilized countries is neither the solar day nor the sidereal day mentioned above. Numerous observations made with accurate instruments and accurate clocks show that the length of the solar day is not always the same; it varies with the seasons. Its length has been measured by a clock for every day throughout the year, and the average of all the values taken. The result is called the **mean solar day**, which is the unit by which time is measured.

The mean solar day is divided into 24 equal parts each called an **hour**. The hour is divided into 60 equal parts each called a **minute**. The minute is divided into 60 equal parts each called a **second**.

In physical science, when we are measuring small intervals of time, the **unit of time** is the **second**, the $\frac{1}{86400}$ part of a mean solar day.

156. MEASUREMENT OF TIME.—A perfect clock is a machine which turns a hand round a circle exactly twice in a mean solar day. By a system or "train" of toothed wheels another hand is made to travel round at the same time twelve times as fast. Thus the hours and the minutes are marked.

All clocks are more or less imperfect. But each astronomical observatory has some of the most accurate clocks that can be made. By means of observations on the sun and stars these clocks are kept right, and all other clocks are regulated by these. The principal observatory for the regulation of time measurement is at Greenwich.

The elementary clock shown in fig. 172 explains the principle by which clocks are made to measure time.

A toothed wheel E called the escapement wheel is rotated by a weight W acting along a cord that is wound round a drum placed on the axis of the escapement wheel. This is the turning mechanism.

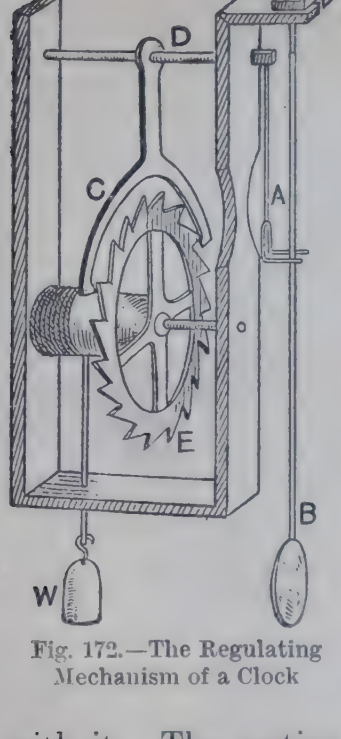

Fig. 172.—The Regulating Mechanism of a Clock

To regulate the rate at which the wheel turns round there is added a regulating mechanism which includes the rest of the apparatus shown in the figure. Above the axis of the escapement wheel there is a spindle D to which is attached a curved piece of metal C called the escapement, which ends in two clutches or pallets that touch the teeth of the wheel. The spindle D carries a forked arm A, and through this fork passes the rod of a pendulum B.

If the regulating mechanism were absent, the weight would rapidly descend to the ground and turn the wheel round very fast. The escapement C divides this motion into very short steps. As the pendulum rod oscillates from side to side it carries the bar A with it. The motion of A causes D to rotate a little, first one way and then the other. In its rotation D carries the escapement C with it, so that one pallet swings outward and the other swings inward. The pallets engage with the teeth of the wheel, so that each oscillation of the escapement allows the wheel to rotate through the distance of one tooth.

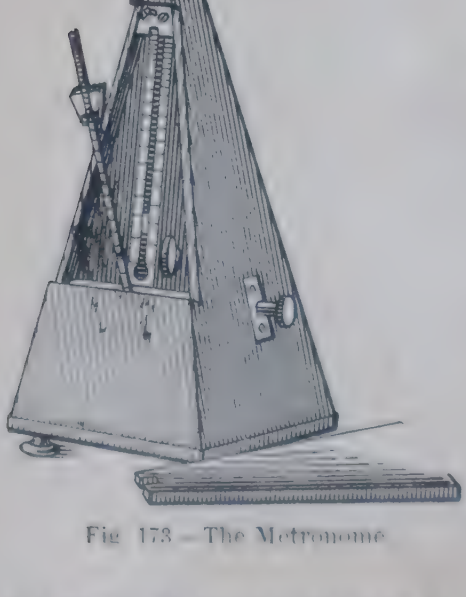
Fig. 173.—The Metronome

The regulating mechanism thus depends upon the pendulum. If the pendulum ticks once a second and the wheel has sixty teeth, the wheel would rotate once a minute. In an ordinary clock there is a train of wheels which turn the hands.

157. METRONOME.—For many laboratory experiments a useful form of time indicator is the metronome (fig. 173), in which a bar oscillates in front of a scale. The bar has a heavy bob at the

bottom, and a pin through it a short distance above the bob. It is kept in motion by a coiled spring. As the bar oscillates it produces a loud ticking. The number of ticks per minute can be altered by sliding a movable mass up or down the bar, the number being indicated on the scale, according to the position of the slider.

158. THE SIMPLE PENDULUM.—We now have to study the pendulum. A simple pendulum consists of a small heavy body called the bob, suspended by a thin thread (fig. 174). The bob is generally a sphere or a double cone; the thread may be of waxed silk or cotton, but it is better to use very fine wire. There must be no "play" at the suspension. This difficulty may be avoided by clamping the thread at the top between two flat metal plates having sharp edges, such as a pair of pliers, or by passing the thread through the eye of a needle stuck into a cork.

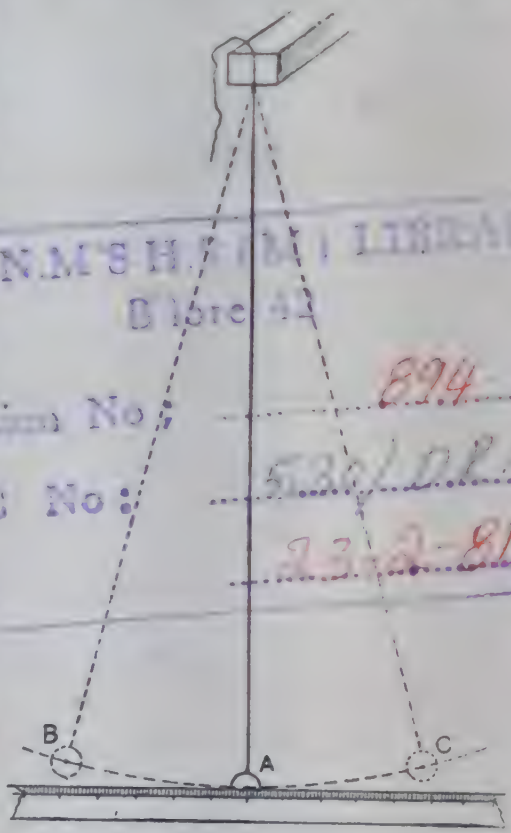

Fig. 174.—The Simple Pendulum

Referring to fig. 174, an **oscillation** is a complete double swing of the bob from B to C and back. The distance from A to B or A to C is called the **amplitude** of the oscillation; the time taken to make one oscillation is called the **period** of the pendulum. The length of the pendulum is the distance from the point of suspension to the centre of gravity of the bob.

Most of the experiments with pendulums require that the period should be determined.

For simple comparative measurements it is sufficient to take the number of seconds occupied by a considerable number—50 or 100 oscillations—and to divide the total number of seconds taken by the number of oscillations.

As there are two operations proceeding simultaneously, two observers are necessary—one to count the oscillations of the pendulum, the other those of the metronome, or in some way to take

the time. The pendulum observer makes a sharp tap on the table when the pendulum passes its middle point, and both begin counting. When the pendulum passes the middle point again in the same direction after 100 oscillations he makes another tap, and the metronome observer gives the number of seconds occupied by these 100 oscillations.

Ex. 258.—To compare the periods of a pendulum having different amplitudes of vibration.

Let the length of the pendulum be about 60 or 80 cm.

Mark on the table or floor the position of the bob when at rest.

Place a centimetre scale level with the middle of the bob (fig. 174).

Draw the bob aside through about 15 cm., and as directed above find its period.

Repeat the operation with starting amplitudes of 12 cm. and of 9 cm. Compare the period obtained in the three cases.

If the experiment has been at all carefully done, the effect of change of amplitude on the period is shown by the values obtained.

Ex. 259.—To find the relation between the length of a pendulum and its period.

(1) Measure the diameter of the bob (supposed spherical), and find its radius.

Place a block under and just touching the bob. Measure the distance from the face of the block to the point of suspension, and subtract the radius of the bob. This gives the length of the pendulum.

With the length about 100 or 120 cm. take the time of 100 oscillations, and hence find the period.

Repeat the observations with several smaller lengths down to 30 or 40 cm.

Writing l for length and t for period, make a table in five columns showing

l	t	$l \div t$	t^2	$l \div t^2$.

Examine the figures, and endeavour to find out a ratio.

(2) Make a graph showing the connection between l and t. As t is small the time of 100 oscillations may be taken in place of t. Such a graph, ABC, is shown in fig. 175, but the numbers from which the graph was made are not given, as they are necessarily the same for all experiments.

The curve is not a straight line. Thus the period of a pendulum is not proportional to its length.

(3) Make another graph showing the connection between the length of a pendulum and the square of its time of oscillation.

This graph DEF, fig. 175, comes out as a straight line, thus showing that the length of a simple pendulum is proportional to the square of its time of oscillation.[1]

(4) Having obtained this second graph note where the graph crosses the line that marks 1 second. Then read off the corresponding length of the pendulum. This length is the length of the seconds pendulum. Its correct value is 99·4 cm.

Set up such a pendulum and compare its time of oscillation with that of a clock (or metronome) ticking seconds.

[1] The two graphs shown in fig. 175 are not on the same scale.

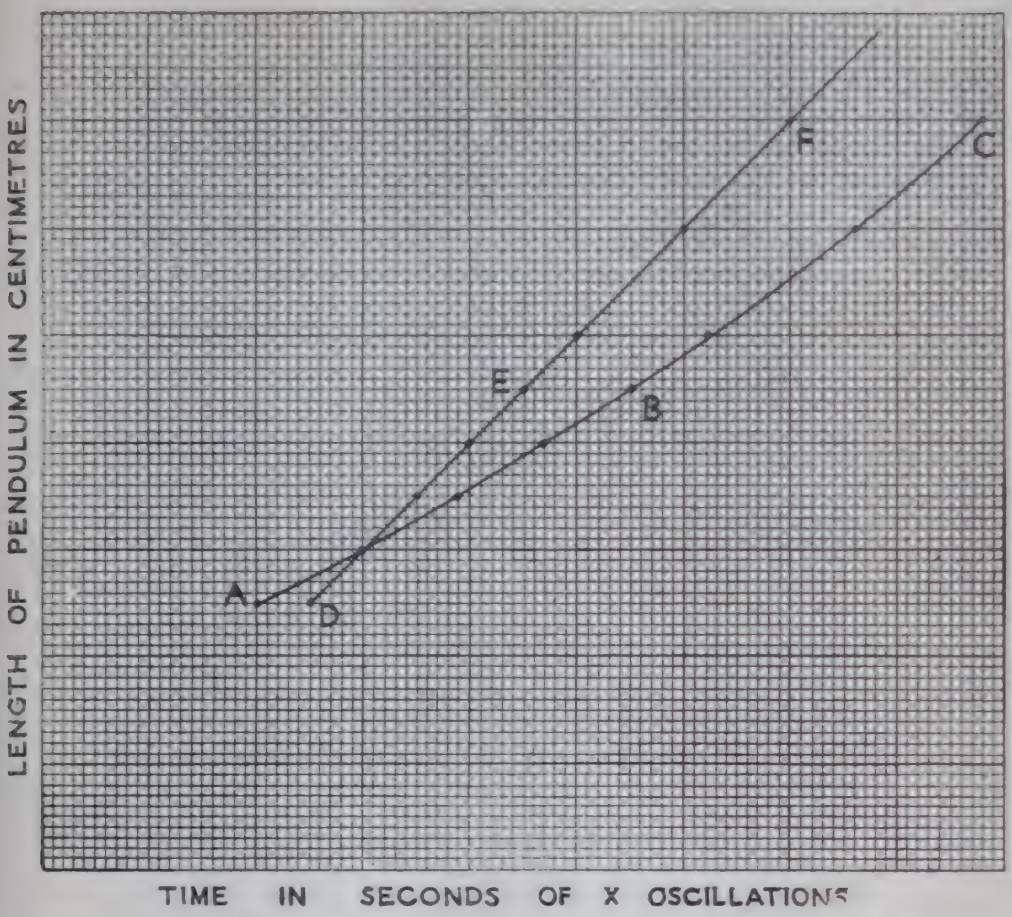

Fig. 175.—Relation between the Length of a Pendulum and (1) its Time of Oscillation (A, B, C); (2) the Square of its Time of Oscillation (D, E, F)

Ex. 260.—To find the effect of the mass of the bob on the period of the pendulum.

Take two pendulums of the same length but with bobs of different mass. It is better for the bobs to be of the same shape and size. Find their periods accurately and compare them.

The experiments show—(*a*) That the time of oscillation of a pendulum does not depend on the substance of the bob nor on its mass.

(*b*) The period is the same whatever the amplitude may be, provided that it is small. This property of a pendulum is called its **isochronism**. The fact that the oscillations are **isochronous**—performed in equal times—is the property on which the value of the pendulum depends for the measurement of time.

(*c*) The period is proportional to the square root of the length of the pendulum.

159. VIBRATIONS OF A ROD OR THIN BAR

Ex. 261.—For this experiment a thin lath of pine wood or a piece of steel spring is necessary. The rod should be firmly fixed at one end, either by clamping it in a vice, or by attaching it firmly to a large block of wood. A vice allows the length to be adjusted more readily.

(1) Pull the free end to one side and release it sharply. The bar springs back and vibrates quickly. If the bar be long the vibrations will be slow enough to count. Count the number made in several seconds, and thus find the period as in the experiment on the pendulum. Measure the length of the vibrating portion of the rod.

Vary the length of the vibrating part of the rod, and find its period of vibration for different lengths.

It will be found that like the pendulum the period is proportional to the square root of the length.

(2) From the figures obtained in your experiment calculate what length of the bar will give 10 vibrations per second.

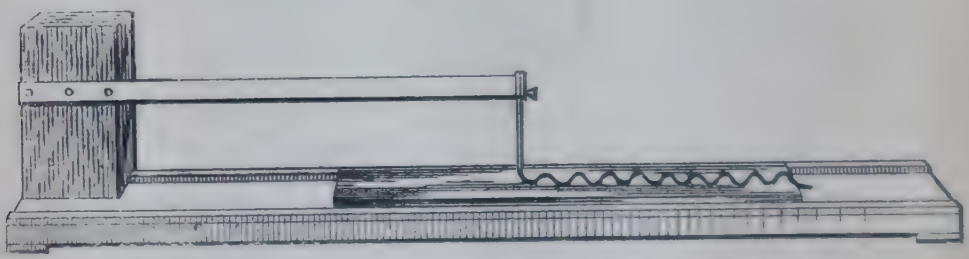

Fig. 176.—Measurement of Time by the Oscillations of a Thin Bar

Ex. 262.—Fasten a light pencil or a small brush dipped in ink to the free end of the rod. Place a strip of paper under the brush so that the brush just touches the paper. Draw the paper along under the brush as it oscillates. A wavy line is traced out. Knowing the period of oscillation, measure a length of 20 or 50 cm. on the paper, count the curves in it, and calculate how long the paper occupied in travelling the measured distance. Such an arrangement is used for measuring distances travelled in small intervals of time.

The vibrations of such a bar are not due to the force of gravitation, but to the elasticity of the bar (Art. 120).

CHAPTER XVII

Uniform and Accelerated Motion

160. RATE OF MOTION.—The measurement of motion involves space and time. A moving body travels a certain distance from some point in a certain time, and we say that it moves faster

or slower according as it takes a longer or shorter time to travel the distance. This is the idea of speed or rate of motion.

In mechanics the word velocity is generally used for rate of motion, and some writers include in the meaning of velocity the idea of the direction in which a body is moving. The **velocity** of motion of a body is $\frac{\text{the distance travelled}}{\text{the time taken}}$, or the distance passed over in unit time.

A body is moving with **unit velocity** when it passes over unit length in unit time, i.e. 1 cm. per second (written 1 cm./sec.). But the velocity of a body may change, hence it is necessary to distinguish between uniform velocity and variable velocity.

A body is said to have a uniform motion, or to be moving with **uniform velocity**, when its rate of motion is constant, that is, when it passes over *equal spaces in equal times.*

A body has a **variable velocity** when it moves through *unequal spaces in equal times.* When this is the case we require to know sometimes its mean velocity during some period and sometimes its velocity at a particular instant.

The **mean velocity** of the body is obtained by dividing the whole distance passed over by the total time occupied.

The **velocity at any instant** is measured by the distance through which the body would travel in unit time if it maintained a uniform motion at the rate at which it was moving at the instant in question.

Questions of motion are often represented graphically.

Below is a time table showing the times at which an express train from A to F reached certain towns on its route:—

Town.		Time.		Distance from A in Kilometres.
A	.....	12 o'clock	.....	
B	.....	1·50 „	.....	160
C	.....	3·40 „	.....	300
D	.....	4·50 „	.....	370
E	.....	6·0 „	.....	450
F	.....	7·50 „	.....	540

From these figures a graph is made (fig. 177), the successive points being joined by straight lines.

If a straight-edge be placed along the graph from A to F it is at once seen that the line is not straight, which means that the speed of the train was not uniform throughout the whole distance- the distance is not proportional to the time.

The line deflects towards the horizontal, which is the direction of the axis of time; that is, the line along which time is measured. This shows that the speed was greatest during the early sections of its journey, and less during the later stages.

Placing the straight-edge along the successive parts of the line, starting with the section A to B, the gradual change in the slope of

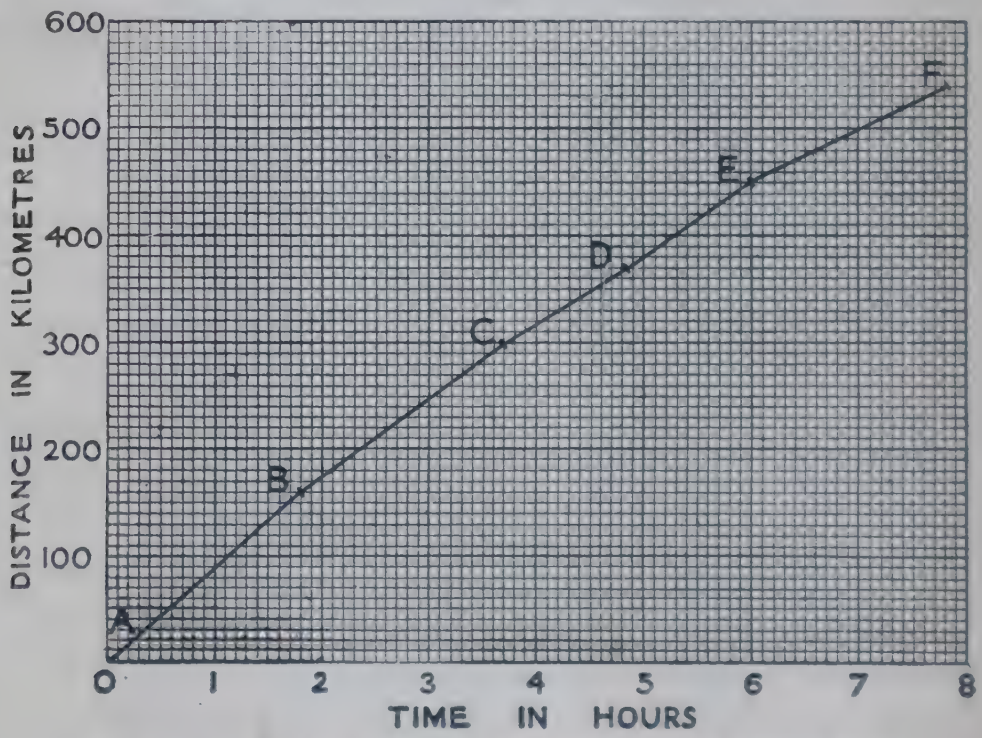

Fig. 177.—Graph of the Motion of a Body moving with Variable Velocity

the line shows that, with one exception (D to E), each succeeding section of the journey was travelled over at a slower speed than the preceding section, the difference being at first small, and in the last section much greater.

Ex. 263.—From the figures in the table given above verify these conclusions by calculating the speed in kilometres per hour for each section of the journey, and the mean speed for the whole journey.

161. ADDITION AND SUBTRACTION OF VELOCITIES.

If a train travelling at 50 miles an hour passes another travelling in the same direction at 30 miles an hour, then we say that the speed of the first train with reference to the second was 20 miles an hour. If the two trains were travelling in opposite directions, then the speed of either train with reference to the other would be 80 miles an hour.

Ex. 264.—(1) A body travels 100 m. in a minute. Express its velocity in centimetres per second.

(2) How far would a body go in 5 min. if its velocity is 30 cm./sec.?

(3) Compare the following velocities:—

(*a*) 60 miles per hour with 22 ft. per minute.

(*b*) 50 Km. per hour with 1000 cm./sec.

(4) The minute hand of a clock is 6 in. long. At what rate does the point move in inches per second?

(5) The hour hand is 10 cm. long. At what rate does the point move?

(6) A steamer is passing up a river at the rate of 12 miles per hour. A man walks up and down the deck lengthwise at the rate of 3 miles per hour. At what rates is he moving with reference to the shore?

162. ACCELERATION.—When a smooth ball starts rolling down a slope, and when an engine starts pulling a train, the velocity of the moving body steadily increases as it proceeds. The motion is therefore said to be **accelerated**. If the brakes are put on a moving train the velocity is diminished; a **retardation** or **negative acceleration** has been communicated to the moving body. We are only concerned with cases in which the acceleration is regular.

Suppose a train to start from rest, and to increase its speed regularly and steadily: suppose that at the end of 1 min. it has travelled 360 m. Then we have:—

Mean Velocity.—Since the train travelled over 36000 cm. in 60 sec. its mean velocity was $36000 \div 60 = 600$ cm. per second.

Final Velocity.—Since the velocity at starting was nothing, the final velocity was double the mean velocity = 1200 cm. per second.

Acceleration.—The velocity increased regularly in 60 sec. from 0 to 1200 cm. per second, therefore every second its velocity was 20 cm. per second greater than in the preceding second.

When the velocity of a moving body regularly increases or decreases the change of velocity that occurs in any second is the measure of the acceleration. The acceleration of the train was 20 cm. *per second per second* (written 20 cm./sec.^{2}).

Confining our attention to bodies that start from rest the above illustration serves for any such body, and we may write the results down in general terms.

Let the space passed over be s,
the time occupied t,
the average velocity during that time u,
the final velocity at the end of that time v,
the acceleration g.

Then by referring to the numerical example above it will be seen that

$$u = \frac{s}{t} \text{ or } s = ut,$$

$$v = 2u.$$

$$\text{(i) } \mathbf{g = \frac{v}{t} \text{ or } v = gt.}$$

From these equations others may be obtained thus:

$$s = ut = \tfrac{1}{2}vt = \tfrac{1}{2}gt^2.$$

$$\therefore \text{ (ii) } \mathbf{s = \tfrac{1}{2}gt^2.}$$

$$v = 2u = 2s \times \frac{1}{t} = 2s \times \frac{g}{v}.$$

$$\therefore \text{ (iii) } \mathbf{v^2 = 2gs.}$$

The equations printed in thick type are those generally required in problems on moving bodies.

Ex. 265.—(1) What is the acceleration of an engine which, starting from rest, gets up a speed of 60 Km. an hour in 4 min.?

(2) What is the negative acceleration or retardation of a train which, travelling at 30 Km. an hour, is brought to rest in 3 min.?

(3) In what time will a body that is moving at the rate of 200 cm. per second have its velocity doubled, if its motion is accelerated at the rate of 5 cm. per second?

163. ACCELERATION PRODUCED BY GRAVITY.—A force that acts on a body which is free to move produces motion and continues to cause an increase in the velocity of the motion as long as it acts; that is, the motion of a body acted on by a force is continuously accelerated.

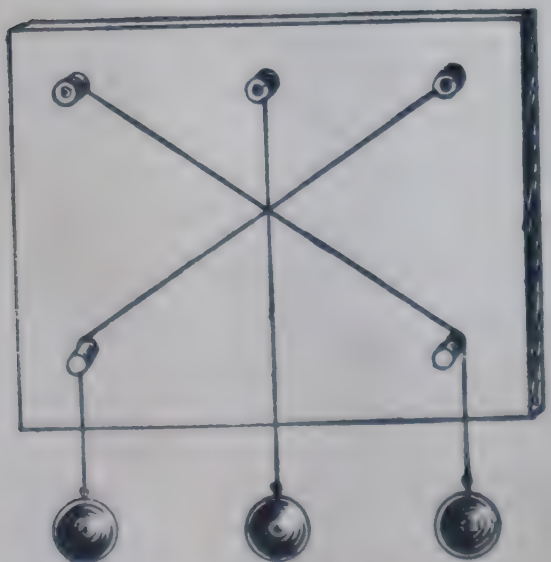

Fig. 178.—Apparatus to show that all Bodies fall at the same Rate

The commonest example of such a motion is that of bodies falling freely under the action of gravity.

Ex. 266.—To show that all bodies fall to the earth at the same rate.

By means of three threads which pass over pins in a board (fig. 178) suspend three balls of different masses—wood, iron, and lead—

suitable. It is better that they should be of the same size. The threads are arranged to pass through one point, and the three masses are at the same height above the ground.

Apply a flame at the point where the threads cross so that the balls are liberated at the same instant.

They strike the ground at the same instant.

All bodies large or small, heavy or light, fall at the same rate, provided that nothing, such as the air, affects one more than another.

Bodies falling freely under the action of gravity move so rapidly that it is difficult to form any correct ideas as to their velocities. The swiftness of the fall is due to the great magnitude of the earth's attractive force. But if it is contrived that a falling body is pulled upwards by a weaker force at the same time that the force of gravitation is pulling it downwards, the motion can be made to take place at any rate we please. Some of the following experiments in which this plan is followed are practicable in any laboratory. All should be made if possible.

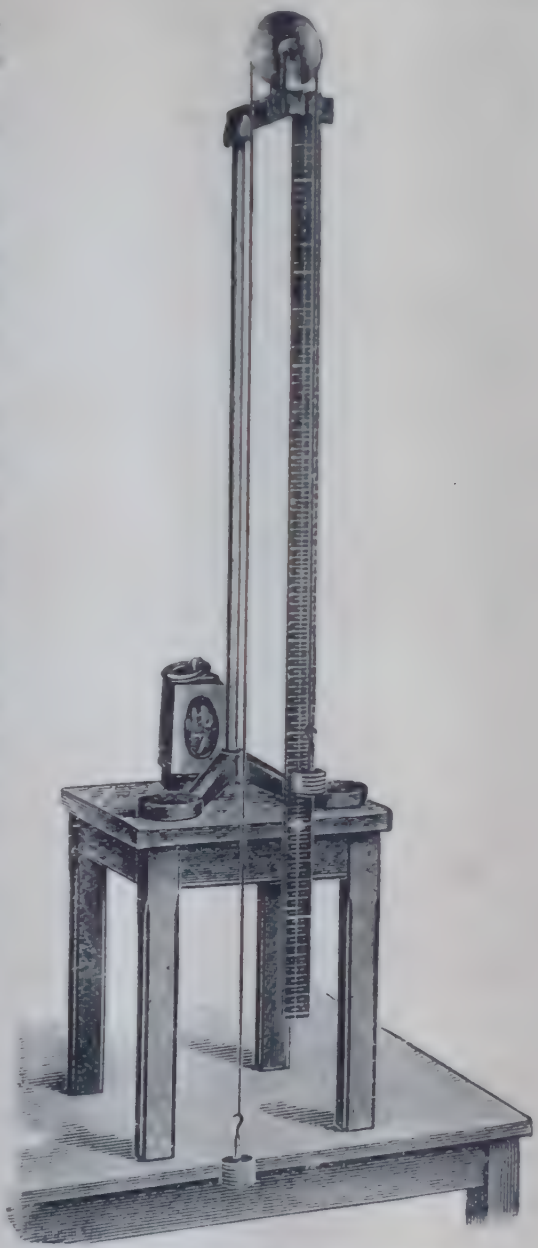

Fig. 179.—Falling Bodies move with Accelerated Velocity

Ex. 267.—To show that a body moving in a straight line under the action of a continuously acting force moves with accelerated velocity.

(1) Set up a single movable pulley in the manner shown in fig. 179, with a metre scale behind the strings. Arrange the weights so that balance is obtained.

Place a metronome or loudly ticking clock beside the apparatus.

To one of the masses add a small weight. Pull this weight up close to the pulley, and hold the string. Liberate the string at a tick of the metronome, and by trial arrange a board so that the descending mass hits the board at the next tick of the metronome.

Make several careful observations, and take the mean of the distances for the space s_1 travelled in 1 sec.

(2) In a similar manner determine the distance s_2 travelled over in 2 sec.

Then s_1 is the distance passed over in the first second.

And $s_2 - s_1$ is the distance passed over in the second second.

The latter is found to be greater than the former.

Ex. 268.—To show that the acceleration is greater when the force is greater.

Repeat the previous experiment with the same masses, but using a little greater added weight.

Find as before s_1 and $s_2 - s_1$. The difference between these spaces is greater than before.

In such experiments the earth's pull upon the added weight is the force: the mass moved is the sum of all the weights and that of the string and the pulley.

The amount of the acceleration is thus seen to depend upon the relation between the moving force and the mass that is moved.

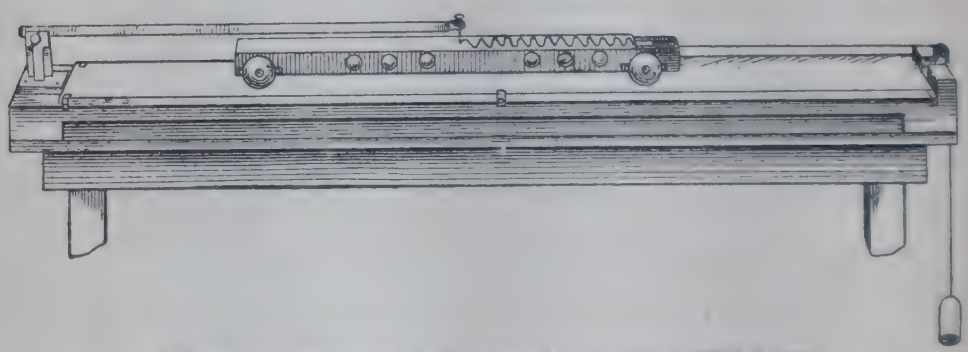

Fig. 180. – Apparatus to Measure Acceleration (Fletcher's)

This relation may be better studied by means of the apparatus shown in fig. 180.

A long heavy trolley, which is the mass to be moved, is placed on a smooth board. The moving force is a weight which is attached to a string that passes over a pulley to the trolley.

On the upper face of the trolley is a sheet of paper. Fixed by an arm to the base board is a vibrating spring which carries a pencil. The length of the spring is adjusted so that it makes (say) ten vibrations a second. When the spring is set in vibration and the trolley in motion a wavy line is traced out on the paper which enables the velocity of motion of the trolley to be found.

Ex. 269.—Weigh the trolley and place weights upon it until the whole mass is say 1 Kg. Put it at its starting point, and exactly parallel to the string. Attach a weight of say 50 grm. to the string, holding the trolley meanwhile. Start the spring and liberate the trolley. Examine the curve traced out.

The line traced out by the pencil is of the form shown in fig. 181, which is drawn on millimetre paper, so as to show on a reduced scale the distances passed over during the first 10 semi oscillations of the spring.

The appearance of the curves at once shows the acceleration.

The following table gives the information to be obtained from the curve:—

Time intervals t = ·1 sec.	Total space s passed over in t intervals	Velocity $v = \frac{2s}{t}$ at end of each interval.	Acceleration g = Velocity added at each interval.
1	·22	·44	
			·24
2	·68	·68	
			·21
3	1·33	·89	
			·21
4	2·2	1·10	
			·24
5	3·34	1·34	
			·23
6	4·7	1·57	
			·23
7	6·3	1·80	
			·22
8	8·09	2·02	
			·23
9	10·11	2·25	
			·22
10	12·33	2·47	

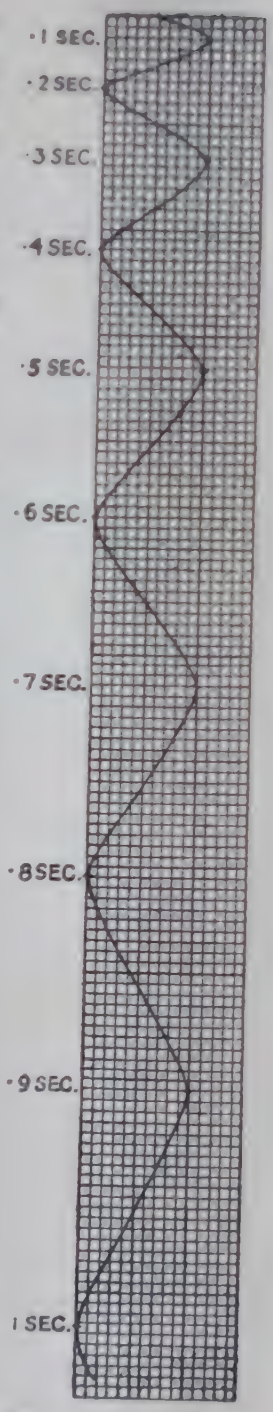

Fig. 181. Curve showing the Acceleration of a Moving Body

The first two columns are obtained from the curve itself. (Verify the numbers.) The third column, which shows the velocity with which the trolley is moving at the end of each interval, is calculated from the values of s and t by the formula $v = 2s \div t$ (Art. 162).

The acceleration is the amount of velocity that is added during each interval, and is obtained from column three by subtracting each velocity from the one that succeeds it. The different values obtained for the acceleration are approximately equal,

the differences being due to inequalities in the apparatus, or to errors in measurement.

Ex. 270.—Repeat Exercise 269 keeping the mass moved the same, but using a different force, say 60 grm.

Find the value of the acceleration as before.

It will be found to be greater with 60 grm. than with 50, in the ratio of 6 to 5.

The acceleration produced is proportional to the force that causes the motion.

164. ATWOOD'S MACHINE.—Fig. 182 shows an Atwood's machine, which enables accurate measurements to be made.

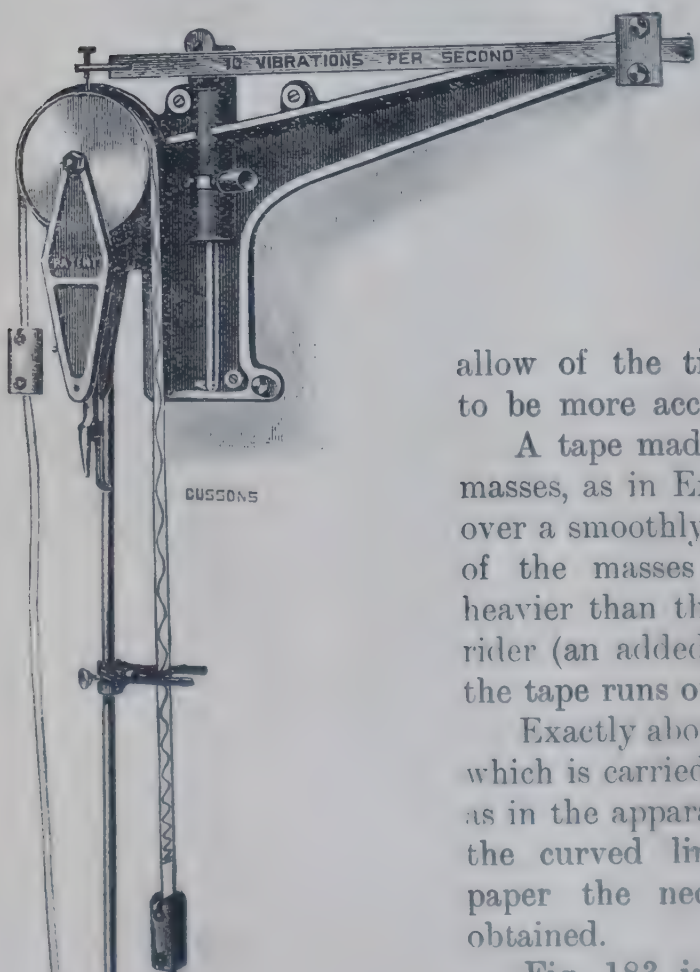

Fig. 182.—Atwood's Machine with Recording Tape (Cussons)

It is like the apparatus of fig. 179, but made so as to reduce friction as far as possible, and to allow of the time taken in the fall to be more accurately measured.

A tape made of paper carries two masses, as in Exercise 267, and passes over a smoothly running pulley. One of the masses being made slightly heavier than the other by means of a rider (an added weight) descends, and the tape runs over the pulley.

Exactly above the pulley is a pencil which is carried by a vibrating spring as in the apparatus of fig. 180. From the curved line traced out on the paper the necessary information is obtained.

Fig. 183 is a copy on a reduced scale of curves obtained in this way. A scale is shown at the side. The vibrator made 10 oscillations per second, so that the time is obtained by counting the bends in the curve; the distance is obtained by measurement of the length of tape.

The first tape shows—

Time.		Distance.
½ sec.	(5 vibrations) ...	8·6 units.
1 „	(10 „) ...	34·2 „
1½ „	(15 „) ...	76 „

The times are in the ratio $1 : 2 : 3$.

The distances are very nearly as $1^2 : 2^2 : 3^2$.

Thus it is shown that the distance fallen is proportional to the square of the time in accordance with the equation $s = \frac{1}{2}gt^2$.

Ex. 271.—Measure the distances in the second tape of fig. 183, and show that the space passed over in 1 sec. is very nearly 4 times that passed over in ½ sec. Repeat for the third tape.

Ex. 272.—Make for each of the curves a table like that from Exercise 269, and deduce the acceleration for each. It is of different value for each curve. The moving weights in the three curves were in the ratio of $1 : 1{\cdot}5 : 2$. The accelerations will be found to be in the same ratio, thus showing that when a force acts on a mass and sets it in motion the acceleration produced is proportional to the force.

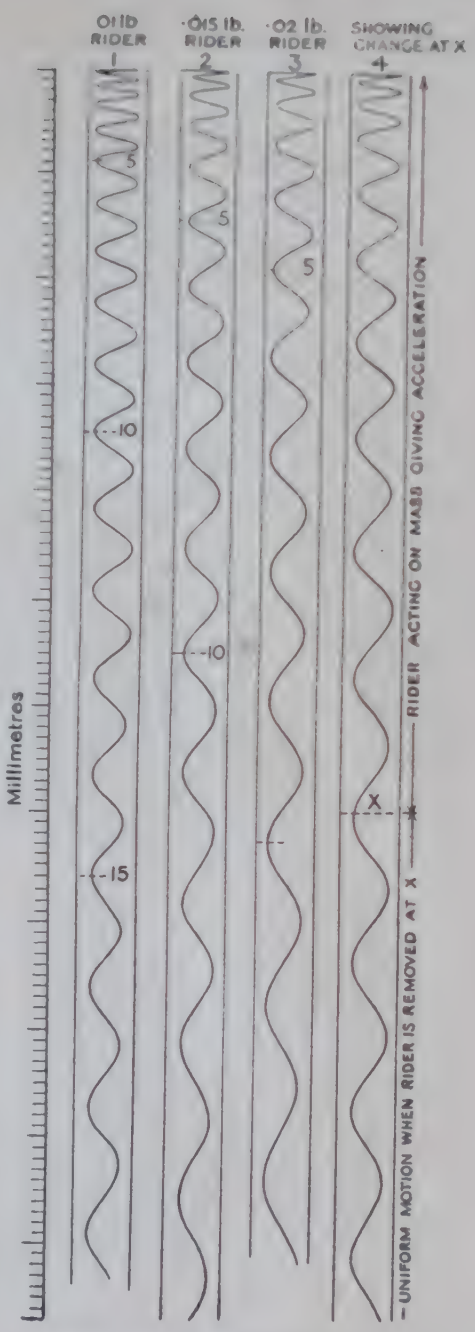

Fig. 183.—Curves taken on an Atwood's Machine. (By permission of Mr. G. Cussons.)

The fourth tape shows the effect when the rider is removed. The removal of the rider took place at the point marked X. Above that point the gradual lengthening of the successive curves shows the acceleration. Below that point the curves are all of equal length, showing that when the force that produced the acceleration was removed, the moving masses continued in motion at a steady uniform rate.

These curves should be carefully studied until they are quite understood.

165. VALUE OF g.—The acceleration of a body which falls

freely under its own weight is called the acceleration due to gravity, and is generally signified by the letter **g**.

From such observations as those above the value of **g** may be calculated.

Thus: In one experiment the two masses were each 340 grm., the wheel was equivalent to 54·5 grm., and the small added weight was 6·8 grm.

The acceleration was found to be 8·97 cm. per second per second.

Then a force of 6·8 grm. acting on a mass of 680 + 54·5 + 6·8 = 741·3 grm. was found to give an acceleration of 8·97 cm./sec.^{2}.

∴ a force of 741·3 grm., i.e. the whole weight of the mass acting on a mass of 741·3 grm. would give an acceleration

$$= \frac{8{\cdot}97 \times 741{\cdot}3}{6{\cdot}8}$$
$$= 978 \text{ cm./sec.}^2$$

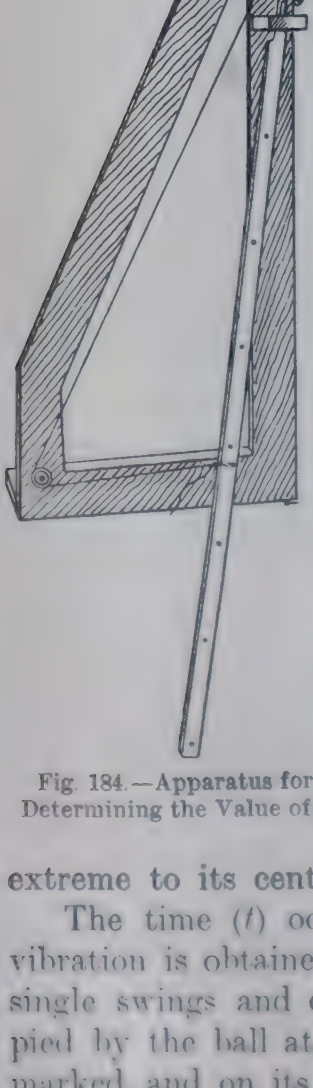

Fig. 184.—Apparatus for Determining the Value of **g**

The accepted value of **g** is 981 cm. per second per second.

The time occupied by a body in falling a known distance under the acceleration of gravity may be ascertained with considerable accuracy by an apparatus such as that shown in fig. 184.

A pendulum in the form of a rod is suspended on a knife-edge. Beside the knife-edge hangs a metal ball. The ball is suspended by a thread which, passing over pulleys, also holds the pendulum at the extremity of its swing. This thread is burned and the pendulum and bob are thus liberated at the same instant. The length of the pendulum and its time of vibration are so arranged that the ball falls the length of the pendulum while the pendulum moves through half a single swing, i.e. from its extreme to its central position.

The time (t) occupied by the pendulum in making this half vibration is obtained by finding the time occupied by 50 complete single swings and dividing this time by 100. The position occupied by the ball at the beginning of the experiment is accurately marked, and on its arrival at the end of its fall the ball marks on

the pendulum where it strikes. The distance between these marks is the distance of falling. The falling ball must be chalked or covered with lampblack to leave a visible trace of its contact.

Then the value of **g** is calculated by placing the values of s and t thus obtained in the equation $s = \frac{1}{2}gt^2$.

Example.—The height of fall in an experiment was 183 cm., and the time of vibration of the pendulum was 1·22 sec.

$$\text{Then } 183 = \tfrac{1}{2}g\,(\cdot 61)^2.$$

$$\therefore g = \frac{366}{(\cdot 61)^2} = 984.$$

In the figure the bob is suspended so as to just touch the side of the rod when the rod is vertical. Then when it reaches the bottom it knocks against the side of the rod and makes a mark.

Greater accuracy is ensured by arranging the top pulley so that the bob is in front of the knife-edge, a plate being fixed on the bottom of the pendulum on which the bob falls. The rod should be long. Its time of vibration may be made suitable by sliding weights that can be held in any desired position by screws.

CHAPTER XVIII

The Second and Third Laws of Motion

166. MOMENTUM.—Suppose that you strike a nail with a hammer. You may hit the nail harder

(i) by making the hammer move faster; or

(ii) by using a more massive hammer.

To give a light blow to a tin tack you use a small hammer, and do not move it very quickly. To split a large hard block a man uses a heavy sledge hammer, and swings it round so as to get up a quick motion.

This difference between a large mass which is moving fast and a small mass which is moving slowly is called a difference in quantity of motion or **momentum** (plural, *momenta*).

All moving bodies possess momentum. A body not in motion possesses no momentum. If a moving body be brought to rest its momentum is destroyed.

Since it is only by the action of force that a body at rest can be

set in motion, or a body in motion be brought to rest, any change in the momentum of a body must be produced by the action of some force.

Ex. 273.—To show the connection of momentum with mass and velocity.

(1) To one end of a piece of string tie a weight, and attach the other end of the string to the hook of a spring balance which is firmly fixed and hangs vertically beside a metre scale. Hold up the weight, and allow it to fall a measured distance. The spring balance shows an extension of the spring.

Repeat for various heights of fall. Record heights of fall and readings of the spring balance. An examination of the readings shows that the longer the fall the greater the extension of the spring. Now, the greater the extension of the spring the greater was the average force that it exercised in bringing the mass to rest; also, the longer the fall of the weight the greater was its velocity of motion when it began to act upon the spring.

Hence the greater the velocity of a moving body the more force is required to destroy its momentum.

(2) Using the same spring balance and the same distance of fall, employ different weights, and record for each the weight and the extension of the spring.

This set of observations shows that the greater the mass of a moving body the greater is the force required to destroy its momentum.

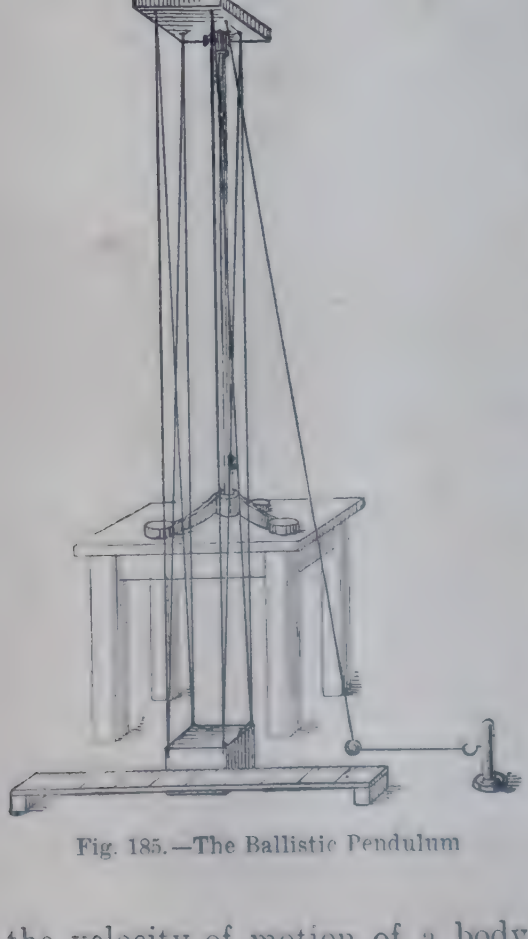

Fig. 185.—The Ballistic Pendulum

The momentum of the moving bodies in Exercise 273 was produced by the force of gravitation, and destroyed by the force of elasticity in the dynamometer. The amount of the momentum of the moving body as shown by its effect on the spring increases with the mass of the body and with the velocity with which it is moving.

The experiments on acceleration in Chap. XVII showed that the velocity of motion of a body which is acted on by a constant force regularly increases with time; its momentum therefore regularly increases with time so long as the force continues to act on it. The product of the force acting, and the time during which it acts, is taken as a measure of the effect of the force in producing momentum. This product is called an *impulse*.

Experiments on momentum may be best made by studying the effects of an impulse of short duration by means of an apparatus such as that shown in fig. 185. A rectangular block of wood whose volume is about a cubic decimetre is suspended by threads of 1 or 1·5 m. in length in the manner shown in the figure.

Beside the block a plumb bob is suspended so that, when hanging freely, the bob just touches a side face of the block.

A horizontal scale is placed alongside the block to measure the distance moved by the block when struck by the plumb bob.

Two or three plumb bobs of unequal masses are required, and their masses must be known.

Ex. 274.—To compare momenta.

(1) Having arranged the ballistic pendulum (fig. 185), draw the plumb bob to one side, and let it fall against the block. A few preliminary experiments will show that the distance through which the block is moved varies according to the distance through which the bob travels. Since the oscillations of a simple pendulum are performed in the same time, the velocity of motion of the bob at the moment when it reaches the lowest point of its path is proportional to the distance through which it has travelled, and is always the same whatever its mass may be; therefore, as we are only dealing with comparative effects, we may enter this distance as a measure of its velocity v. Thus the distance through which the bob travels along the scale is entered as v.

Using the plumb bob whose mass is m, make a careful set of observations with different values of v, and for each read the displacement of the block, repeating each observation two or three times, and taking the mean in order to obtain a more accurate result.

(2) Take a similar set of observations with another plumb bob whose mass is M, arranging your values of V so as to get *the same set of values for the displacements of the block* as before.

Record the results in a table, thus:—

	Displacement of Block.	First Plumb Bob.			Second Plumb Bob.		
		Mass m.	Velocity v.	Mass × Velocity mv.	Mass M.	Velocity V.	Mass × Velocity MV.
1.	... cm.	...	...	...	..	...	...
2.	...	...	...	...	...	...	...
..	...	...	...	...	...	...	

(3) Compare the values of mv and MV in the first line of the table. The displacement of the block was the same in the two cases. Are the products mv and MV approximately the same?

Similarly, compare the pairs of products of mv and MV in the succeeding lines of the table.

Usually none of them quite agree, because of errors in the observations. The errors will probably to some extent cancel each other if you add up the values of mv and the corresponding values of MV.

Experiments of this nature show that if a mass m, moving with a velocity v, strike a body that is at rest, and communicate to the body the same motion as is communicated by another mass moving with a velocity V, then $mv = MV$.

The general idea of momentum given above may now be replaced by the statement:—**The momentum of a moving body is equal to the product of the number of units of mass that it contains by the number of units of velocity with which it is moving.**

The **unit of momentum** is that possessed by unit mass moving with unit velocity. This unit has no universally accepted name.

167. A body that is in motion, and which therefore possesses momentum, may have a new motion given to it and therefore a new momentum. Thus, suppose a lamp is at the bottom of the mast of a ship in motion: the lamp is moving with the ship and possesses momentum in that direction: now suppose it to be hauled steadily to the top of the mast. A new momentum has been given to it by the force which is pulling it up. The total momentum of the lamp has been altered, and its direction has also been changed. The change of momentum is proportional to the force that produced it, just as if the ship had been at rest.

Experience shows further that any change in the momentum of a body is equal to the product of the force that produces it by the time during which the force acts.

$$\text{Momentum} = \text{Force} \times \text{Time}.$$

All these considerations are included in the statement known as the *Second Law of Motion*, which is:—

The change of momentum of a body is equal to the impulse which produces it, and takes place in the direction in which the force acts.

168. ABSOLUTE UNIT OF FORCE.—This law furnishes us with a new method of measuring force, viz. by the momentum that it imparts to a body.

In the previous chapters, forces were measured by comparing them with the weight of 1 cub. cm. of water. But this weight differs in different places. Hence, although practically this unit

force is very convenient, it does not fulfil the scientific requirement of constancy.

But the same impulse acting on a body always produces the same change of momentum. We therefore define the **absolute or dynamical unit of force** as that force which, acting on unit mass for one second, changes its momentum by one unit.

Employing C.G.S. units the same statement may be made in this form:—The absolute unit of force is that force which, when applied to a mass of one gramme during one second, imparts to the mass a velocity of one centimetre per second. This unit of force is called a **dyne**.

The connection between the dyne and the weight of 1 grm. may be thus shown. Suppose a mass of 1 grm. be allowed to fall for 1 sec. Then its velocity at the end of the second is 981 cm. per second; to give it a velocity of only 1 cm. per second there would only be required a force $= \frac{1}{981}$ grm.

$$\text{Thus 1 dyne} = \frac{1}{981} \text{ the weight of 1 grm.}$$

In the same way the absolute unit of force in the British system, which is called the **poundal** $= \frac{1}{32{\cdot}2}$ the weight of a pound, because 32·2 ft. = 981 cm.

Example.—The following example illustrates the connections between the various quantities. Suppose that a force of 20 dynes has acted for 30 sec. on a mass of 10 grm. originally at rest; then we have:—

(*a*) Impulse = Force × time = 600 units.
$\therefore$ Momentum = 600 units.

(*b*) Momentum = Mass × velocity
$\therefore$ Velocity = 600 ÷ 10 = 60 cm./sec.

(*c*) Velocity = Acceleration × time
$\therefore$ Acceleration = 60 ÷ 30 = 2 cm./sec.^{2}

(*d*) Space passed over = Mean velocity × time
= 30 × 30 = 900 cm.

Ex. 275.—(1) Calculate the momentum of a mass of 20 grm. which has fallen from rest for 5 sec.

(2) Compare the momentum of a cannon ball whose mass is 100 Kg., moving with a velocity of 600 m. per second, with that of a locomotive engine whose mass is 50000 Kg., moving with a velocity of 40 Km. an hour. (Reduce the quantities to grammes and centimetres.)

(3) Calculate the acceleration produced and the momentum acquired in half a minute—

(*a*) When a force of 40 dynes acts on a mass of 1 grm.
(*b*) When a force of 32 poundals acts on a mass of 64 lb.

(4) A mass of 1 Kg. is moving at the rate of 10 m. per second. A force is then set to act on it, and 1 min. later the body is found to be moving at the same rate in the opposite direction. Calculate the value of the force that was introduced.

169. ACTION AND REACTION. — When one body acts forcibly on another we often direct our attention only to one of the bodies. But both are affected, and forces are called into play in opposite directions.

If you strike an object with your hand you feel that the effect is the same as though the object struck you.

If a glass flask be knocked against a stone the effect is the same as if the stone were knocked against the flask.

The experiments in Chap. X show that when pressure is exerted on a confined mass of air the air presses back.

In Experiment 124, where a light body was pushed down into water, the water exerted a backthrust.

When a force is exerted on the spring of a dynamometer so as to extend it, the spring exerts a force in the opposite direction.

In many cases it is not so obvious that there is a second force acting against the first. A book placed on a table presses on the table. But the table must press back against the book; otherwise the book would fall under the influence of the force of gravitation. If a man stand on a piece of board, upward from which project two handles, he cannot by lifting vertically raise himself and the board. The upward pull which he exercises with his arms is exactly counteracted by an increase in the downward pressure transmitted through his feet.

Ex. 276.—Repeat Exercise 190, first attaching the dynamometer to the keeper, and having the magnet fixed; and secondly having the keeper fixed, and applying the dynamometer to the magnet. Compare the pulls in the two cases. If gravity is acting on one of the bodies, its effect (i.e. the weight of the keeper or magnet hanging on the dynamometer) must be subtracted from the pull registered by the dynamometer, in order to get the magnetic force alone.

Thus not only does the magnet attract the iron, but the iron attracts the magnet exactly as much. The attraction is mutual.

This experiment illustrates the action of the earth on the bodies we deal with. The earth pulls them down. They pull the earth up. We see bodies fall under the influence of this force, but we cannot see the earth rise to meet them, because, owing to its great mass, the distance through which it moves is extremely small.

In the same way, if one sits in a boat on a lake and pulls at one

end of a rope attached to another unmoored boat, both boats move visibly towards each other. But if the object at the other end of the rope is a small block of wood, only the smaller mass will be seen to move. In reality both move, one much, the other little.

Ex. 277.—Take a tall vessel with a hole closed by a cork or tap near the bottom of the vessel, fig. 186.

Fill it with water and float it upright in a larger vessel of water.

On opening the tap, water flows out, and the vessel moves backward in a direction opposite to the flow of the water.

The pressure of the fluid in the vessel forces out a jet of water, and at the same time a pressure is produced in the opposite direction.

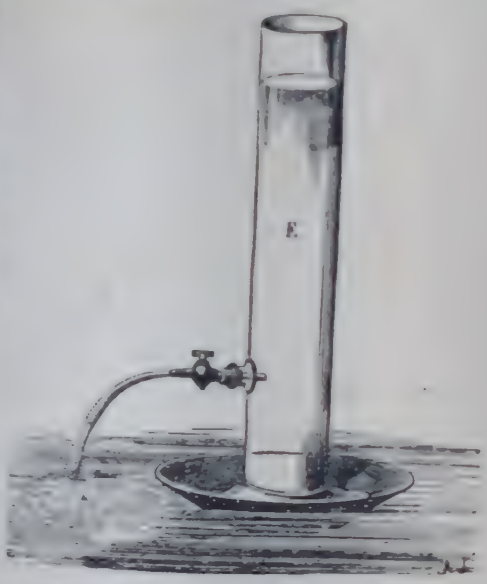

Fig. 186.—Recoil of a Discharging Vessel

This result is similar to that which occurs in the firing of a gun loaded with ball. The ball is propelled forward and the rifle backward. The "kick" back of a rifle is very decided. In large guns, such as those used on war ships, the recoil of the gun is very great.

This principle is expressed in the *Third Law of Motion*:—

To every action there is always an equal and contrary reaction.

Ex. 277*a*.—Point out how the third law of motion is concerned in the act of walking, in the starting of a cart wheel, and in the propulsion of a bicycle.

CHAPTER XIX

Principle of Work

170. WORK.—Suppose a man carries a load from the bottom to the top of a building, or wheels a barrow along a road, or digs the earth, or adds up columns of figures, we ordinarily say he does work.

In mechanics we are not concerned with mental work, but only with work which is done by the exercise of force on matter.

And whatever exercises force, whether it is man, animal, earth or magnet, as described in Chap. XI, is said to do work, when the matter acted on moves against the action of the force.

Suppose you push hard all day against a wall, in mechanics we should say that you have done no work. If you push a truck along a level road all day, we should say that you had done work (against friction).

The simplest example of work is that done in raising a weight. A man lifts 1 Kg. weight up through a height of 1 m.: he does a certain amount of work. If the height is 2 m., he does twice as much; if the weight is 2 Kg., he does again twice as much.

In this way work is measured. The practical unit of work is the amount of work done when 1 grm. is raised through a height of 1 cm. This unit is called a **gramme-centimetre**.

In English units this statement becomes: A unit of work (called the **foot-pound**) is done when the weight of 1 lb. is raised through 1 ft.

Ex. 278.—Find the number of units of work done in the following cases:—

(1) A weight of 25 grm. is raised through a height 16 cm.

(2) A man draws 16 buckets of water up a well. The bucket weighs 3 lb. and holds 10 litres. The depth of the well is 25 m.

(3) By a single fixed pulley a man raises 1000 bricks, each weighing 3 lb. on to a scaffold 52 ft. above the ground.

We have seen in Chap. XI that the reason why matter has weight is that the earth is pulling it. Therefore in the instances given in the previous exercise the reason why work had to be done in raising the masses was because there was a force opposing the motion.

Work is done whenever a body is moved against an opposing force.

Ex. 279.—Drag a body steadily along a table by means of a spring balance. Record the distance the body moves (x cm.), and the reading of the dynamometer (y grm.). Then the work done is xy grm.-cm.

In this example there is no thought of the mass that is moved. The result might have been just the same if the mass had been different and the smoothness of the table also different.

Thus the essential idea of work is that a force is exercised through a certain distance. And *the number of units of work done = the number of units of force × the number of units of length through which the force has worked.*

Ex. 280.—Calculate the number of units of work done in the following cases:—

(1) A man weighing 70 Kg. climbs a mountain 4000 m. high.
(2) A steam engine working a pump raises 10000 litres of water from a well 50 m. deep.
(3) A boy throws a stone weighing 300 grm. to a height of 20 m.
(4) An engine pulls a train for 30 miles against a resistance equal to the weight of 900 lb.

171. WORK DONE ON AND BY A MACHINE.—We are only concerned here with the principle of work so far as is necessary to an understanding of the simple machines when they are at work.

Ex. 281.—Set up the simple arrangement of pulleys shown in fig. 166.

Arrange the values of F and W so that they remain at rest in any position. Then measure their heights above the table.

Allow F and W to take up another position, F having descended and W ascended, and again measure the heights.

F has descended a distance p, W has ascended a distance q. Find the product $F \times p$ (the work done by the effort), and the product $W \times q$ (the work done on the load).

Repeat the experiment with different distances of movement and different values of F and W. In each case find the products $F \times p$ and $W \times q$. Write them all down in two columns, and compare them.

(*a*) The numerical results obtained in this experiment show that the product of the effort and the distance through which it has moved is equal to the product of the weight and the distance through which it has moved; or, **the work done by the effort is equal to the work done on the load**. This is a very important principle of machines.

(*b*) Find the values of the ratios $\frac{p}{q}$ and $\frac{W}{F}$. The ratio $\frac{p}{q}$ $= \frac{\text{distance moved by the effort}}{\text{distance moved by the load}}$ is called the **velocity ratio** of the machine. The results show that the velocity ratio, if friction is absent, is equal to the mechanical advantage (Exercise 247).

Ex. 282.—Wheel and Axle.—Using the apparatus of fig. 158, make a set of experiments exactly as in the last exercise, obtaining a number of values of F and p and the corresponding values of W and q. Then—

(*a*) Compare $F \times p$ with $W \times q$, that is, the work done on each side.

(*b*) Obtain the velocity ratio $\frac{p}{q}$, and compare it with the mechanical advantage obtained in Exercise 241. Are they equal?

Ex. 283.—The Lever. Obtaining exact balance, as in fig. 153, displace the

beam a little from the horizontal, so that the load at the end of the shorter arm ascends and the effort at the end of the longer arm descends.

Measure the distances moved through, and make the same determinations and comparisons as in the two previous exercises.

172. The hydraulic or Bramah press is a machine by means of which very great resistance is overcome by a small effort.

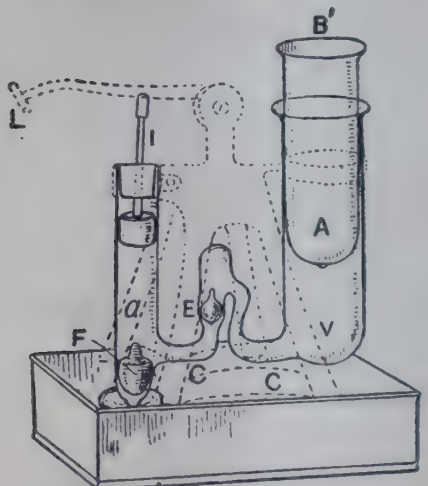

Fig. 187.—Model of a Hydraulic Press

Fig. 187 shows a glass model, with the iron framework indicated by dotted lines, and fig. 188 a section of a working type of hydraulic press.

One side of the press consists of a pump, the piston or plunger of which works in a small cylinder *a*. When the piston rises, water enters through a valve shown at F in the model. This valve opens upwards. When the piston of the pump descends, the valve F closes, and the water is forced along the tube C, in which there is a valve E, that allows it to pass but not to return.

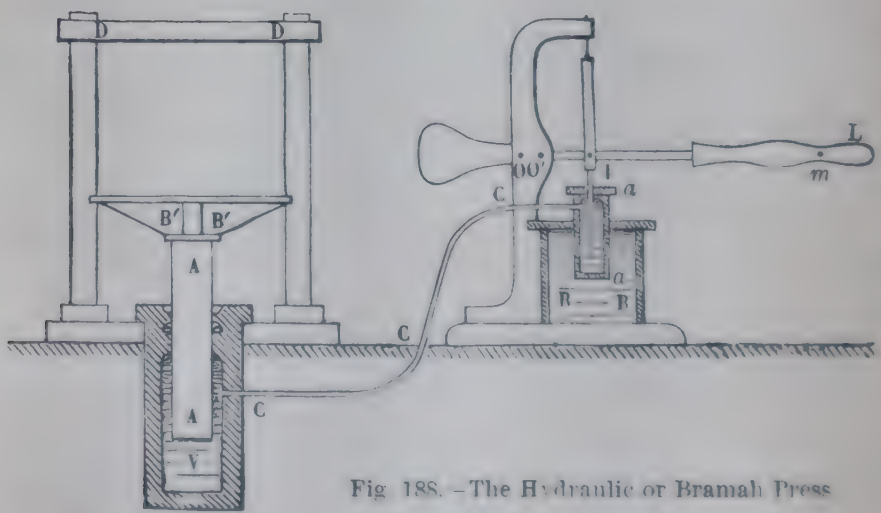

Fig. 188.—The Hydraulic or Bramah Press

Thus at each stroke of the pump water passes along C into the strong chamber V. Into this chamber there passes a plunger which carries at its top a platform B′. Thus the action of the pump, owing to the transmission of the pressure by the water, causes A to rise, and a heavy weight on B′ may thereby be raised. The sectional area of the plunger A is much larger than that of

If A has 100 times the area of *a* then a pressure of 1 lb. on *a* produces a pressure of 100 lb. on A. The lever L allows of considerable pressure being exercised on *a*, and therefore of very great pressure on A. The platform B′ (fig. 188) often has a framework round it. Any object placed on B′ is thus pressed against the top D with great force.

The mechanical advantage of the hydraulic press may be made very large indeed by making the arms of the lever very unequal and applying the effort to the long arm, and by making one piston much larger than the other.

But the work done by the effort is here, too, as much as or more than that done by the resistance. For many strokes of the pump have to be made in order to force up the large piston through a small distance.

173. The above experiments illustrate and to some extent demonstrate what machines do.

Exercise 281 shows that when a man raises a weight by means of a pulley he does the full amount of work required to raise the weight. Suppose he has to get a ton weight up from the bottom of a hole. He cannot move it alone. He attaches a rope to it and uses a pulley of many sheaves. He pulls, and a great length of rope passes through his hands. He exerts a small effort through a long distance, and the heavy weight is raised a short distance. The pulley does no work. The man does the work.

Similarly with the other machines. An effort is applied to some part of the machine, and work is done at some other part of the machine; but this work done is never more (and is generally less) than the work done by the man or engine that exerts the effort.

No machine multiplies the work done by the agent.

The uses of a machine are (*a*) to enable force that is applied at one point to move matter at some other point; (*b*) to enable an agent exerting a small effort rough a long distance to overcome a great resistance through a short distance.

PART IV.—HEAT

CHAPTER XX

General Effects of Heat on Matter

174. COMBUSTION OF FUELS.—Ordinary experience shows that some things will burn and others will not

Ex. 284.—Light a candle. Note the appearances of the flame as it travels down the wick. A new process now begins. A cup-shaped hollow is formed partly filled with liquid.

Observe the flame and the wick—the colours of different parts (fig. 189). From these appearances, which is the hottest part of the wick? which the coldest?

Blow out the candle. Note the long trail of whitish smoke. Quickly put a lighted match in this smoke about an inch off the wick. What is the result?

Thus the effect of heat upon the material of which the candle is made is to turn the solid into liquid, and the liquid into smoke which can be set fire to. The candle itself is slowly disappearing, "burning away", and heat and light are given out.

What becomes of the material of which the candle is made? To answer this question, hold the bottom of a flask full of cold water above the flame so as to just touch the wick.

What appears on the bottom of the flask? What on the sides?

Fig. 189.—Structure of a Candle Flame

We have got soot (carbon) and water from the burning of a candle. But the candle has gone and cannot be recovered.

Ex. 285.—(1) Place some pieces of solid paraffin in a test tube fitted with

a cork and delivery tube, as in fig. 190, and heat the substance carefully until it boils. Light the gas that issues from the delivery tube.

The flame does not fill the test tube. The issuing gas only burns when air is present.

(2) Place a narrow tube over a lighted candle, so that no air can get in at the bottom of the tube.

Does the candle burn?

Most substances will not burn unless air is supplied. (Ex. 325.)

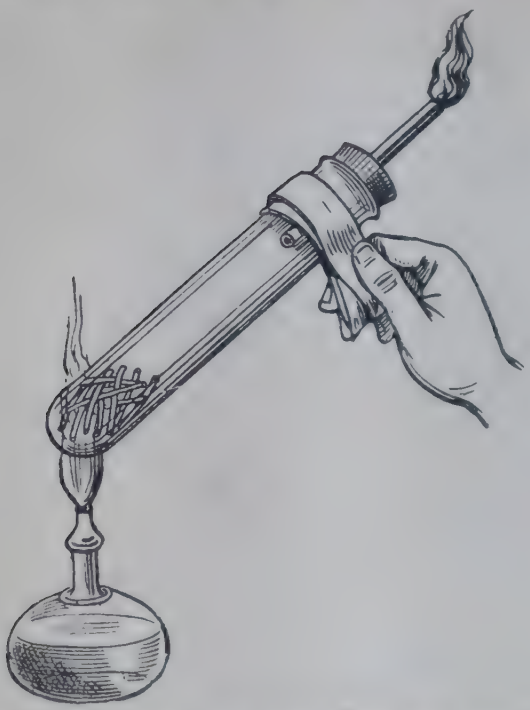

Fig. 190.—The Burning of Gas from Solid Paraffin

An examination of a lighted lamp shows the same general results, the main difference being that the material used is liquid already and requires no melting. The liquid is converted into a gas and the gas burns.

The substances burned in candles and most forms of lamps are fat, tallow, and oil. Tallow is the name given to fat procured from certain animals. Oil is obtained from various sources; some from animals, some from vegetables, some from mineral substances.

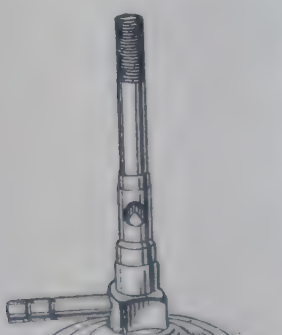

Fig. 191.—The Bunsen Burner

Ex. 286.—Coal Gas. Light a Bunsen burner. This heating apparatus, fig. 191, consists of a fine gas jet inside a tube, at the bottom of which are holes to admit air.

Examine the flame; note the colour and the small amount of light it gives out. Put a piece of iron wire in the flame. The effect on the wire shows that the flame is very hot.

Hold a flask of water over the flame as before, but not too near. What appears on the flask?

Close the air holes at the bottom of the burner by means of the metal slide. Note the change in the appearance of the flame. Place a cold flask over this flame and compare the effect with that previously obtained and also with that obtained in Exercise 284.

Place a piece of iron wire in the flame, and try to judge whether the yellow flame is as hot as the blue flame.

This experiment confirms the last, that air plays an important part in the burning of bodies.

Ex. 287.—Coal. Into a clay pipe place enough coal dust to about three-fourths fill the bowl. Cover in the top with a plug of moist clay, and place the pipe bowl in a flame or fire (fig. 192).

Note the smoke issuing from the stem. Hold a flask containing cold water in the smoke. What is the nature of the deposits on the cold surface?

Apply a lighted taper to the smoke. Hold a cold flask just above the flame. Examine the deposits on the flask.

Continue the heating until no more smoke issues from the stem. Allow the pipe to cool. Shake out and examine the contents. We have a mass of charcoal (carbon). The gas that burned is substantially the same as that supplied to light towns, but contains impurities.

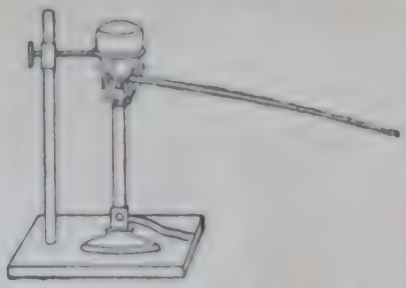

Fig. 192.—The Manufacture of Coal Gas

Repeat the experiment with the apparatus shown in fig. 193. A tarry liquid condenses in the cold part of the apparatus. From the gas with which towns are supplied the tar has been extracted at the gas works.

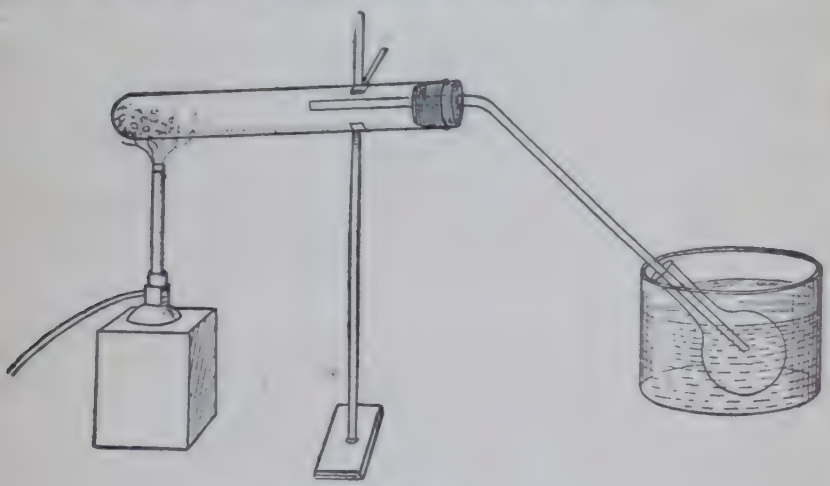

Fig. 193.—A Distilling Apparatus

The apparatus shown in fig. 190 is really the same as that shown in fig. 192. Most of the experiments in this section can be made with a similar apparatus. In some cases, instead of burning the gas that comes off, it is desirable to condense it. This may be done by altering the arrangement to that shown in fig. 193, where the delivery tube is shown leading into a small flask immersed in cold water. Such an apparatus is really an elementary distilling apparatus.

The arrangement shown in fig. 190 can be converted into that of fig. 193 at any moment by an indiarubber connection. The test tube is best fixed nearly horizontal.

Ex. 288.—Wood. Using the apparatus of fig. 190, fragments of wood or

sawdust are placed in the tube and strongly heated. The wood becomes charred. Gas is given off: light it as it issues from the delivery tube.

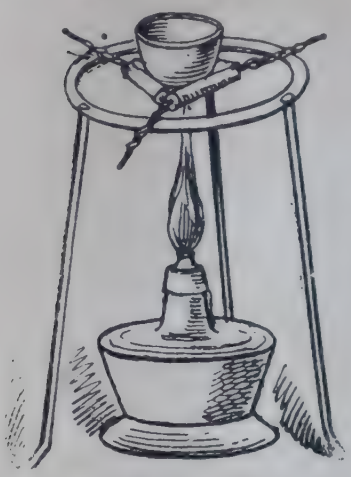

Fig. 194.—Heating Matter in a Crucible

Blow out the flame, and add the condensing apparatus, fig. 193. Note the formation of water, and of a brown liquid in the cold flask. The latter contains tar.

After a time, gases cease to come off from the wood, and charcoal is left in the tube. If the charcoal be taken out, placed in an iron spoon, and kept white-hot for a time in a fire or Bunsen flame, it will gradually get smaller until nothing is left but a greyish-white ash, which is not carbon.

Leaves of plants—green or dried—may be treated in the same way. Compare the amounts of water given off in the two cases. The residue left is mainly carbon, as was the case with wood.

A small plant, such as a root of grass free from earth, may be placed on an iron dish or in a porcelain crucible (fig. 194) and heated slowly. A large quantity of water vapour is given off; the plant turns black and shrivels up, the remains being chiefly carbon.

175. ORGANIC SUBSTANCES. — Animal matter and foodstuffs may be treated in much the same way, the changes taking place generally with less heating.

Ex. 289.—Lean Meat. Placing some small pieces in the apparatus of fig. 193, apply heat gently. The meat gives off a quantity of vapour, which may be collected on a cold surface, and proves to be water.

A charred black mass is left.

Ex. 290.—Blood. Use the apparatus of fig. 193. Place a small quantity of blood in the test tube, and heat it gradually and not too strongly. Liquid condenses in the cold vessel C. Note its appearance and smell.

There remains in the test tube a black solid mass, which, when dry, is brittle. Powder a portion of this black mass; put the powder into an evaporating dish and heat it. It may at first swell up, but after a time most of it burns away, leaving only a small quantity of ash that will not burn.

Ex. 291.—Bone. Take a piece of fresh bone, such as the leg bone of a fowl. Hold it by tongs in a hot fire or flame. Vapours are given off. When these cease to rise from it remove the bone from the flame, and allow it to cool. The substance that remains is called bone ash: it is white and brittle, of much the same nature as egg-shell.

Bread may be treated as in Exercise 288.

Ex. 292. Sugar. Crush some sugar into powder. Place it in an evaporating dish (fig. 195), and heat it gently.

The sugar melts into a pale-yellow liquid which gradually darkens in colour. Continue the heating until the liquid boils. Clouds of vapour are given off

Place a lighted match in the vapour. It is extinguished by the water that is present.

When the water has all been driven off, again apply a lighted match above the mass. At a certain stage the whole takes fire, and swells up into a spongy mass. This is carbon.

Fig. 195

Ex. 293.—Starch. Starch is an important article of food. Rice, potato, arrowroot all contain starch.

Using the apparatus of fig. 193, place a small quantity of dry powdered starch in the tube and heat it. The starch becomes charred, and water is driven off into the cold vessel. Again we have left in the tube a mass which is mainly carbon.

Ex. 294.—Milk. Place a small quantity in a porcelain dish. Heat it slowly and carefully, stirring it occasionally with a glass rod, and taking care that the milk does not quite boil. Vapour comes off. Hold a cold flask in the vapour. Water condenses on the flask.

If the heating be continued nearly all the water is driven off. The solid mass that is left contains carbon, and small quantities of iron and other substances.

176. Many mineral substances are changed by heat.

Ex. 295.—(1) Heat a piece of gypsum in the apparatus of fig. 193. Water is given off, and a white powder—plaster of Paris—is left.

(2) Heat a piece of marble strongly in an iron spoon in a fire. Gas is driven off; the marble loses weight; and quicklime is left.

177. METALS.—Metals do not as a rule burn under ordinary conditions.

But zinc, lead, and iron in the state of fine powder will burn if small quantities of the powder are taken between the finger and thumb and allowed to fall into a hot flame, such as that of a Bunsen burner. Zinc shavings also burn readily. Even iron burns vigorously in large quantities when air is forced through it in a Bessemer furnace.

The metal that burns the most easily is magnesium. If a short piece of magnesium ribbon be held with tongs and one end placed in a hot flame, the metal burns rapidly and gives out a brilliant light.

But although metals do not usually burn they are greatly affected by heat.

Ex. 296.—Take a small strip of copper foil, and another of sheet iron, and clean them with emery paper. Then hold them by tongs in a flame or fire so that they become strongly heated.

Note the colours that appear on the bright surfaces. After some time the copper will appear black and the iron grey. When this has taken place allow the

metal to fall into cold water. Scales may fall off. If they do not, scrape the surface. The detached parts are not at all like the original metal, but the scraping shows the bright metal underneath.

By weighing before and after the heating it is found that the metal increases in mass.

Ex. 297.—Heat a quantity of scraps of tin in a porcelain crucible. The tin soon melts. Note the colours on the surface, and the formation of a film. Scrape the film to one side a little and disclose the metal.

Allow the mass to cool a little until a surface crust has formed. Pierce the crust, and pour out the liquid. Examine the under surface of the crust. Tin is crystalline in structure.

178. TEMPERING AND ANNEALING.—Steel is made by mixing very small quantities of other substances with iron when it is in the condition of a hot liquid, and there are many different kinds of steel differing in composition.

Ordinary steel of which tools are made is iron that contains a little carbon, and has been prepared in a special manner.

Ex. 298.—Treat a few small pieces of clean bright steel (we will call them A, B, C, D for facility of reference), such as needles, knitting needles, pieces of watch spring, in the following manner:—

(A) Make red-hot, and allow to cool slowly. When cold this steel is rather soft, and may be scratched by a knife or a sharp piece of glass. It will bend rather than break.

(B) Make red-hot, and cool suddenly by quickly dropping it into water. Such steel is very hard, and will be found to scratch iron and glass. It will not bend, but breaks.

Prepare several pieces, B, C, D, of this hard steel.

Take three pieces of hardened steel, brighten their surfaces, and heat them slowly and gradually, watching the surface carefully. The steel changes colour, becoming first of a light straw colour, then, as it gets hotter, yellow, brown, and blue.

When B first shows the faint straw colour, drop it quickly into cold water. Do the same with C when it is brown; and with D when it is blue. The whole process through which B, C, and D have gone is called **tempering**. The three pieces now show different degrees of hardness. B is of the temper of which tools are made for cutting metal; C is of the temper suitable for knives and chisels; D is of the temper suitable for making springs, saws, and magnets (see Art. 307).

Ex. 299.—Wind some thin steel wire round a circular ruler, and thus make two similar coils or springs. Temper them, one as B above and one as D. Suspend them by one end, hang a small weight on each, and allow them to oscillate up and down. Note the differences in their stretching, and in the rate of their vertical oscillations.

179. GLASS.—Experiments showing the effect of heat on glass may be made on glass tube, the elementary manipulation of which is useful in a laboratory.

Ex. 300.—Take a piece of soft quill tube, and hold it lightly in the fingers over a flame. The flame should be flat and long, such as that of a paraffin lamp or a bat's-wing gas burner.

Bring the tube slowly up into the flame, as shown in fig. 197, and steadily rotate it. The glass becomes red-hot and softens, and may be bent, as shown in fig. 196, where (*a*) is a proper even bend, and (*b*) is a bad one.

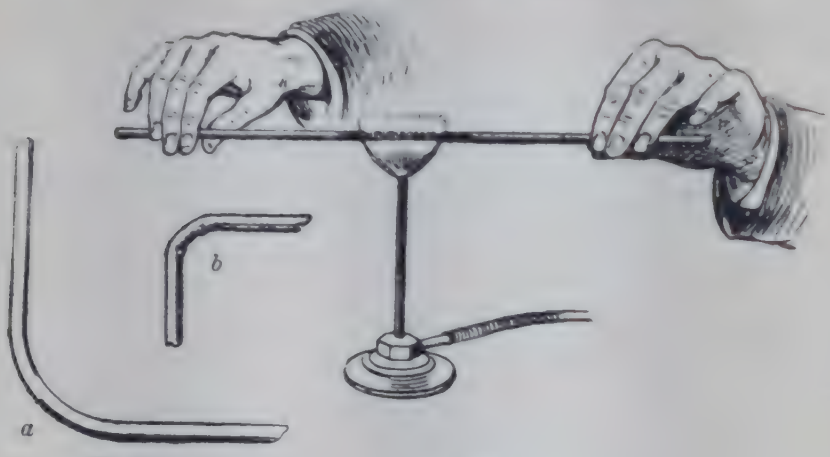

Fig. 196 Fig. 197

How to Bend Glass Tube

Remove the tube slowly from the flame, and allow it to cool slowly.

Repeat the experiment, suddenly plunging the hot bend into cold water. The tube will probably break.

Ex. 301.—Take another piece of tube, heat it in the flame as before, and when the glass is soft or plastic draw the hands apart horizontally. The hot part of the tube is drawn out, and by breaking it at the narrowest part when it is cold, two fine nozzles are obtained.

All ordinary glass vessels are after manufacture allowed to cool very slowly in special ovens. This process is called **annealing**, and has the effect of rendering the glass less easily broken.

Fig. 198.—A Rupert's Drop

Glass that has been suddenly cooled is in a strained condition. Pieces of glass thus prepared, called Rupert's drops (fig. 198), are sold by instrument makers. When the tail of a Rupert's drop is broken off the whole drop is violently reduced to powder.

180. CHANGE OF PHYSICAL STATE PRODUCED BY HEATING AND COOLING.—We have now to watch the processes by which a solid substance is changed into a liquid and the liquid into a vapour by means of heat, and reconverted into solid by means of cold. Many substances will show this effect.

Ex. 302.—Sulphur. Using the apparatus of fig. 193, place some small pieces of solid sulphur in the tube. Heat gently. The sulphur melts. Continue the heating. Note the change in the colour of the liquid. As it boils, the sulphur vapour coming off reaches the colder parts of the apparatus. Note the condensation of the vapour into liquid, and the passage of the liquid into solid.

Ice and many other substances thus treated behave in the same manner.

Ex. 303.—Water. Gently distil a quantity of spring water by the same method, and watch the whole process. Note the bubbles of air that at first collect on the sides of the tube, the formation of bubbles of steam, their varying size, the invisibility of the steam itself in the flask. The steam when cooled condenses into "distilled" water, which is free from air and impurities. A small quantity of solid matter, probably chalk and gypsum, is left behind after the water has all boiled away.

Ex. 304.—Camphor. Place a piece of camphor in the tube (fig. 193), and warm gently.

Without liquefying, the solid passes into the gaseous state, and the gas condenses again into a solid in the cold tube.

This process is called **sublimation**. Iodine, white arsenic, sal ammoniac, and calomel behave in a similar manner.

181. EXPANSION.—The volume and density of most substances is changed by heating them.

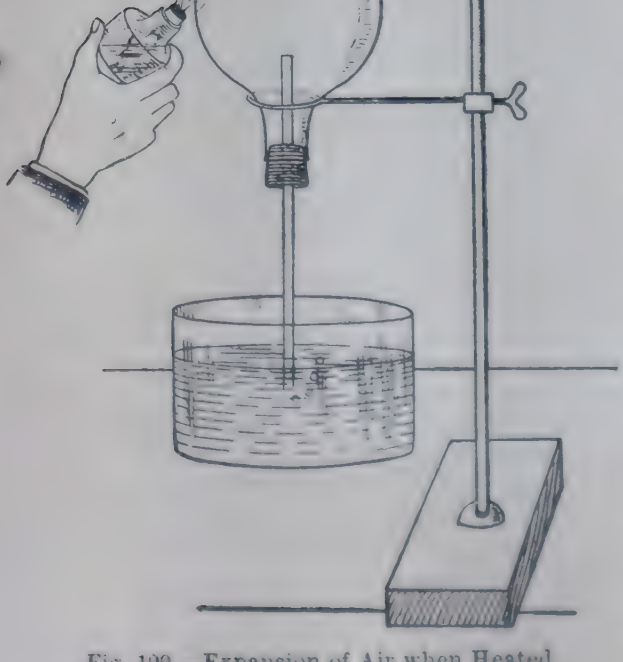

Fig. 199.—Expansion of Air when Heated

Ex. 305.—Gases. (1) Fit a flask with a well-fitting india-rubber stopper and straight delivery tube, and place the mouth of the delivery tube under water (fig. 199). Warm the flask by a burner, or by placing it in the sunshine or near a fire. Note the escape of bubbles of air.

(2) Place a partially inflated bladder near a fire or in hot sunshine. The increase in volume is considerable. Does the enclosed air return to its original volume on cooling? What would be the effect of making the bladder very cold?

Ex. 306.—Liquids. Take the flask used in the last experiment. Fill it up to the brim with water. Screw in the stopper with the tube already in it, so that no air is left in the flask. Tie a millimetre scale to the glass tube, as

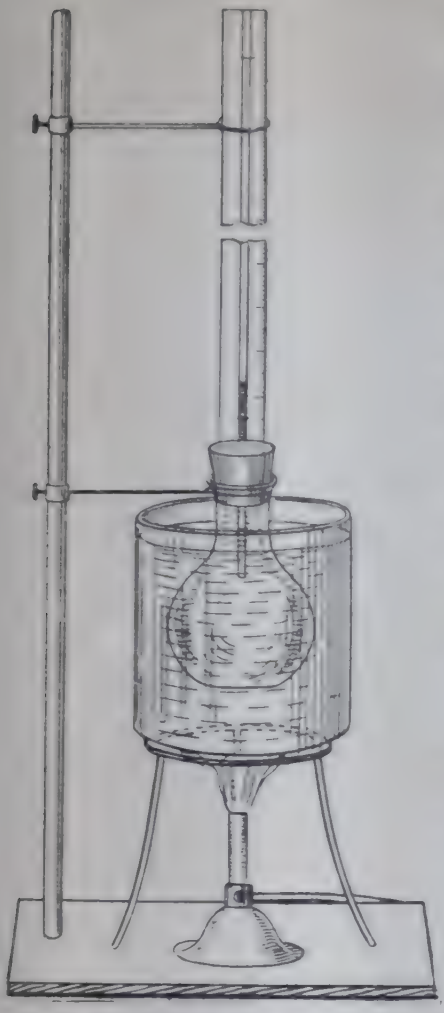

Fig. 200.—Expansion of a Liquid when Heated

support the flask on a retort stand in a beaker of water, as shown in fig. 200. Note the level of the water in the tube.

Warm the beaker. Note the steady continuous rise of the water in the tube as heating proceeds. Do not allow the water in the beaker to boil.

Take away the flame, and note the steady fall of the water in the tube as it gets cooler.

Ex. 307. — Solids. (1) The apparatus shown in fig. 201, called a Gravesande's ring, will demonstrate the expansion of solids when heated.

Supported on a stand is a metal ring, the hole through it being of just the right size to allow a solid metal ball to pass through it when cold. Heat the ball strongly in a fire or over a burner. It will then, as indicated in the figure, be too large to pass through the hole in the ring.

(2) The same result may be shown by the apparatus shown in fig. 202, consisting of a rod and gauge. These are carefully made, so that when they are both cool the rod exactly fits the jaws of the gauge. If the rod be heated it will not go into the gauge.

Ex. 308.—Stretch pieces of thin wire side by side, between the rings of two retort stands or similar suitable uprights, so that the wires are straight.

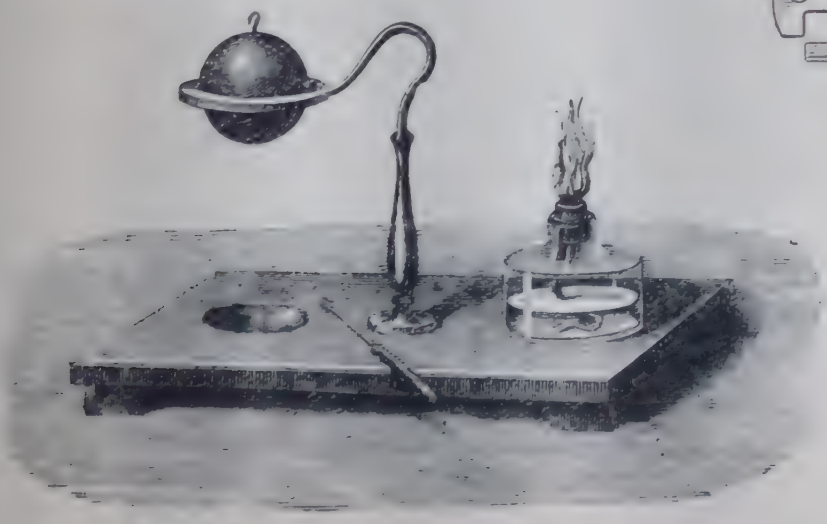

Fig. 201

Fig. 202

Expansion of a Solid when Heated

Pass a flame along the wires; note that they bend or sag like telegraph wires.

If the wires are of different materials the amount of bending will not be the same in the different wires.

Ex. 309.—Take the flask and tube fitted up for Exercise 306 (fig. 200).

Fill it with cold water or other liquid, so that the liquid stands up the stem of the tube.

Place the flask in a beaker of hot water. Note the fall of the liquid in the tube.

The fall is due to the expansion of the flask, which being in contact with the hot water at once expanded, before the liquid inside had time to get hot.

182. CHANGE OF ELASTICITY.

Ex. 310.—Support one of the steel springs made in Exercise 299 at one end, and hang a small weight on the other end. Heat the spring by a flame. Does the weight move? Does the spring recover on cooling? The heat has produced a change in the **elasticity** of the spring. If it has been only slightly heated, the spring recovers its elasticity on cooling; if strongly heated it may not recover.

Ex. 311.—Place a fully inflated toy balloon in front of a fire or in hot sunshine. Test the pressure at intervals with the finger. What is the effect of heating a quantity of gas in a confined space when no bursting occurs? Does the pressure of the gas return to its original value on cooling?

183. SUMMARY.—The experiments made in this chapter show some of the chief effects of heat on matter.

(*a*) Some substances, such as fat, oil, coal, wood, easily burn, producing heat and light. When sufficient heat is applied to one part of such a body, the whole body gradually wastes away and disappears. We found, however, that these substances gave off gas which was inflammable, and often carbon in the form of soot, while water could sometimes be obtained from the gas that passed off.

Other substances which are not generally regarded as inflammable—for example, some metals—burn when suitable arrangements are made for burning them.

The burned substances disappear, but other substances are formed during the burning, so that the matter of which they are made is not destroyed; it changes its form only. When the substances that are formed are collected and weighed, it is found that none has been lost. *Matter is indestructible.*

Changes of this character are called **chemical** changes. Many substances are composed of several others called **elements**, and in a chemical change these elements make themselves up into new groups or **combinations**, which means a different kind of substance.

(*b*) Other **organic** substances—animal and vegetable substances,

bone, blood, lean flesh, milk, sugar, bread, starch — when heated only to a moderate degree, changed their weight and appearance, gave off gases, and left material very different from that with which the experiment was started.

The substances in this group underwent the same kind of change as those in the previous group—a chemical change. Most organic substances are very complicated in their structure, and under the influence of heat their elements form different combinations resulting in the production of different substances, among which are usually carbon and water.

(*c*) The metals behaved rather differently, but in one important respect the same. In place of the metal after heating we had a different substance, produced by the chemical change that the metal had undergone by combination with oxygen in the air.

(*d*) Another class of substances—glass, sulphur, water, camphor—on heating were not destroyed or changed into other substances, but passed into a different condition. The glass became soft, the sulphur melted, the sulphur, water, and camphor passed into vapour. But in these cases, on cooling, we again obtained the substance with which we started, and not a different substance. Many solids when heated pass into the liquid state; and many liquids when heated pass into the gaseous state. But if the stuff of which they are made remains unchanged, then the change is called a **physical** change or **change of state**, not a change of composition.

(*e*) Experiments 305 to 309 illustrate the fact that nearly all substances—solid, liquid, or gaseous—expand when heated. The same quantity of material occupies a larger volume when warm than when cold. If cooled, it shrinks and occupies less volume. Thus the density of a substance is not always the same; it is smaller when the body is hot, and larger when the body is cooled. The value of the densities measured in Chap. VI therefore are only correct at ordinary temperatures.

(*f*) Experiments 310 and 311 illustrate the fact that the elasticity of a substance is changed by heating it.

CHAPTER XXI

Temperature. The Thermometer

184. TEMPERATURE.—In the previous chapter we heated bodies more or less, making some warm, others hot, and others very hot, using merely the general impressions which everyday experience gives as to the practical meaning of such terms.

The scientific way of saying that a body is hot or cold is that the body is at a high or low temperature. Temperature is the state of a body with regard to heat. But temperature is not heat. If we take a spoonful of hot sweet liquid from a larger quantity, the small quantity is as sweet as the larger, but does not contain as much sugar. So, too, the small quantity is as hot (is at the same temperature) as the larger quantity, but does not contain so much heat. One cubic centimetre taken from a litre of hot water is at the same temperature as the remainder, but only contains $\frac{1}{1000}$ of the heat contained in the litre.

Fig. 203.—Arrangement for Filling a Thermometer

Quantity of heat is dealt with in Chap. XXIII.

Change of temperature is a change in the condition of a body that sometimes follows upon the addition or withdrawal of a quantity of heat.

Within certain limits we can by our sensation tell which bodies are at a higher temperature than others. But the sensation is painful if the temperature is above or below those limits, and it often gives unreliable results. To give accurate intimations—that is, to measure temperature—an instrument is used called a **thermometer**. The department of science that deals with the measurement of temperature is called **thermometry**.

185. THE THERMOMETER.—The principle of the thermometer and its mode of construction may be illustrated by making an alcohol thermometer.

Ex. 312.—The general idea has been learned in Exercise 306. A glass bulb is provided with a long tubular stem. To the top of the stem, by means of indiarubber tube, fit a small funnel (fig. 203). Fix the tube vertical, and in the funnel pour alcohol in sufficient quantity to fill the bulb and tube; it may conveniently be coloured with aniline magenta

Bring up a beaker of boiling water, so that the bulb is immersed in it. What effect does this produce?

After a short time remove the hot water and allow the bulb to cool. Why does alcohol enter the bulb?

When alcohol ceases to descend bring up the hot water again as before. The alcohol in the bulb boils, and its vapour carries out air with it, as in Exercise 152.

After a short time remove the hot water. More alcohol enters the bulb. Continue these heatings and coolings of the bulb, until all the air is driven out and the bulb and stem are completely full of alcohol. Seal the top of the stem with cement, sealing wax, or by the blowpipe. Allow the apparatus to cool. As the temperature decreases, note the fall of the alcohol in the stem of the tube.

Whenever such an instrument is heated the liquid rises in the tube, when it is cooled the liquid falls, so that rise of temperature brings about a rise of the liquid column in the thermometer, and fall of temperature a fall of the column. Since there is no means of measuring temperature directly, it is measured indirectly by the change of volume of the liquid.

186. THE MERCURY THERMOMETER. —The liquid ordinarily used is mercury, which possesses the advantages of not adhering to the glass, and remains in the liquid state at all temperatures in ordinary daily use. Fig. 204 shows two mercury thermometers, one such as is ordinarily used in houses, mounted in a wood case, the other such as is employed in laboratory work. The chamber formed by the hollow bulb and stem is closed at the top when the instrument is hot, so that when the instrument is cold there is a vacuum above the mercury. The bore of the tube must be as uniform as it can be made.

Fig. 204. — Forms of Mercury Thermometer

187. SCALES OF TEMPERATURE. — Examine the divisions marked on the mercury thermometer. They are called **degrees**, marked °, and we speak of a temperature of 20°, 80°, &c.

To form the scale it is necessary to determine two fixed points on the stem of the instrument, showing the position of the top of the mercury column at two fixed temperatures. The two temperatures chosen are the freezing and boiling "points" of water.

Ex. 313.—To test the freezing-point of a mercury thermometer. Place a funnel in the ring of a retort stand, and underneath it an empty beaker. In the funnel place a mercury thermometer (fig. 205) and the alcohol thermometer made in the last experiment.

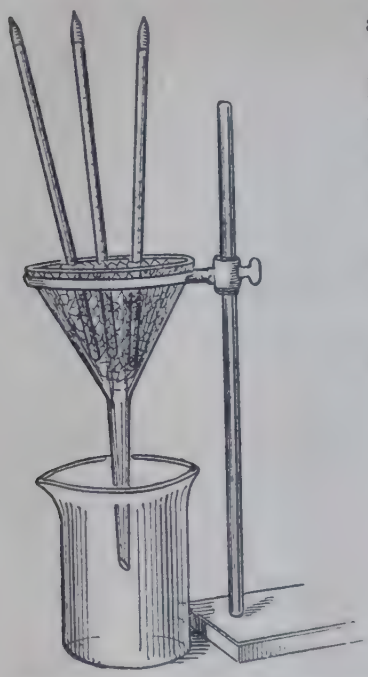

Fig. 205.—Testing the Freezing-points of Thermometers

Prepare some small pieces of ice and wash them. Pour them into the funnel until they cover both bulbs up to the level marked 0° on the mercury thermometer, and nearly up to the top of the alcohol column. A little distilled water should be added to the ice.

Leave the thermometers in this position for several minutes to make sure that they take the temperature of the melting ice.

Take the reading of the mercury thermometer accurately. This may not be quite at the zero. In old thermometers the position of the surface of the mercury at freezing temperature is often a little above 0°. This is due to a gradual contraction of the bulb, which forces the mercury higher up the tube. The glass having been subjected to great changes of temperature during the manufacture of the thermometer, becomes strained, and takes some time to recover. The gradual return from this state is accompanied by a diminution of the volume of the inner chamber.

With a small file make a mark on the tube of the alcohol thermometer to record the same temperature (0° C.).

As melting ice is always at the same temperature, it is agreed to take this temperature as the starting-point in making a scale of temperature.

Record the zero error of the mercury thermometer employed in this experiment.

Another fixed point is necessary: and for this the temperature at which water boils is chosen.

Ex. 314.—To test the boiling-point of a thermometer accurately, a **hypsometer** is used. A simple form is shown in fig. 206. It consists of one tube inside another.

Fix the tubes vertically, with the thermometer passing into the inner tube, which is open at both ends. Pass the lower part of the smaller tube through the stopper of a flask in which water is boiling, so that the steam passes up the smaller tube, round about the thermometer, out at the open top of the small tube, into the outer tube, escaping finally into the air at the bottom of the large tube. The outer tube forms a steam jacket for the inner one.

Fig. 206.—Testing the Boiling-point of a Thermometer

When the whole apparatus is hot, and steam is escaping from the exit tube, push the thermometer down through the indiarubber stopper at the top, until the top of the mercury column is just above the stopper. When steam has been thus passing for several minutes, read accurately the level of the mercury and also the height of the barometer.

The temperature at which water boils is not the same at all times and places (Art. 213), and the point marked 100° may therefore not be that indicated by the thermometer in the steam proceeding from the boiling water. Thermometers are made to indicate 100° when the barometer stands at 760 mm. at the level of the sea.

The space between the fixed points on a mercury thermometer may be subdivided in one of three ways, thus (fig. 207):—

Name.	Freezing-point Marked.	Boiling-point Marked.	Number of Divisions between Freezing-point and Boiling-point.
Centigrade (C.) ...	0°	100°	100
Fahrenheit (F.) ...	32°	212°	180
Réaumur (R.) ...	0°	80°	80

Fig. 207.—Comparison of Temperature Scales

The Centigrade scale is now generally used in scientific work. The Fahrenheit scale is in popular use in Britain.

The degrees are marked both above and below freezing-point. In any scale those below zero (0°) are marked minus thus, −16° R., which is the same as −20° C. or −4° F.

Fahrenheit marked his zero 32° below freezing-point; the Centigrade and Réaumur zero is at freezing-point. This makes the comparison of Fahrenheit readings with Centigrade or Réaumur readings a little complicated.

Let f, c, r be corresponding temperature readings on the F., C., and R. scales. Since 180 Fahrenheit scale divisions occupy the same space as 100 Centigrade or 80 Réaumur divisions, the numbers expressing any temperature, if they all started from the same zero, would be inversely as the above numbers, i.e. f, $\frac{9}{5}c$, $\frac{9}{4}r$, would be the

numbers indicating the same temperature. But since the Fahrenheit scale starts at 32° below freezing point the relation becomes:

$$f - 32 = \tfrac{9}{5}c = \tfrac{9}{4}r.$$

This is for temperature *readings*. For comparison of *intervals* of temperature it need only be noticed that 9 Fahrenheit, 5 Centigrade, and 4 Réaumur divisions occupy the same space.

Mercury thermometers are made which show temperatures up to 300° or 400° C. They contain (above the mercury) nitrogen gas under a high pressure.

Ex. 315.—To complete the alcohol thermometer immerse it side by side with the tested mercury thermometer in a water bath, and slowly heat the bath. At every 10° rise of the mercury mark the level of the alcohol up to 60°. The spaces between these marks may be equally divided to mark single degrees.

188. MAXIMUM AND MINIMUM THERMOMETERS.—It is often required to know what has been the highest or lowest temperature during any period. Instruments constructed to afford this information are called *maximum and minimum thermometers*, and

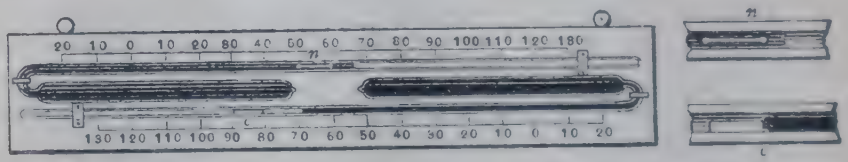

Fig. 208.—Maximum and Minimum Thermometers

are made to be self-registering. Fig. 208 shows *Rutherford's* maximum and minimum thermometers. These are entirely distinct instruments although mounted on one stand.

In the minimum thermometer the liquid used is alcohol, in which there is placed a small dumb-bell-shaped index (n) of glass or enamel. The alcohol wets this index, and the tension of the alcoholic surface (Art. 78) exercises a sufficient force on n to keep it inside the liquid. When the alcohol contracts, therefore, the index keeps just inside the surface, but when the liquid expands the index is left behind. Its position thus indicates the lowest point reached.

In the maximum thermometer the liquid used is mercury, and the index c, which is of steel, is placed outside the mercury. Since mercury does not wet steel, the index is pushed in front of the mercury when the liquid column advances, and is not carried back when that column retreats.

These instruments should be placed horizontal.

189. DIRECTIONS FOR USE OF THERMOMETERS.

1. When not in use keep them in their cases.

2. Do not plunge them into very hot liquids if it can be avoided. See that the mercury never closely approaches the highest graduation mark.

3. Before taking the temperature of a liquid, see that the liquid is thoroughly stirred.

4. Before taking a reading see that it is steady. Estimate to tenths of a degree.

Ex. 316.—(1) Draw a graph showing the connection between the Fahrenheit and Centigrade scales of temperature. A portion of the graph round the freezing-point temperature (marked F.P.) is shown in fig. 209.

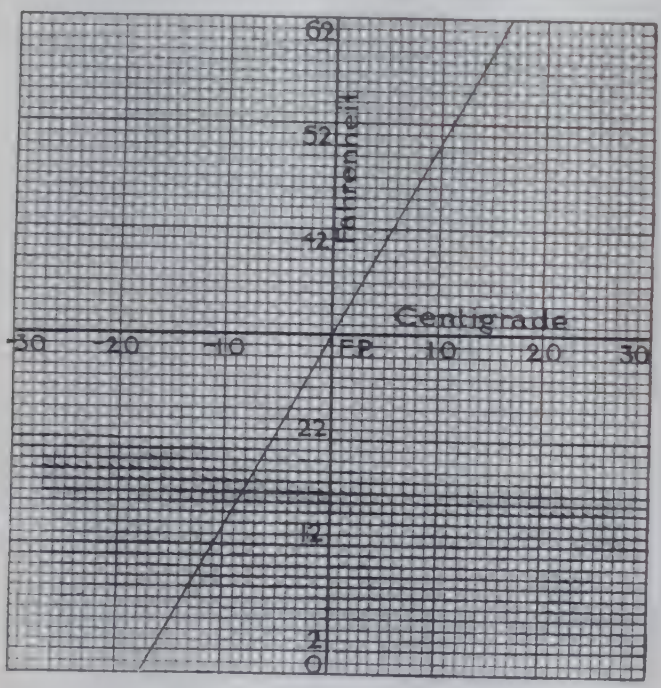

Fig. 209.—Conversion of Centigrade and Fahrenheit Temperature Readings

(2) By means of the graph, and also by calculation, find the equivalent—

(*a*) On the Fahrenheit scale of 25° C., 35° C., −25° C., −40° C.
(*b*) On the Centigrade scale of 77° F., 95° F., −13° F., −40° F.

(3) Take the temperature of a room for a period at regular intervals, and show graphically the changes of temperature. (See instructions for Exercise 162 (6).)

(4) The temperature of a body is raised through 52° C.; its original temperature was 45° F. What is its final temperature on the F. scale?

(5) Find the first two coincident scale divisions above freezing-point on the F. and C. scales.

CHAPTER XXII

Conduction, Convection, and Radiation of Heat

190. Previous experiments, as well as everyday experience, sho that heat can pass from one body to another. There are thre different modes by which this transference of heat may take plac

191. CONDUCTION.—If one end of a pin, knitting needle, similar piece of any metal be placed in a flame or fire, those par of the metal that are not in the fire are felt to become hot or war according to their distance from the flame. The heat slowly trave along through the substance of the metal. This process is call **conduction**: it is the way by which heat is transmitted throu solids.

Ex. 317.—Place rods or strips of several different materials, such as copp iron, glass, wood, lead, ebonite, all about of the same size and length, with o end close to the edge of a flame. Do not let the rods touch each other.

After a few minutes feel the cooler ends. They are not all equally war Write down the list of substances in order of the hotness as you estimated it the cooler end.

The experiment shows that the various materials do not condu heat equally well. Those that felt the hottest are good conducto and are said to possess high **conductivity** or **conducting powe** those that felt the coolest have lower conductivity. Metals are t best conductors of heat.

A rough classification of solids into good and bad conducto may be made by touch. In a room where there is no fire to ma one part hotter than another, if different articles be touched so will feel colder than others. Thus any piece of metal feels col than any piece of marble or earthenware; this feels colder th any object made of wood; and this colder than the carpet or a piece of woollen material. The explanation of this lies in the f that these substances vary in conducting power. Those that f the coldest are the best conductors; those that feel the warm are the worst. But in reality the substances are all equa warm. The good conductors feel cold because the part touch by the hand rapidly transmits the heat to the rest of its substan all of which begins to grow warmer. Much heat is therefore tak

from the hand; hence the sensation of cold. The bad conductors do not feel cold, because they only take from the hand enough heat to warm just the part touched.

If these same substances were touched in a *very hot* room the effects would be exactly opposite. The metal would feel the hottest, because it would readily give out its heat to the hand; the carpet would feel the coldest, because heat cannot travel through its substance to the part touched.

A piece of dry wood can be handled without pain on a frosty night, when a piece of metal cannot, because the wood, being a bad conductor of heat, abstracts heat from the hand but slowly, whereas metals abstract heat rapidly and thus produce a sensation of great cold.

Most solids, except the metals, are bad conductors of heat.

If the bottom of a kettle be covered thickly with soot it may be placed on the hand when full of boiling water without causing pain, since soot is a very bad conductor of heat.

If a sheet of fairly stiff paper be folded, and then opened into a cone, and water poured into the cone, it may be placed on the top of a fire. The cold water prevents the outside of the paper from becoming hot enough to take fire.

Ex. 318.—Hold a piece of wire gauze over a jet of gas, and light the gas. The appearance of the flame will be like the left-hand figure (210) if the gas be lighted

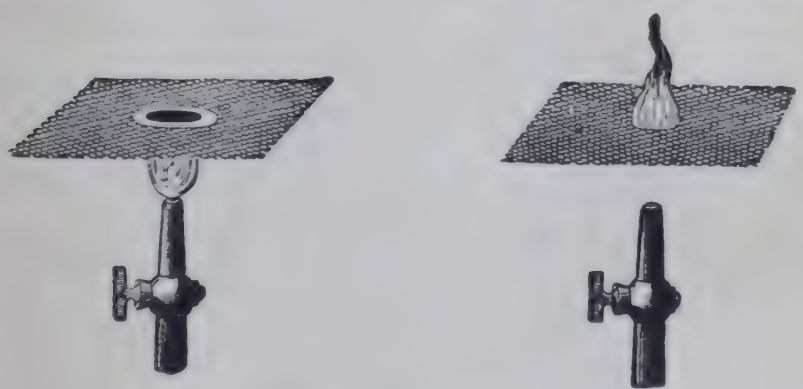

Fig. 210.—Action of Wire Gauze on Flame

below the gauze, and like the right-hand figure if lighted above the gauze. The reason is that the gauze conducts away the heat so fast that the gas on the side opposite to where the flame is applied does not get hot enough to ignite.

To make the experiment with a spirit lamp, light the lamp and hold the gauze over the wick. Blow out the flame and rapidly relight it above the gauze.

This fact is made use of in the miner's **safety lamp** (fig. 211),

which is an ordinary lamp with the flame surrounded by wire gauze. In coal mines gas that will burn is often present in the air. Such gas passes through the wire gauze and burns *inside* the lamp, but the flame does not pass outside. Thus the miner knows when inflammable gas is present.

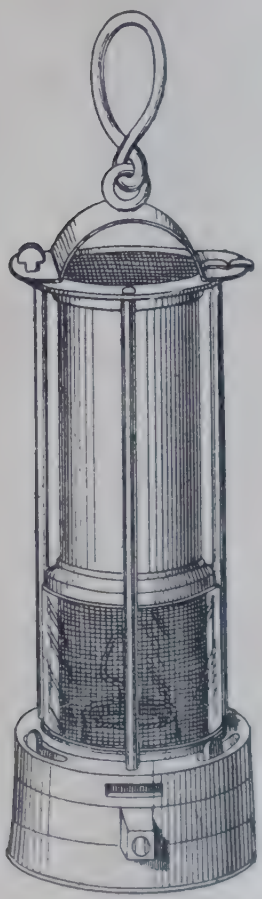

Fig. 211.—A Miner's Safety Lamp

Ex. 319.—If a piece of tissue paper be tightly stretched over the bowl of a metal spoon a flame playing on the paper does not set it alight.

If a rod of metal be fastened to a rod of wood (fig. 365), both covered tightly with a sheet of paper, and a flame played on them, the paper in contact with the wood becomes scorched much sooner than that in contact with the metal.

In these and similar cases the heat is conducted rapidly away by the metal, so that the material in contact with the flame does not get very hot.

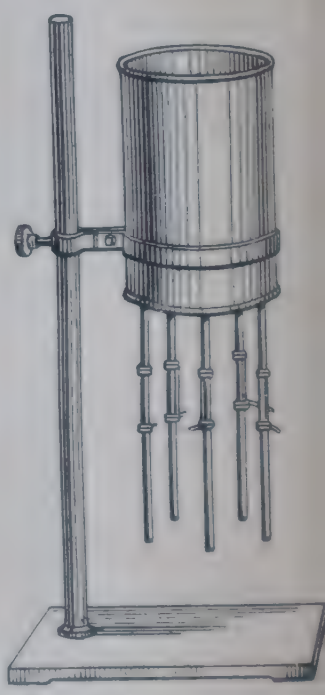

Fig. 212.—Comparison of the Conductivity of Metals (Edser's Apparatus)

192. COMPARISON OF THE CONDUCTIVITIES OF METALS.

—This may be effected by the apparatus shown in fig. 212.

Ex. 320.—Rods of different metals have one end passing into a metal vessel. The rods are coated with paraffin wax, and each one carries a small coil of wire.

A quantity of hot water or other liquid is placed in the vessel. As the heat is conducted along the rods the wax melts, and the coils of wire slip down as far as the wax melts on each rod. These lengths are different.

The apparatus is left for some time, and the distances from the bottom of the vessel to the bottom of the coils are measured. The conductivities of the rods are proportional to the *squares* of these lengths.

193. CONDUCTIVITY OF LIQUIDS.

Ex. 321.—Tie a piece of metal or wire gauze to a lump of ice, and put it at the bottom of a long test tube or boiling tube. Fill the tube with water, and

apply heat to the top of the tube (fig. 213). The water at the top may be made to boil while the ice at the bottom is unmelted. The water conducts very little heat down the tube; the bottom remains quite cold.

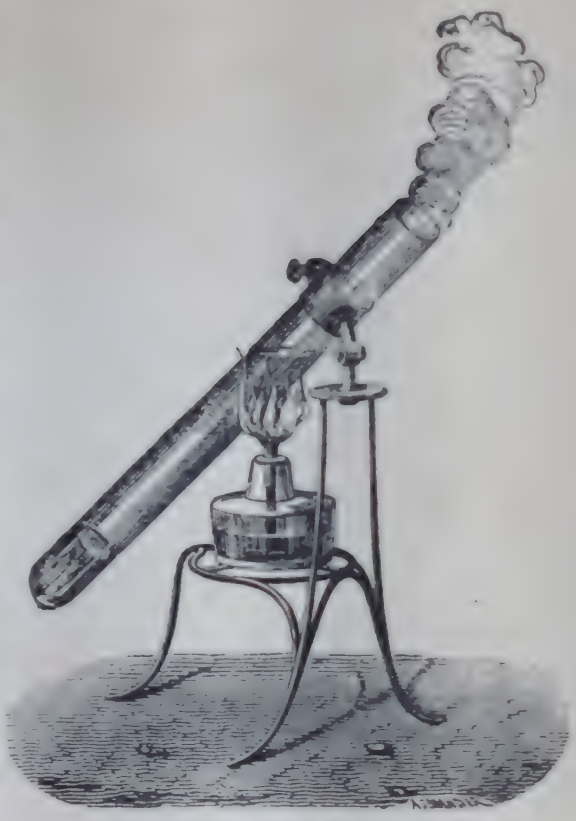

Fig. 213.—Boiling of Water over Ice

Most other liquids (not metals) are still worse conductors, but mercury is a good conductor.

194. CLOTHING.—The chief function of clothing is to prevent the natural heat of the body from escaping. The heat of the body results from a process similar to that which goes on in an ordinary fire, namely, the combination of carbon and hydrogen with oxygen. The oxygen required for this process comes from the air, the carbon and hydrogen is contained in our food.

Warm clothing prevents the escape of this heat, cool clothing allows it. Clothing which prevents the escape of heat from the body does so mainly by keeping the body surrounded with air, which, like all other gases (except hydrogen), is a very bad conductor of heat. This air is contained in the small cavities of the material. A bird in a cage may be observed on a cold night to stick out its feathers so as to surround itself with a thicker layer of non-conducting air, a process very similar to adding an extra blanket to a bed. The air contained in snow plays a similar part. Snow lying loosely on the ground contains ten or eleven times as much air as it does ice, and this layer of ice and air acts like a blanket, and keeps the earth warm.

Similarly, ice is kept from melting by being wrapped in flannel or felt, which is a bad conductor of heat, and does not allow warm air to come in contact with the ice.

195. CONVECTION.—When a quantity of liquid is heated in a vessel the heat does not pass from the bottom to the top in the manner in which it passes along a rod.

Ex. 322.—Place some water in a large beaker, and under one side of the beaker put a small flame as in fig. 214.

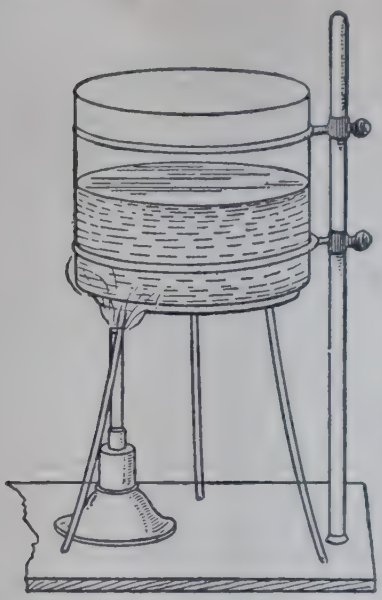

Fig. 214.—Arrangement for showing Convection Currents

Place in the water some material that will show its movements, such as small pieces of bran or fine oak sawdust; or just above the flame a coloured crystal that will dissolve.

A continual circulation of the water takes place, currents passing up the warm side and down the cold side. The water immediately over the lamp becomes heated, expands, and being thus bulk for bulk lighter, rises to the top, while the colder denser water at the top sinks to the bottom. This in its turn is warmed and rises to the top again, and so on.

It will be seen that this process depends on the fact that liquids expand when heated. The energy with which it takes place is greater in fluids in which such expansion is great.

The effect is increased by placing a lump of ice in contact with the cool side of the beaker.

Ex. 323.—By arranging a series of tubes which may be joined at the angles with indiarubber tubing in the form of fig. 215, interference of the currents is avoided. If such a circuit be filled with turbid water containing small particles, it is seen that water flows steadily up that vertical tube which is warmed and descends through the colder tube.

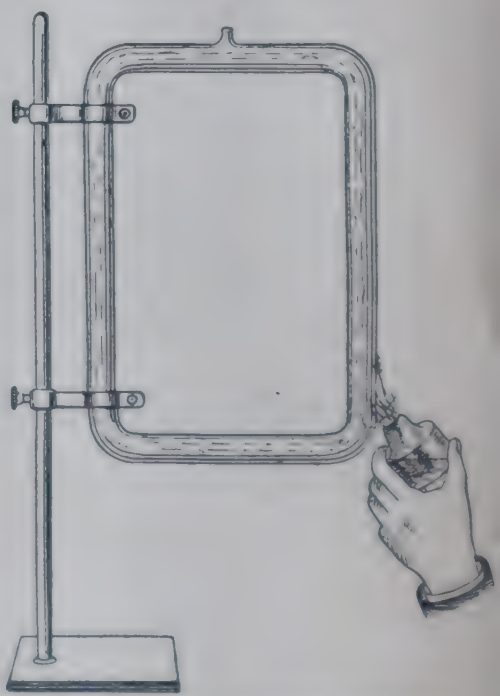

Fig. 215.—The Circulation of Liquid in a Continuous Tube

196. OCEAN CURRENTS.—These currents form a very complicated system, owing to the obstacles offered by the large land masses to their free flow. They are caused by the winds and by the inequality of temperature in the different parts. From the equatorial regions the warm surface water flows outward north and south towards the polar regions. There it becomes chilled and sinks to the bottom, along which it slowly creeps towards the tropics, there to rise and recommence

the same round. A general circulation is thus always going on (fig. 216). Compare with Exercise 323.

The Gulf Stream.—This is one of the most remarkable of ocean currents. It flows from the Gulf of Mexico north-eastward across the Atlantic Ocean. When the Gulf Stream issues from the Straits of Florida its waters are of a high temperature. It travels fast, and at first keeps quite distinct from the waters of the Atlantic, through which it flows like a river between its banks. Far out in the Atlantic, however, it loses this definite character and becomes a general diffused drift of water, which divides into two parts. One of these currents turns south-east towards the coast of Africa, the other continues towards the north-east and washes the shores of Great Britain, Ireland, and Norway. To these countries it gives out vast quantities of heat, and thus renders their climate much warmer than it otherwise would be.

Fig. 216.—General Circulation of Ocean Waters

197. GASES.—The movements of the air are common experience. They are convection currents.

Ex. 324.—Apply a lighted taper or match to the crack round a room door which is very nearly closed.

The movement of the flame will show any movement of air in or out of the room. Notice its movements.

These will vary to some extent according to the temperature of the room and the temperature outside, but generally there will be found to be a draught in at the bottom, and a draught out at the top of the door. If one room be warmer than an adjacent room the flow is towards the warmer room at the bottom of the door, and in the opposite direction at the top.

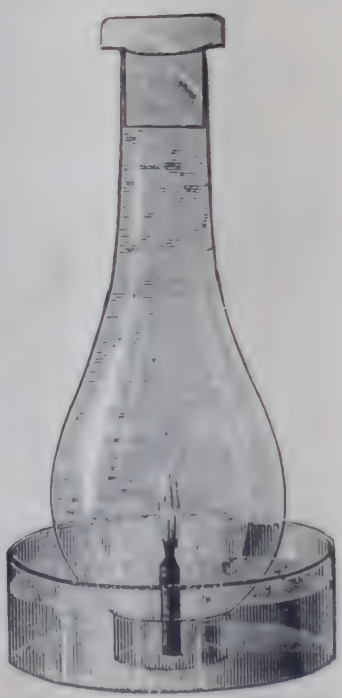

Fig. 217.—Circulation of the Air set up by Heat

Ex. 325.—Set up a lighted candle standing in shallow water surrounded by a lamp glass or any other tube not of large diameter (fig. 217). The candle not being able to obtain a supply of air, is soon extinguished.

Repeat the experiment with a partition of a T-shaped piece of cardboard or tin in the top of the glass.

Examine the air currents on each side of the partition by means of a lighted match. What is observed?

The action of chimneys is an illustration of convection in the air. When a fire is lighted in a grate the air above it is heated, expands, becomes bulk for bulk lighter than the surrounding air, and therefore passes up the chimney, its place being taken by other air from the room, which in its turn is replaced by air from outside. If air cannot get in from outside the smoke cannot ascend the chimney.

Light a lamp and note the effect of putting on the chimney. The lamp requires a great deal of air in order to burn brightly. When the chimney is put on, a stream of hot air comes out at the top and a stream of cold air enters at the bottom.

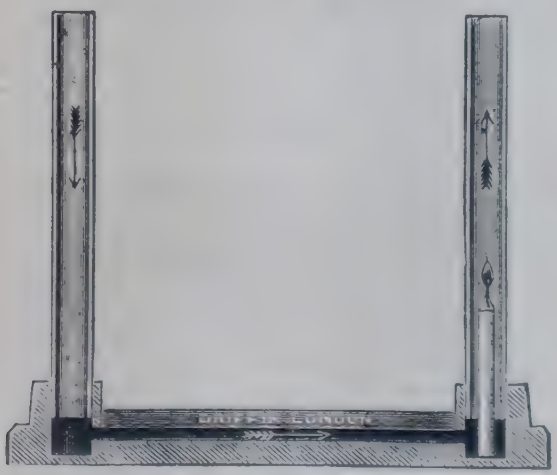

Fig. 218.—Ventilation of a Mine

The taller the chimney the stronger the draught. Hence factories that require a large hot fire build tall chimneys, and some lamps require taller chimneys than others.

Mines are often ventilated by this means, as illustrated in fig. 218. Two shafts are bored, one at each end of the workings. At the bottom of one shaft a fire is kept burning, and consequently up this shaft there is always a current of warm air. Cold fresh air passes down the other shaft, circulates through the passages, and ultimately reaches the fire, where it is warmed and ascends.

Convection takes place much more energetically in gases than in liquids, owing to the large amount of expansion produced by heat and to the freer movement of gaseous particles among one another.

198. WINDS.—Winds are currents of air on a large scale. They are produced mainly by the heating effect of the sun. The air in the region of the equator is being continually heated; it expands, rises to the higher regions, and there flows away on both sides towards the poles (fig. 216). The place of this air thus

removed is taken by air which flows from temperate regions along the earth's surface towards the equator. Thus the general atmospheric circulation is brought about.

The Trade Winds.—The trade winds are steady winds that blow along the surface of the earth towards the equator for about 1600 miles on each side of it.

The direction of these winds is in the northern hemisphere from N.E. to S.W., and in the southern hemisphere from S.E. to N.W. If the earth were at rest the directions would be from N. to S. and S. to N. respectively; in each case straight from the pole to the equator. But the earth turns from W. to E., and the air over any large area at any time, speaking roughly, goes round with that area. If the air does not go round so fast as the land underneath it, there is a wind from the east.

As the earth revolves on its axis every point on its surface except the poles (so far as this motion is concerned) describes a circle, a point near the equator a large circle, a point near the poles a small one. Since each of these circles is described in 24 hours, it follows that an area near the equator travels faster than an area near the poles. As air then moves from the pole towards the equator, it is at each moment coming on to parts that travel faster to the east than it does. It therefore lags behind and seems to an observer to come somewhat from the east; while it still keeps on its journey towards the equator. The result of these two motions is that it comes from the N.E. relatively to an observer in the northern hemisphere, and from the S.E. in the southern hemisphere.

Variable Winds.—But there are many local differences on the earth's surface which interfere with the regularity of the process. When the air above any region of the earth's surface is at a different temperature from that in an adjacent region, a wind generally blows from the colder region to the warmer at a low level, and in the opposite direction at a higher level, as illustrated in fig. 216.

In time, however, the air that has risen from the warmer area becomes chilled, as it is a general fact that the higher we go into the air the colder it is.

Hence this upper air gradually descends to the surface of the earth, and the regularity which takes place near the equator is disturbed. Temperate latitudes have more variable winds.

Land and Sea Breezes.—At the seaside a wind often blows from the sea towards the land during most of the day, and from

the land towards the sea during most of the night. The former is called a sea breeze, the latter a land breeze, and their origin is to be found in the fact that the sea cools and heats more slowly than the land on account of its great capacity for heat (Arts. 203 and 204). In the daytime the temperature of the land is higher than that of the water, consequently the air above the land becomes more heated, expands and rises, and the cooler air from over the sea flows in. During the night the land cools faster than the sea, so that the air over the sea is then the warmer. This air in its turn ascends, and the colder air from over the land flows towards the sea.

The **monsoons** are land and sea breezes on a large scale. In the summer when the large area of Central Asia is hot, a regular wind blows towards it from the Indian Ocean. In the winter when the land is cooler than the ocean the wind blows in the opposite direction.

Cyclones are winds which pursue a circular or curved path. Owing to the motion of the earth, a mass of air in the northern hemisphere that starts moving southward is gradually deflected more and more towards the west, while a mass moving northward is deflected towards the east. Consequently, when from any cause the atmospheric pressure over an area E (fig. 219) falls, and air begins to move in towards the low-pressure area, the air to the north of E has a westerly tendency, and that to the south of E an easterly tendency. This starts a rotatory movement in which air rushing in from other quarters becomes involved. The nearer the air gets towards the centre E the faster it travels. Such a rotating storm area is called a cyclone.

Fig. 219.—The Formation of a Cyclone

In the southern hemisphere the direction of rotation is opposite to that shown in fig. 219.

The centre of the storm area moves along at the same time.

The rotatory motion acquired by water flowing from a bath into the waste pipe illustrates the motion of the air in a cyclone.

199. RADIATION.—All heated matter parts with heat by

radiation, which is the most general of the modes of heat transference.

Ex. 326.—Boil a flask full of water and remove the flame.

Hang two thermometers outside the flask with their bulbs near the middle of it, and on opposite sides of it—one at a distance of about a centimetre, the other at a distance of two or three centimetres from the flask.

Note the readings of the thermometers from time to time, and compare them.

Heat is reaching the thermometers from the flask, but not by conduction or convection. The more distant thermometer receives much less heat than the nearer one.

If the hand be held in front of a fire the warmth is instantly felt, although a draught of air is proceeding from the hand to the fire.

This process is called **radiation**.

The heat of the sun reaches the earth by the same process. If you shield yourself by an umbrella, the radiant heat is stopped by the umbrella, and you feel a very great difference. If it were that the air is made very hot by the sun, the putting up of the umbrella would make very little difference.

Heat, like light, comes from the sun in rays that pass through empty space, but are stopped by most solid and liquid substances and to some extent by gases.

Ex. 327.—Hold your hand *near* a quantity of ice. Does it feel chilled? Does the position of your hand make any difference?

What is the behaviour of a thermometer whose bulb is held close to but not touching ice?

It would appear from this experiment that cold is radiated as well as heat. But all the bodies we deal with contain some heat; when we say they are cold we mean less hot. This experiment with ice shows that all bodies are radiating heat whether they are what we call hot or cold. The heat radiated from a body falls on all other bodies that it can reach. When the hand was held in front of the fire, the fire radiated heat to the hand and the hand to the fire. The hand received more than it gave out, and thus felt a sensation of warmth.

When the hand was held in front of the ice, the hand gave out more heat to the ice than it received from the ice, hence the hand felt chilled.

The heat received by the earth from the sun must be given out again by the earth or it would get continuously hotter. The earth radiates heat into space in large quantities. The operation takes

place at night so fast that often after a hot day the nights are cold.

Bodies are considered to be always exchanging heat by radiation, and this explanation is called the **theory of exchanges**.

200. In the heating of buildings by hot water, all the modes of heat transference are employed.

Ex. 328.—The process may be illustrated by an apparatus such as that shown in fig. 220, which may be built up on a retort stand, the upper vessel consisting of an inverted bell jar or a lamp glass. Note the positions of the ends of the connecting tubes.

Fill the apparatus with water containing small particles of light solid material until the level of the liquid is above the mouth of the highest tube.

Apply heat at the bottom of the lower reservoir. Notice the method of circulation of the water. The warm water rises up the straight tube into the top of the upper reservoir, which represents the cistern at the top of the house, and at the same time cold water descends from the bottom of this reservoir, through the bent tube, to the "boiler" at the bottom.

If the heating be maintained the process continues, unless all the water should reach the same temperature. If that condition be reached, the circulation will cease.

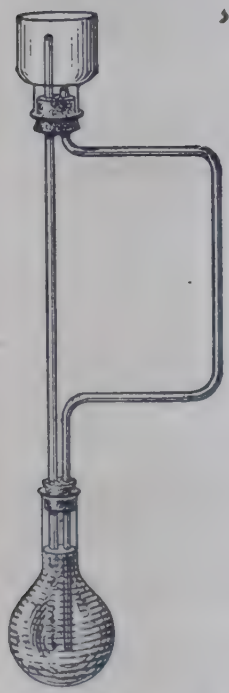

Fig. 220.—How a Building is Heated by Hot Water

In buildings the hot-water pipe passes through a number of cisterns of special shape which gives them a large surface. These are the **radiators**, where the water gives out much heat by radiation, so that the water always arrives at the bottom much colder than when it started on its round, and the rooms have the heat which left the water principally at the surface of the radiators.

201. From the above illustrations it will be seen that conduction is the passing on of heat from particle to particle of a substance, the particles themselves not travelling about. In convection the portions of hot material move about and carry their heat with them. Thus convection can take place in fluids (liquids and gases), but not in solids. The process of radiation is quite independent of matter altogether, because it takes place across what we call empty space or a vacuum, such as exists between us and the sun, in which space there is no continuous matter of any kind, as we usually understand the word matter.

CHAPTER XXIII

Quantity of Heat. Specific Heat

202. QUANTITY OF HEAT.—In most experiments in this subject heat passes from one body to another. It is only when such a passage of heat occurs that it can be measured. There is no method of measuring the quantity of heat that a body actually has in it.

Ex. 329.—(1) Place 200 cub. cm. of water in a beaker, and heat it to about 30° C. In another beaker heat an equal mass of water to about 50° C.

Take the temperatures of both accurately by a thermometer. Call them $x°$ and $y°$.

Pour the hotter water into the colder, stir it with the thermometer, and at once take its temperature $z°$.

Is there any relation between z and $(x + y)$?

(2) Repeat the operation with about 150 cub. cm. of water in each beaker, raising one to about 40° and the other to about 70°, this time pouring the colder water into the hotter.

From the results of these experiments—remembering that the beakers took a little of the heat, and that some escaped by radiation—when two equal masses of water at different temperatures are mixed, what is the relation between the temperature of the mixture and the original temperatures?

Here we have hot water giving out heat to cold, and cold water taking in heat from hot water. The changes of temperature indicated the passage of heat. Since the masses of water were equal, their changes of temperature were also equal.

The same principle applies when the masses are not equal.

Ex. 330.—Mix together 300 grm. of water at 20° C. and 100 grm. at 40°. Note the final temperature. It is $x°$ above 20 and $y°$ below 40. What is the ratio between x and y? Is this ratio nearly the inverse of the ratio between the masses of water?

Ex. 331.—Make several experiments as above, mixing different unequal masses of water at different temperatures.

Record the results thus:—

Hot Water.			Cold Water.		
Mass, M grm.	Fall in Temperature, T°.	Product, M × T.	Mass, m grm.	Rise in Temperature, $t°$.	Product, $m \times t$.
...	...	...	...	...	...
...	...	...	...	...	...

Compare line by line the values of M T with those of $m\ t$. To assist in

eliminating error add up all the values of M T and those of $m\,t$. If the experiments have been carefully made the products obtained are nearly equal, and

$$\frac{M}{m} = \frac{t}{T}.$$

Thus in a perfect experiment, in which masses and temperatures were quite accurately taken and losses all allowed for, we should have figures such as these:—

Heat given out by hot water,	Heat taken in by cold water,
$150 \times 10 = 1500.$	$200 \times 7{\cdot}5 = 1500.$

It is convenient to express this by saying: The hot water gave out 1500 units of heat to the cold water, and the cold water took in 1500 units of heat from the hot water. Thus we have a way of measuring quantities of heat. The products MT and *mt* above represent quantities of heat.

The **unit of heat** is the quantity of heat given out or taken in by 1 grm. of water when its temperature changes 1° C. This unit is called a **calorie** or (by some writers) a **therm**.

Thus, to find in calories the quantity of heat that has passed into or out of water in any operation similar to those above, we must determine (*a*) the mass of the water in grammes, (*b*) the number of degrees (Centigrade) of change of temperature, and multiply the numbers together.

Ex. 332.—(1) Heat a known mass (about 200 or 300 grm.) of water from the temperature of the room to about 80° C. Stir the water, and read the temperature at several equal intervals of time.

Calculate how many calories entered the water during each interval. Plot a graph of the observations, marking the times along the horizontal axis, and the calories along the vertical axis. Examine the graph, and write down any conclusion that you can draw from it.

(2) Make a piece of metal hot either by placing it in hot water, or by holding it over a flame. Then place the metal in a known mass of cold water that contains a thermometer. From the rise of temperature of the water and its mass, calculate the number of calories given out by the metal in cooling.

(3) Into a known mass of warm water of known temperature drop a few small pieces of ice. Find out how many calories were taken from the water. What has this quantity of heat done?

(4) Fit a flask containing water with a delivery tube that passes into a known mass of cold water contained in a beaker (fig. 225).

Boil the water in the flask and let some steam pass into the water in the beaker. Find the number of calories given out by the steam to this water.

Ex. 333.—(1) Calculate the temperatures of the following mixtures of warm and cold water.

(i) 30 grm. at 50° C. with 60 grm. at 62° C.
(ii) 120 grm. at 18° C. with 90 grm. at 46° C.
(iii) 256 grm. at 59° F. with 184 grm. at 81° C.

(2) If 120 grm. of water at 80° C. are mixed with 300 grm. at x° C., and the temperature of the mixture is 36° C., find x.

(3) Find experimentally by the method indicated in the previous exercise the temperature of some cold water; and (separately) of some warm water.

(4) A litre of water at 60° F. is heated to 200° C.; how many calories has it absorbed?

203. THERMAL CAPACITY.

Ex. 334.—Weigh a mass of iron or brass (x grm.); tie a thread to it, and heat it in water to 100° C. Transfer it by the thread into an equal mass (x grm.) of water in which there is a thermometer that indicates about 20° C. Move the iron about by the thread, and when the water has ceased to rise in temperature, read the thermometer again. The rise in temperature will be found to be about 8° or 9° C.

From the previous experiments we know that if, in place of the metal, water had been used, the rise in temperature would have been about 40° instead of 8° or 9°. There is therefore a great difference between the quantities of heat given out by equal masses of hot water and hot iron when they are placed in cold water of the same temperature.

Ex. 335.—Make a similar experiment to the last with various materials, such as glass, marble, glycerine, turpentine, mixing x grm. of the hot substance at 100° with an equal mass x grm. of cold water at about 20°. x may be a different number in the different experiments.

Liquids may be contained in small flasks, the flask transferred from the hot to the cold water, and shaken about to assist the transfer of heat.

Make a table, showing the substances and the rise of temperature from each.

The results of such experiments show—

(*a*) That the substances vary very much in the quantity of heat given out.

(*b*) That all ordinary substances give out much less heat than an equal quantity of water which cools to the same extent.

This is expressed by saying that each substance has its own special thermal capacity, and that the thermal capacity of each substance tried is less than that of water.

The thermal capacity or **capacity for heat** of any body is measured by the number of calories required to raise its temperature through 1° C., or the number of calories it gives out in cooling through 1° C. The greater the capacity of a substance for heat, the more heat does it require to warm it and the more heat will it give out in cooling.

204. GREAT CAPACITY OF WATER FOR HEAT.—*The*

capacity of water for heat is greater than that of any other common substance, so that more heat is required to raise the temperature of a pound of water one degree than is required to raise the temperature of a pound of any other common substance one degree and the pound of water gives out more heat in cooling than a pound of any such substance.

This fact has a most important effect on climate. The waters of the ocean receive vast quantities of heat without becoming sensibly warmer, and give out vast quantities without becoming sensibly colder. During periods of heat large bodies of water take in heat, which is given out again during periods of cold. They thus act as moderators of climate, rendering summer cooler and winter warmer.

The high capacity of water for heat renders it specially suitable for storing heat in carriage foot-warmers, &c.

205. CALORIMETERS.—It is seen from Art. 203 that the thermal capacity of a body is the same as the number of grammes of water that would give out or take in the same quantity of heat as the body, when its temperature changed by an equal amount, that is the **water equivalent** of the body. The water or other liquids that are used in such experiments as those above must be contained in a vessel of some kind. It is usual to employ small beakers of brass or copper for this purpose when heat quantities are being measured. Such a vessel is called a *calorimeter*. The calorimeter changes in temperature with the liquid which it contains. Hence the calorimeter must be included in the measurements, and it is necessary to find its water equivalent.

Ex. 336.—To find the water equivalent of a calorimeter.

(1) Place the calorimeter on the table with a thermometer in it. Do not handle the calorimeter. After a few minutes take its temperature (t_1).

In a beaker rather larger than the calorimeter place a known mass m (about 100 grm. or 150 grm.) of water, and heat it to about 90° C. Remove the heat, flame and take the exact temperature (t_2°).

Quickly place the calorimeter in the beaker, pressing it down with a piece of wood. Read again the temperature (fig. ire water (t_3°).

The insertion of the calorimeter has lowered the temperature of the water how many degrees? ($t_2 - t_3$).

How many calories has the hot water lost? $m(t_2 - t_3)$

If cold water had been added instead of the cold calorimeter, how much would have produced the same effect? That amount is the water equivalent the calorimeter.

Repeat the experiment, and take the mean of the values obtained.

Example.—Temperatures:— hot water : calorimeter : both
99·5° : 15° : 95·4°
Calories:— given out : taken in :
4·1 × 100 : 80·4 × x :

where x is the water equivalent of the calorimeter.

Then $80{\cdot}4x = 410$, whence $x = 5{\cdot}1$. The calorimeter was equivalent to 5·1 grm. of water.

(2) Find the water equivalent of the same calorimeter by pouring warm water into it.

Example.—Into a calorimeter whose temperature was 15° there was poured 100 grm. of water at 99·5°. The temperature of the water in the calorimeter was then found to be 95·4°. The calculation of the water equivalent is the same as in the previous exercise.

There are causes of error in both methods. In elementary practical work it is better to calculate the water equivalent from the value of the specific heat of the material (Art. 206). For elementary work, the water equivalent of a copper calorimeter may be taken as $\frac{1}{10}$ of its mass, and of a glass vessel as $\frac{1}{5}$ of its mass.

206. SPECIFIC HEAT. — In the previous articles we have found the thermal capacities of bodies of any mass. In comparing different materials it is necessary to make the comparison between equal masses, and of course unit mass is chosen. Thus in the above example the calorimeter was found to be equivalent for heat purposes to 5·1 grm. of water. Suppose the calorimeter was made of copper and weighed 53·7 grm. Then the heat required to raise 1 grm. of copper through 1° C. is $\frac{5{\cdot}1}{53{\cdot}7} = {\cdot}095$. The number ·095 is called the specific heat of copper. The **specific heat** of a substance is the number of calories required to raise the temperature of unit mass of the substance from 0° C. to 1° C.

Ex. 337.—To find the specific heat of copper or brass in a copper or brass calorimeter. (The specific heats of copper and brass are nearly the same.) The method is that employed in the former experiments, called the **method of mixtures**, but certain precautions are necessary.

It is better for the calorimeter to consist of two copper vessels, one inside the other, as shown in the upper part of fig. 225, the inner vessel standing on two or three short pieces of glass rod.

Weigh the inner vessel empty (m_1 grm.), and also with a quantity of cold water in it (total mass, m_2 grm.). Make the mass of cold water less than that of the mass of metal whose specific heat is to be found.

This mass should not be too small; about 100 or 150 grm. is suitable. The metal is best in small pieces; copper nails or brass screws are convenient. Suppose the mass of metal is M grm.

Place it in a heater such as is shown in fig. 221. The heater consists of one vessel inside another. Water is placed in the outer vessel, which is provided at the top with an outlet for steam. The body to be heated is placed with a thermometer in the inner vessel.

Boil the water for several minutes.

When all is ready, read the temperature of the cold water in the calorimeter (t_1).

Gently shoot the contents of the inner vessel into the calorimeter. In the second form of heater shown in fig. 221 this can be done by merely raising the upper part of the inner tube, which slides up inside the lower part, and thus leaves the sloping tube open at both ends.

Stir the water in the calorimeter, and when its temperature has ceased to rise, record it (t_2).

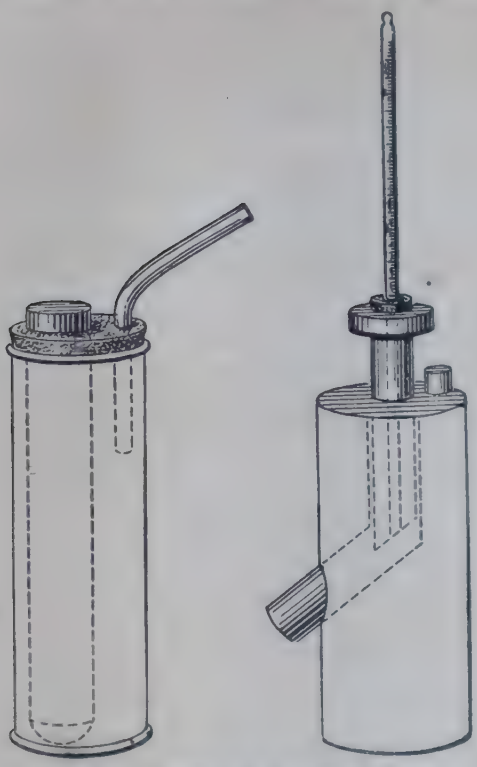

Fig. 221.—Forms of Heater by which Bodies are kept Dry

Then, assuming that the boiling water was at 100°, we have—

(a) M grm. of metal of specific heat S cooled from 100° to t_2°: therefore the heat given out was $M \times S \times (100 - t_2)$.

(b) m_1 grm. of the same metal warmed from t_1° to t_2°: the heat taken in was $m_1 \times S \times (t_2 - t_1)$; also $(m_2 - m_1)$ grm. of water warmed from t_1° to t_2°: heat taken in was $(m_2 - m_1)(t_2 - t_1)$.

Thus the total heat taken in was—

$$\{m_1 \times S \times (t_2 - t_1)\} + \{(m_2 - m_1)(t_2 - t_1)\}$$

Hence, writing the loss and gain of heat equal to each other, we have—

$$M \times S \times (100 - t_2) = \{m_1 \times S \times (t_2 - t_1)\} + \{(m_2 - m_1)(t_2 - t_1)\}$$

whence S can be calculated.

Example.—A mass of 147·5 grm. of copper was heated to 100° and placed in 117·2 grm. of water contained in a copper calorimeter of mass 54·2 grm., and at a temperature of 18° C. The temperature of the mixture was 26·3°. Find the specific heat of copper.

(a) Heat given out by 147·5 grm. of copper of specific heat S in cooling from 100° to 26·3°

$= 147{\cdot}5 \times S \times (100 - 26{\cdot}3) = 10871 \times S$ calories.

(*b*) Heat taken in by calorimeter of mass 54·2 grm. in being heated from 18° to 26·3°

$$= 54{\cdot}2 \times S \times (26{\cdot}3 - 18) = 450\ S \text{ calories.}$$

Also heat taken in by 117·2 grm. of water in being heated from 18° to 26·3°

$$= 117{\cdot}2\ (26{\cdot}3 - 18) = 973 \text{ calories.}$$

$$\therefore 10871\ S = 450\ S + 973$$

$$\therefore S = {\cdot}093$$

Ex. 338.—Knowing the specific heat of the material of which the calorimeter is made, find the specific heat of iron and of lead by the same method.

207. The specific heat of liquids may also be obtained by the methods indicated above.

Ex. 339.—In small similar flasks place the same mass (*x* grm.) of water, alcohol, turpentine, &c., at the temperature of the room. Place a thermometer in each flask.

Raise the temperature of a mass of water in a large beaker or other suitable vessel to about 50° C. In this water place the flasks. Agitate the liquids at frequent intervals, and after a short time read their temperatures. Then the specific heats of the liquids are inversely proportional to the rises in temperature.

Thus, suppose the rises of temperature after five minutes to be—

Water	Alcohol	Turpentine
10°	16°	21·5°

Then the specific heat of alcohol is $\frac{10}{16} = {\cdot}62$, and of turpentine $\frac{10}{21{\cdot}5} = {\cdot}46$.

Ex. 340.—Find the specific heats of mercury and of turpentine by the method of mixtures, as in Exercise 337. This method may only be used for such liquids as do not enter into chemical combination with water.

Ex. 341.—Raise *z* grm. of copper, whose specific heat is ·095, to a temperature of 100° C. in the heater. Immerse it in about ·4*z* grm. of alcohol at, say, 20° C., contained in a calorimeter whose water equivalent is *m* grm. Suppose the rise in temperature to be *t*° C.

Then heat given out by copper $= z \times {\cdot}095 \times (100 - t)$ calories.

Heat taken in by alcohol and calorimeter $= ({\cdot}4zS \times m)\ (t - 20)$.

Equate these two quantities, and hence find S, the specific heat of alcohol.

The value of the specific heat of various common substances is given in the table on p. 409.

Ex. 342.—(1) Small cubes of aluminium, iron, and copper are heated to 100° C., and then placed on a block of ice. Which will sink in deepest? Why?

(2) A thin plate of beeswax is supported on the ring of a retort stand, and on it are placed hot cylinders of different metals. Taking the metals given in (1) above, write down the order in which they will fall through the plate.

(3) If it took 5 min. to heat 1000 grm. of water in a beaker over a flame through 20° C., how long would it take to heat 1000 grm. of lead shot through 20° C. in a similar beaker over the same flame?

CHAPTER XXIV

Latent Heat of Fusion

208. MELTING-POINTS.—Fusion is melting—the process by which a solid is converted into a liquid. Fusion is accomplished by heating the solid.

In Exercise 313 the zero of the thermometer was taken as the position of the mercury surface when the instrument was placed in melting ice.

Ex. 343.—Place a thermometer in a beaker or large test tube, and pack it round closely with small pieces of ice. Place the beaker in a vessel of cold water. Read the thermometer.

Gently warm the water, and read the thermometer at intervals.

Can ice be warmed above the temperature marked as zero?

When the ice has all melted what happens to the temperature of the inner vessel?

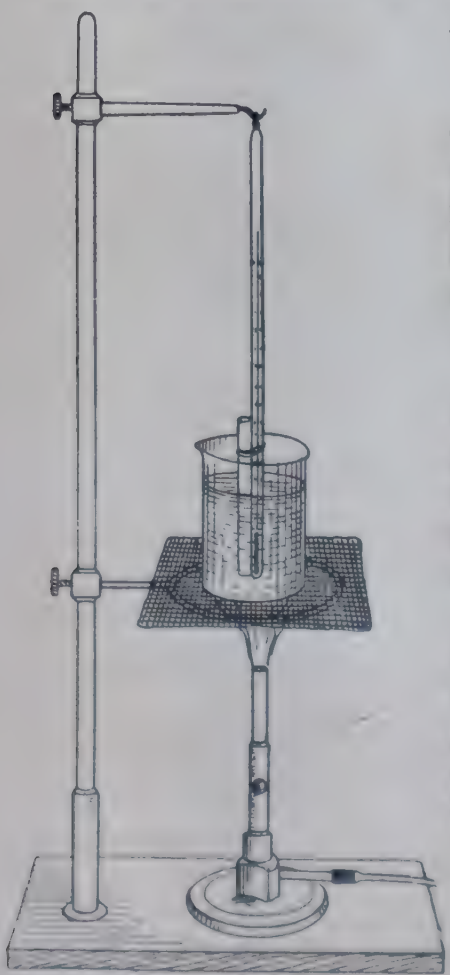

Fig. 222.—Determination of the Melting-point of Wax

The temperature at which under ordinary pressures ice melts, and above which it cannot be raised, is its **melting-point**. Multitudes of observations have shown that ice cannot be raised above that temperature: it can be made colder.

Many other substances behave in the same way. Below a certain temperature they are solids. When heated to that temperature they melt. This temperature is the melting-point of the solid.

The melting-point of any solid is determined by heating a quantity of the solid that contains a thermometer, and reading the temperature when the process of liquefaction is taking place, as in the last exercise.

Ex. 344.—The following method is suitable for such substances as paraffin wax, beeswax, stearic acid, sulphur.

Melt some paraffin wax, and in it dip a fine-bore tube, and suck up some wax inside the tube. Tie the tube to a thermometer (fig. 222),

and suspend the thermometer in a beaker of cold water, so that the water covers the paraffin and the mercury column of the thermometer. Heat the water slowly, watching the appearance of the paraffin. When it becomes transparent the thermometer records approximately the melting-point. Suppose it is 50°. Raise the temperature of the bath to about 53°, and then allow it to cool. Note accurately the temperature when the liquid solidifies. Slowly warm the bath again, and note accurately the temperature when the solid liquefies. Keep the substance oscillating slowly in this way above and below its melting-point, and take several pairs of observations. Take the mean of all the values.

Ex. 345.—*Second Method.* For some substances, such as naphthalene, beeswax, sulphur, the following method is preferable.

Place sufficient substance in a large test tube to nearly fill the tube. Place the tube in a bath, and heat the bath until the substance is all melted. At the same time warm a thermometer. Put the warm thermometer in the liquid. Allow the whole apparatus to cool slowly, and record the temperature at equal intervals of time, e.g. every minute. Make a table showing time and temperature. Make a graph of the observations, as in fig. 223, which shows the results obtained in an experiment on naphthalene. The curve obtained is called a cooling curve. Note its shape.

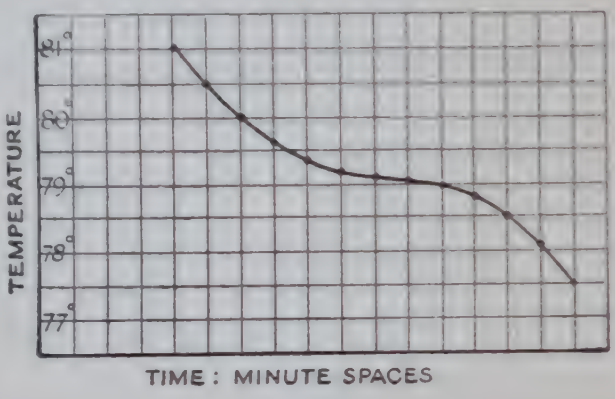

Fig. 223

There is a period of time during which the thermometer is stationary, and the curve at this part is nearly or quite horizontal. The temperature during that time is the melting-point.

Fig. 223 shows that the melting-point of naphthalene according to the thermometer employed is 79° C. The thermometer may have a zero error (Exercise 313). If the zero error in this thermometer was 1°, what is the corrected melting-point of naphthalene?

Some substances, such as sealing wax and glass, have no definite melting-point. If an experiment similar to No. 345 be performed with such substances, there is no straight part in the cooling curve. They pass through an intermediate stage between solid and liquid in which they are in a viscous condition.

The melting-points of many substances, e.g. the metals, cannot be determined with a mercury thermometer. Other means of determining the temperature are employed.

209. LATENT HEAT OF FUSION.—In Exercise 343, during the period of melting, although heat was passing into the test tube from the bath the contents of the tube got no hotter. Heat

that passes into a substance without making it hotter is called *latent* heat. This heat has its effect in changing the physical state of the substance. Conversely, in Experiment 345, during the process of solidification, although heat was leaving the test tube the contents got no colder. Thus in the process of melting, a solid absorbs heat, and during solidification gives out heat.

The number of units of heat thus absorbed or given out when unit mass of the substance changes its state from solid to liquid or from liquid to solid is called its **latent heat of fusion.**

Ex. 346.—To find the latent heat of fusion of ice.

Weigh a small calorimeter or glass beaker, and find its water equivalent (Art. 205).

In the calorimeter place about 100 or 150 cub. cm. of water; weigh again, and find the mass of the water only.

Warm this water to a temperature about 8° or 10° above that of the laboratory, and read the temperature accurately (t_1).

Place ten or twelve lumps of ice, each roughly about 1 cub. cm. in volume, on blotting or filter paper; dry them; shoot them quickly into the water. Stir the water with the thermometer, and when all the ice has melted take its temperature accurately (t_2), and remove the thermometer.

Weigh the calorimeter and contents, and find the weight of the ice added.

From these observations obtain—

		Example.
(*a*) Original mass of water plus the water equivalent of the calorimeter ...	a	125·4 grm.
(*b*) Mass of ice melted	b	11·9 grm.
(*c*) Fall of temperature of the calorimeter and contents ($t_1° - t_2°$) ...	$t_1° - t_2°$	23·5° − 14·6°
(*d*) Final temperature of the whole ...	$t_2°$	14·6°

Then, if L is the latent heat of fusion of ice, we have—

I. Heat taken in (in calories)—		
By ice in melting	$L \times b$	11·9 L
By water from the melted ice in rising from 0° to t^2 ...	$t_2 \times b$	11·9 × 14·6
II. Heat given out (in calories) ...	$a \times (t_1 - t_2)$	125·4 × 8·9

Neglecting any heat that has been lost by radiation, equate the heat given out and taken in.

$$b(L + t_2) = a \times (t_1 - t_2),$$

whence the value of L can be calculated. From the example above

$$11{\cdot}9\,(L + 14{\cdot}6) = 125{\cdot}4 \times 8{\cdot}9$$
$$11{\cdot}9\,L = 942$$
$$L = 79.$$

The latent heat of fusion of other solids that have a definite melting-point may be found in the same manner, but the calculation involves the specific heat of the substance.

The latent heat of fusion of ice is higher than that of any other substance. This fact has an important effect on climate, since it renders water difficult to freeze, and ice correspondingly difficult to melt. The change from one state to the other therefore takes place slowly, and during the change a large quantity of heat is taken in or given out. If the latent heat of water were very low, unless there were some compensating effect, a lake or sea might be liquid one day and solid the next; thaws would be followed by sudden and disastrous floods.

210. The presence of another substance often effects a considerable change in the melting-point of a body.

Ex. 347.—To find the effect of salt on the melting-point of ice.

(1) Set up the apparatus shown in fig. 205, filling the funnel with a mixture of ice (or snow) and common salt—one-fourth salt, three-fourths ice—and using a thermometer graduated to read below 0° C.

Note the lowest reading of the thermometer.

(2) Place in the ice-salt mixture two test tubes, one containing fresh water, the other salt water. Note the temperature at which ice *begins* to form in the test tubes. Do not allow the thermometer to be frozen into the ice.

(3) Place some water and a thermometer in a test tube, and take the reading of the thermometer.

Add some powdered ammonium nitrate, stir, and read the thermometer.

Add some ammonium chloride, stir, and read the thermometer.

211. FREEZING MIXTURES.—The mixtures of substances made in Exercise 347 are called *freezing mixtures.*

When a solid dissolves in water the solid is converted into a liquid, and during the process of liquefaction heat is absorbed or rendered latent. This heat is abstracted from the water in which the solid dissolves, and the temperature of the water is accordingly lowered. In some cases "sensible" heat is by such a process converted into latent heat so *rapidly* that a very low temperature results.

CHAPTER XXV

Latent Heat of Vaporization

212. EVAPORATION AND BOILING.—The process of converting a liquid into a vapour or gas is called **vaporization**. A **vapour** is a gas which can easily be condensed into a liquid, as steam can be.

But the conversion of water into water vapour, as cold steam is generally called, is nearly always going on when water is freely exposed to the air. Thus pools dry up, and a wet cloth hung in the air becomes dry. Such common facts as these indicate that the air readily takes up water vapour, and therefore always contains some.

Ex. 348.—(1) Place a quantity of water in a shallow vessel and weigh it. By means of a small flame maintain the water for about 20 min. at a temperature of 30° or 40° C., then weigh again.

The loss in weight shows that some water has passed out of the vessel. The process that took place is called **evaporation**.

(2) Heat the remaining water to 70° C., and maintain it at that temperature for the same time as before. Weigh again.

The loss in weight is greater than during the previous period.

Evaporation, then, takes place faster at high than at low temperatures.

Ex. 349.—Place some water in a dish with a glass tube dipping into it. Weigh the vessel and contents.

Pump air through the tube for a few minutes. Weigh again. The loss in weight shows that the air in passing through the water took up vapour from it, although it was cold.

Similar experiments made with various liquids show that they evaporate in a similar manner (Exercise 359).

If a vessel of water be exposed for several hours on different days in the same position away from any source of heat, it is found that more water evaporates on dry days than on damp days. And more evaporates from a large surface than from a small surface.

Confining our attention to water, evaporation is thus shown to be a process that takes place at the surface that separates water from air. It takes place faster at high than at low temperatures; faster in dry air than in air which already contains much water

vapour; and faster when fresh air continually passes over or through water than when the same air remains stagnant in contact with it.

Boiling or **ebullition**, on the other hand (Exercise 303), takes place by the formation of bubbles of steam inside the water, and only occurs (for the same pressure) at one fixed temperature, and takes place exactly the same whether the air above it be dry or damp.

Corresponding statements are true about other liquids and vapours.

213. BOILING-POINTS. — Vapours, like gases, are compressible, tend to expand, and exercise pressure, just as was found to be the case with air in Chap. VIII.

In the ordinary operation of boiling water the steam comes off quietly. From the safety valve of a steam engine it often rushes out with great force. The cause of this is that the steam in the boiler is under a great pressure, and is much compressed as air is compressed in a bicycle tyre.

Such steam, as well as the water in the boiler in contact with the steam, is very hot, much above 100° C. Water can be made to boil at almost any temperature according to the pressure of the air above it. When that pressure is the standard pressure of the atmosphere water boils at 100° C.; when the pressure is lower than that, the boiling-point is lower; when the pressure is greater the boiling-point is higher.

Fig. 224.—Water Boiling below 100° C.

Ex. 350.—Let a flask be half-filled with water and heated till the water boils. After a short time, when the upper part of the flask is filled with steam, let it be well corked and inverted (fig. 224). If now cold water be poured over the flask, or a cold wet cloth wrapped round it, the water will again boil, and continue to do so for some time, although its temperature is considerably below 100°.

The steam that arose from the boiling water expelled much of

the air and itself filled the space above the water. Some of this steam was liquefied when the cold was applied, and thus the pressure on the surface of the water was diminished. Owing to this diminution of pressure on its surface the water was enabled to boil at the lower temperature.

If, on the other hand, the pressure on the surface of water be in any way increased, the water must be raised above 100° C. before it can boil. Water boils at about 119° in a diving bell 33 ft. below the surface of water, owing to the additional pressure.

When we speak simply of the *boiling-point* of a liquid it is understood to be under atmospheric pressure simply.

Ex. 351.—To determine the boiling-point of a liquid.

(1) Salt solution. Use a thermometer reading to 120° C. Boil a considerable quantity of water in a flask, through the stopper of which passes the thermometer and an exit tube. While the water in the flask is heating, weigh and place on separate pieces of paper several quantities of common salt, each 10 grm.

Add 10 grm. of salt to the water, and take the temperature of the thermometer (*a*) in the water, (*b*) in the steam. Note the difference. The reading (*b*) is to be taken as the boiling-point.

Add another 10 grm. of salt, and take the readings again.

Continue to add the salt—10 grm. at a time—and take the readings of the solution and of the steam. Do not exceed the limit of the highest reading marked on the thermometer.

Record the height of the barometer at the time of making the experiment.

(2) Make a graph showing the variation in the boiling-point of water, with the quantity of salt that it contains.

(3) Benzene, alcohol, turpentine, and glycerine are other suitable liquids for experiment. For liquids other than water use the apparatus of fig. 206, and connect the vapour delivery tube with a condensing arrangement, as in fig. 193.

For turpentine and glycerine use a thermometer that reads up to 350° C.

214. LATENT HEAT.—In these experiments the same fact is noticed as occurred during fusion. While the substance is changing its state—boiling away—it gets no hotter, although heat is continuously supplied to it, and absorbed by it. There is then a latent heat of vaporization. *The latent heat of vaporization of a liquid is the number of calories required to convert 1 grm. of the liquid into vapour without change of temperature.*

Ex. 352.—To find the latent heat of vaporization of water or the latent heat of steam.

(1) Place about 50 cub. cm. of water in a small flask or a boiling tube, and take its temperature (*t*° C.).

Place a flame under the flask, and note the *exact time* (*a*) of doing so. The flame should only play upon the bottom of the flask, and should remain steady during the experiment.

When the water *begins* to boil (suppose at 100° C.) again note the time (b).

Let the water all boil away, and as the last of it disappears again note the time (c) and remove the flame.

Find the number of seconds in the intervals ($b - a$) and ($c - b$).

Then approximately

$$\frac{\text{Number of calories taken in by the water at 100° in evaporating}}{100 - t} = \frac{c - b}{b - a};$$

or writing L for the latent heat of steam,

$$\therefore \text{L} = (100 - t)\frac{c - b}{b - a}.$$

(2) In this experiment read the thermometer to a fraction of a degree, and weigh accurately.

Fit a flask E with a tube bent approximately as ABC in fig. 225. The tube should be thin quill tubing, and the arm BC not more than 12 cm. in length. The part BD should slope towards the flask. Boil water in the flask.

Weigh a small calorimeter or beaker, and support it inside a larger one either by threads or by standing it on small pieces of glass rod.

Place about 200 cub. cm. of cold water in the calorimeter and weigh again. When steam has been issuing for a few minutes from C, take the temperature of the water in the calorimeter, wipe away any water there may be at C, and bring up the calorimeter so that the end of the tube C is nearly at the bottom of the water within it. Stir the water continuously with the thermometer, and when the temperature of the water has risen about 20° remove the calorimeter or the steam tube and take the temperature of the water. Again weigh the calorimeter and contents.

Record as in the following example:—

A. Masses in grammes.

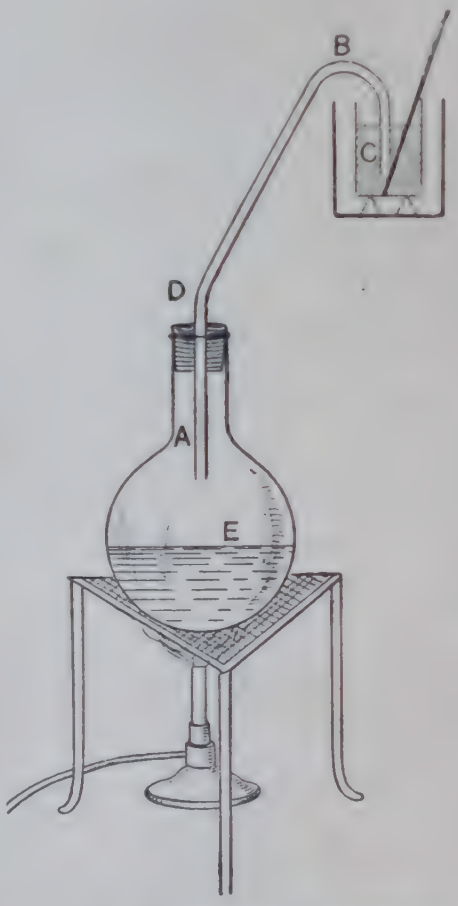

Fig. 225.—Apparatus for Determining the Latent Heat of Steam

Calorimeter only.	Calorimeter + Water.	Calorimeter + Water + Condensed Steam.	Water only.	Water + Calorimeter Equivalent.	Condensed Steam.
39·2	241·6	247·4	202·4	206·3	5·8

B. Temperatures of calorimeter: Initial, 14·2° C.; final, 31·3° C.

Then we have—

(i) Heat (number of calories) given out—

(a) By steam in condensing.....5·8L

(b) By condensed water in cooling from 100° C. to 31·3°...5·8 × 68·7

(ii) Heat (number of calories) taken in by water and calorimeter in being heated from 14·2° to 31·3°.....206·3 × 17·1

$$\therefore 5{\cdot}8L + (5{\cdot}8 \times 68{\cdot}7) = 206{\cdot}3 \times 17{\cdot}1$$

whence $L = 540$.

The latent heat of other vapours is determined in the same manner. They are all much less than that of steam.

215. COLD PRODUCED BY EVAPORATION. — The heat that is rendered latent when a liquid passes into a vapour is obtained either from surrounding objects or from the liquid itself, and if there is no source of heat to keep these at a constant temperature, they must become colder.

Ex. 353.—(1) Repeat Exercise 349 with ether or alcohol in a test tube containing a thermometer. Note the fall in temperature. By rapid pumping a deposit of hoar frost is sometimes produced on the outside of the tube.

(2) Wrap a piece of muslin round the bulb of a thermometer. Take its reading. Wet the muslin with ether. What is the fall in temperature?

Swing the thermometer gently to and fro. The faster the evaporation the lower the temperature produced.

Repeat with alcohol and with water.

If the hand or wrist be wetted with water and swung to and fro in the air, the cold produced may be readily felt. A similar application of ether will produce numbness due to the cold.

If the air in a room be hot and dry it will be considerably cooled by sprinkling water over the floor, even though the water be as warm as the room. The reason is, that the water evaporates quickly into the dry air, and in so doing takes heat from the air which is thus cooled. A damp cloth exposed to a draught becomes very cold for the same reason.

If two vessels containing water be set side by side exposed to the air, one made of glazed non-porous earthenware and the other porous, the water in the porous vessel will often be considerably colder than that in the non-porous vessel, owing to the evaporation from its surface. If there is no such difference of temperature, it is because the air is already nearly saturated and the evaporation is therefore slow.

Since *evaporation takes place much faster in a vacuum than in air*, Leslie's experiment can be readily understood. He placed a little water in a shallow dish over another dish containing strong sulphuric acid under the receiver of an air pump (fig. 226). The air was then pumped out of the receiver, and the water began to evaporate very quickly, but the vapour was absorbed by the sulphuric acid as fast as it was formed. The heat required to form the vapour was taken from the water itself, which, after a time, became so cold that it froze.

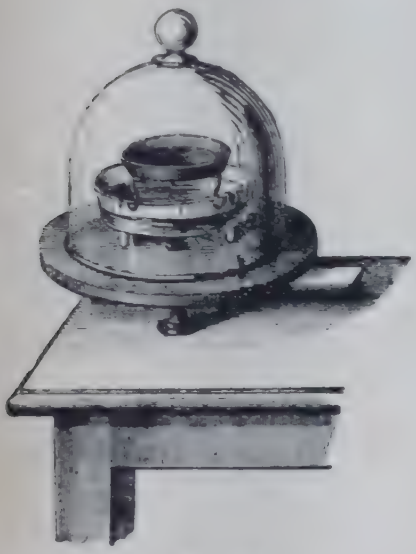

Fig. 226

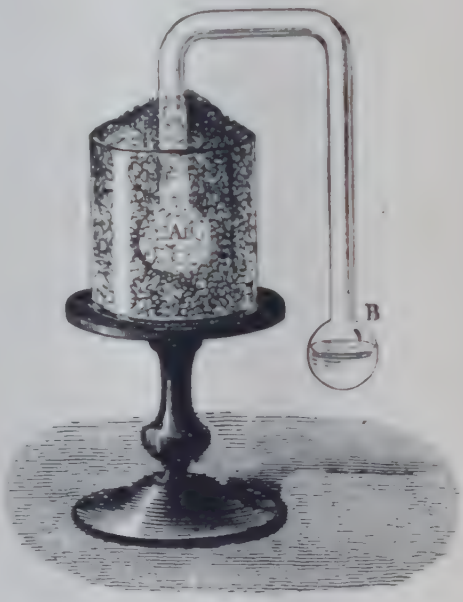

Fig. 227

Water Freezing owing to the Cold produced by its own Evaporation

This experiment of freezing water by the cold produced by its own evaporation may be more easily performed with

Wollaston's Cryophorus.—This consists of a U tube with a bulb at each end, as shown in fig. 227. The air has been nearly all driven out from the apparatus by boiling water in one of the bulbs, and sealing the instrument while the space not occupied by water was full of steam. The object of this is to enable the water to evaporate more quickly than it could do were much air present. If all the water in the instrument be made to pass into bulb B, and bulb A be then placed in a freezing mixture, such as powdered ice and salt, the vapour in A is quickly condensed. The water in B then quickly evaporates, owing to the diminished pressure of the vapour on its surface; the heat required for the evaporation is taken from the water in B, which is thus chilled until it freezes.

216. The heat given out in the condensation of a vapour into a liquid has been shown in Exercise 352 (2).

This evolution of heat explains why the temperature is often high on wet days. The vapour in the atmosphere in condensing into rain gives out a large quantity of heat which warms the air.

Ex. 354.—(1) How many grammes of steam at 100° C. would just melt 20 grm. of ice, the final temperature of the whole mass being 0° C.?

(2) If 5 grm. of steam at 100° C. are passed into a chamber containing ice, how much ice would be melted?

(3) If 2 grm. of steam at 100° C. are passed into 30 grm. of water originally at 20 C., to what temperature would the water be raised?

(4) If you drop a volatile liquid on your hand you feel cold. What is the reason? Why is the sensation of cold greater with ether than with water?

(5) Explain why, in order to cook food at the top of a high mountain, some method must be employed different from that used at the sea-level.

(6) What is meant by the statement that the latent heat of steam is 540?

CHAPTER XXVI

Water Vapour in the Atmosphere

217. VAPOUR PRESSURE. — In Art. 213 it was seen that the amount of pressure exercised by water vapour was very different under different conditions. It must be remembered that water vapour, whether cold or hot, is an invisible gas. The visible cloud formed when steam rushes out into the air is not vapour.

All gases and vapours when sufficiently compressed and chilled pass into the liquid state. The differences in the behaviour of steam and air are mainly due to the fact that air requires great pressure and cold to liquefy it, whereas water vapour can be liquefied by ordinary pressures at ordinary temperatures.

If we have a vessel at any ordinary temperature and pass steam into it, the steam exercises a pressure on the walls of the vessel; if more and more is forced in, there comes a point at which the steam that is forced in cannot exist there; it condenses into water. The space inside the vessel is then said to be **saturated**: it has in it as much water vapour as it can hold.

If the vessel be heated the space can hold more steam; and if more be pumped in, of course the pressure increases. The pressure exercised by saturated water vapour thus changes with the temperature.

Ex. 355.—(1) Set up a U tube on the general model of fig. 228, with the shorter arm A capable of being opened or closed. With both arms open pour in mercury; pour water into A nearly up to the top, and close the opening. Place the tube as shown in a bath of cold water D. Warm the bath.

As the heating of the bath proceeds, some of the water in A evaporates; a space containing this vapour appears above the water layer C. This vapour exercises a pressure on the liquid below it, and the levels become as shown in fig. 228. The pressure of the water vapour forces the mercury out of A into the tube B, and the amount of this pressure is measured by the difference of level of the mercury surfaces. The pressure of the layer of water may be neglected.

A thermometer in the bath gives the temperature of the water vapour. As the heating slowly proceeds, take at regular intervals (*a*) the temperature of the vapour, (*b*) its pressure, and record them in two columns.

(2) Make a graph of the readings, plotting the temperatures along the horizontal axis and the pressures along the vertical axis.

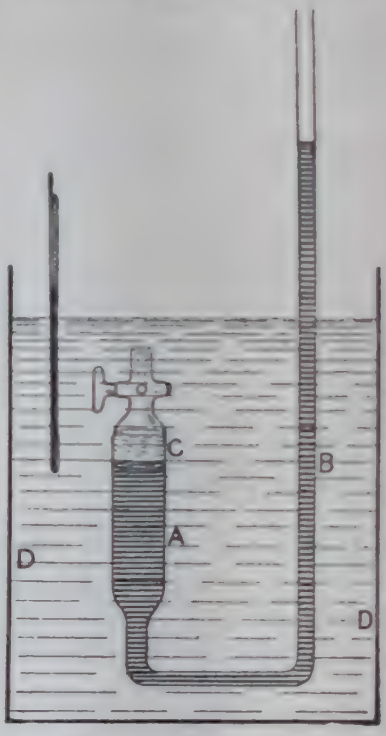

Fig. 228.—Apparatus to show the Pressure exercised by a Vapour

Throughout this experiment the space above the water was saturated with vapour: it could hold no more. The amount of vapour present, and therefore the pressure of the vapour, increased with rise of temperature, as is the case with all gases and vapours; but there is no simple ratio between the pressure and the temperature.

The graph shows that at the temperature at which water boils, the pressure of the vapour is equal to the pressure of the atmosphere. A liquid boils when the pressure upon its surface is equal to the pressure which its vapour exercises at the same temperature.

218. There is always more or less water vapour present in the atmosphere.

Ex. 356.—Dissolve a little cobalt chloride in water, and paint a piece of paper with the solution. When the paper is moist its colour is pink. Dry it; the colour is blue.

Expose the paper to the air on various days. Its varying colour shows that it is more moist on some days than on others. The moisture has been absorbed from the atmosphere.

Ex. 357.—Weigh a small quantity of anhydrous copper sulphate—a white powder—in a small porcelain dish. Expose it for some time to the air. Note the change in colour. Weigh again. The addition of weight is moisture that the salt has absorbed from the atmosphere.

Ex. 358.—Place about 6 cub. cm. of strong sulphuric acid in a beaker and weigh. Leave the acid exposed to the air for a few hours. Weigh again. The increase in weight is due to the absorption of moisture from the atmosphere.

Dry calcium chloride or dry phosphoric anhydride give similar results.

Such experiments show that on any day in almost any place water vapour is present in the atmosphere.

219. DEW.—If a glass of cold water be taken into a warm room, the outside of the glass often becomes covered with a thin film of water. This is **dew**, which has been formed by the condensation of water vapour present in the air. The effect is better seen if the cold water is contained in a cup of polished metal.

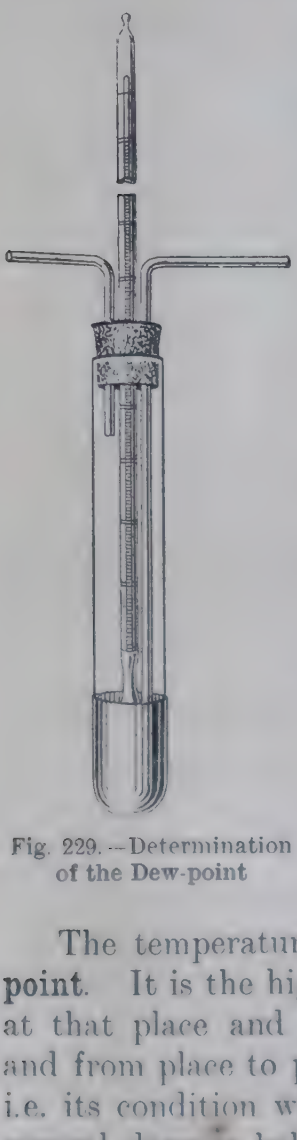

Fig. 229.—Determination of the Dew-point

Ex. 359.—(1) Put cold water in an aluminium cup until it is two-thirds full. Place in the water a thermometer and a stirrer, and put a sheet of glass between yourself and the cup. Drop a few small pieces of ice into the water, stir it thoroughly, and immediately dew *begins* to form on the bright surface of the cup read the thermometer.

Repeat the experiment two or three times, and take the mean of the temperature readings.

(2) Fit a large test tube, as shown in fig. 229, with an indiarubber stopper through which there pass a thermometer, and two bent glass tubes, one of which goes nearly to the bottom of the tube.

In the tube place some liquid ether, so that the thermometer dips into it.

Fix a small compression pump on the longer tube and slowly force air bubbles through the ether. When a film of dew *begins* to form on the bottom of the tube cease passing air through it and read the thermometer.

Repeat the experiment, and take the mean value of the temperature readings.

The temperature obtained in Experiment 359 is called the **dew point**. It is the highest temperature at which dew can be deposited at that place and time. The dew-point varies from day to day and from place to place according to the *hygrometric* state of the air, i.e. its condition with regard to moisture. When the dew-point is several degrees below the temperature of the air, the atmosphere is comparatively dry; when the dew-point is only a degree or two below the temperature of the air, the atmosphere is nearly saturated with water vapour, that is it contains nearly as much vapour as it can hold.

Example.—Thus on a certain day the mean of four readings of the dew-point was 14·5° C., and the temperature of the laboratory was 17° C.

Since dew began to be deposited at 14·5°, there was enough water vapour present in the air just to saturate it at that temperature. Its pressure must then have been (p. 409) $\frac{1}{2}(11{\cdot}88 + 12{\cdot}67) = 12{\cdot}27$ mm. of mercury.

The pressure of water vapour required to saturate the air at 17° is (p. 409) 14·39 mm. of mercury. Hence the proportion of vapour present was $\frac{12{\cdot}27}{14{\cdot}39}$, or 85 per cent of the largest amount that the air could actually hold.

This fraction is called the *relative humidity* of the air.

Ex. 360.—From your observations of the dew-point and the tables on p. 409 determine the relative humidity of the air at the time when and place where the observations were taken.

When air that contains so much vapour as to be nearly saturated, is chilled by contact with some cold object, the vapour in the air is more than sufficient to saturate it at the lower temperature, and water is therefore deposited on the cold object. Thus on a clear evening in summer, dew is often deposited freely on grass. There has been much evaporation during the day, and the air contains much moisture, but is not saturated because it is warm. When the sun sets, however, the surface of the earth becomes much colder by radiation. This cold surface acts in the same way as the glass of water; it reduces the temperature of the air near it to below the dew-point, and dew is deposited on the cold surface.

The chief circumstances that influence the deposition of dew are: (1) *the amount of moisture in the air*, (2) *wind*, (3) *the presence or absence of clouds*, and (4) *the character of the surface on which the dew is deposited*. The influence of the first of these circumstances is manifest. A high wind hinders the formation of dew, by preventing any layer of air from remaining in contact with the cold surface sufficiently long for its temperature to be reduced to the dew-point. The effect of the presence of clouds is to check radiation from the earth's surface, and thus to keep the surface warm. As to the character of the surface, more dew is deposited on grass than on gravel, because grass radiates its heat easily, and being a bad conductor does not get a fresh supply easily from the earth.

220. HYGROMETERS.—The aluminium cup used in Exercise 359 and the tube shown in Fig. 229 are simple forms of *hygrometer*, which is an instrument for determining the humidity of the air.

In *Daniell's hygrometer* (fig. 230), a ∩ tube has a bulb at each end. The bulb A contains liquid ether and a thermometer; B is surrounded by muslin.

Some liquid ether is dropped on to the muslin, evaporates quickly, and cools bulb B, inside which the ether vapour begins to condense. Owing to the diminution of pressure, evaporation of the liquid in A at once commences; this process cools bulb A, which in its turn chills the external air in contact with it. The air thus chilled cannot retain its moisture and dew appears on the outside of A, which is made of black glass in order that the appearance of the film of moisture may be more easily seen.

The temperature indicated by the thermometer when this film begins to form is the dew-point.

The *wet and dry bulb hygrometer* consists of two similar thermometers mounted side by side on a stand (fig. 231). One of these thermometers has its bulb covered with muslin, which is kept moistened with water by means of threads that dip into water contained in a small cup placed a few inches below. The water on the muslin is continually evaporating, and thus chills the bulb, so that the left-hand thermometer indicates a lower temperature than the right, which is simply exposed to the air.

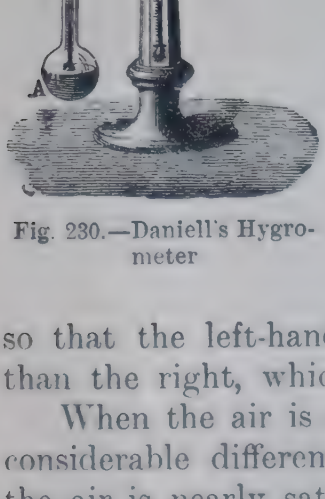

Fig. 230.—Daniell's Hygrometer

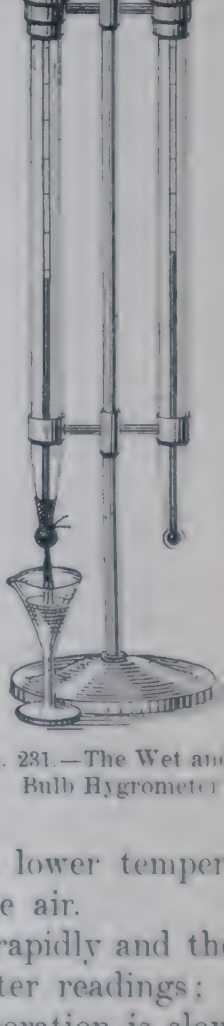
Fig. 231.—The Wet and Dry Bulb Hygrometer

When the air is dry, evaporation proceeds rapidly and there is considerable difference between the thermometer readings; when the air is nearly saturated with moisture, evaporation is slow and the temperatures are nearly the same. The instrument does not directly give the dew-point, but this can be obtained from tables that have been drawn up by comparison with Daniell's hygrometer.

221. The aqueous vapour in the air condenses into various forms.

Cloud and Mist.—These are really the same: a mist is only a cloud touching the ground. Water *vapour* is invisible. Clouds

are visible because they consist not of vapour, but of very small globules of water which can reflect the light. A cloud or mist is always formed when water vapour is chilled to condensation point. This often occurs in low valleys in the evening.

On the tops of mountains clouds are formed by vapour-laden air coming in contact with the cold mountain.

Clouds are formed or dissolved in the air when currents of vapour-laden air pass into regions of lower or higher temperature.

The very small globules of water in a cloud begin to fall as soon as they are formed, but owing to their small size they fall very slowly, and often pass into vapour before reaching the ground. New ones are continually being formed, so that a cloud is continuously changing. Under some conditions the very small drops condense more vapour on their surfaces and increase in size. The larger they are, the faster they fall. Hence they often reach the surface of the earth, and form *rain*.

Snow and Sleet.—When water vapour passes rapidly into the solid state snow is formed. When this snow becomes partly melted

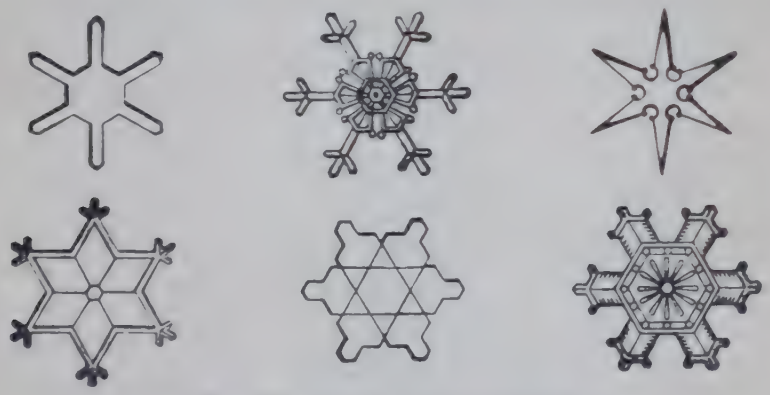

Fig. 232.—Forms of Snow Crystals

in falling it is known as sleet. Snow is simply a collection of ice particles arranged in a crystalline form as shown in fig. 232.

When this rapid condensation takes place at the surface of the earth, *hoar frost* is deposited.

The formation of a hailstone is a complicated process not fully understood. A hailstone has a core of snow surrounded by layers of ice and snow, which are supposed to be deposited round the snow core in different strata of air through which the hailstone has been whirled by air currents.

CHAPTER XXVII

Measurement of Expansion

222. COEFFICIENT OF EXPANSION OF GASES. — The general facts of expansion as a result of heat have been dealt with in Art. 181.

We have now to measure the amount of this expansion.

Gases

Ex. 361.—A quantity of dry air is confined within a narrow tube A (fig. 233) by a thread of mercury B about 2 cm. long. The tube should be not less than

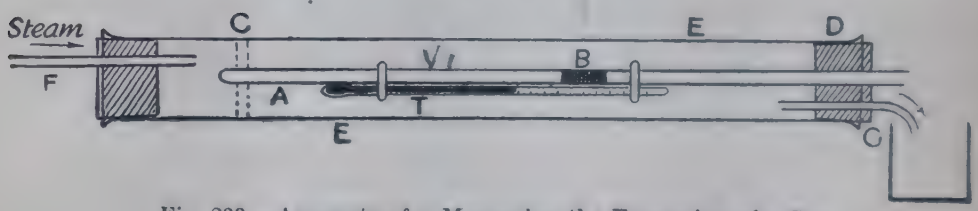

Fig 233.—Apparatus for Measuring the Expansion of a Gas

half a metre long, open at one end and closed at the other. A barometer tube is suitable. A thermometer T is tied to the tube A, which is supported by a perforated cork C, and a stopper D, inside a larger glass tube E. Through the stoppers of the outer tube there pass small tubes F, G, to serve as inlet and outlet for steam.

The whole apparatus should be suitably supported in a horizontal position. The object of the experiment is to find the change in the volume of the air in A when its temperature is changed by a certain known number of degrees.

(1) The whole apparatus being at the temperature of the laboratory, tap the tube to assist the mercury thread B to take up its proper position; with a millimetre scale outside the tube E read the length of the column of air enclosed in A by the mercury. For comparative purposes this length may be taken as the volume (v_1). Also read the thermometer T. Call the temperature t_1.

(2) Pass steam in at F for several minutes. When the whole apparatus is at the temperature of the steam, tap the tube, and read again the length of the air column and the temperature. Call these quantities v_2 and t_2.

(3) Make a graph as in the following example:—

$$t_1 = 15^\circ \text{ C.}; \; v_1 = 19{\cdot}65; \; t_2 = 99^\circ \text{ C.}: \; v_2 = 25{\cdot}1.$$

The graph is shown in fig. 234, where the point A corresponds to a volume of 19·65, and a temperature 15° C.; and the point B to a volume of 25·1 and temperature 99° C.

Produce the graph to the point C, corresponding to 0° C. The volume at this temperature would have been 18·7.

Produce the graph similarly to D, corresponding to 100° C.; the volume at this temperature would have been 25·15.

$$\text{Then we have } \frac{\text{Volume at } 100^\circ \text{ C.}}{\text{Volume at } 0^\circ \text{ C.}} = \frac{25{\cdot}15}{18{\cdot}7} = \frac{1{\cdot}35}{1}$$

Therefore (the expansion taking place regularly), according to this experiment, a volume = 1 at 0° C., increases by ·0035 when raised to 1° C.

The correct value of this quantity is ·00367 or $\frac{1}{273}$: this is the **coefficient of expansion of air.** It is the amount by which unit volume of air at 0° C. expands when its temperature is raised to 1° C.

Since air expands regularly, it follows that if V_0 be the volume of a mass of air at 0° C., and V_1, V_2 the volumes at some higher temperatures t_1, t_2, then

$$V_1 = V_0(1 + \cdot00367t_1),$$
$$V_2 = V_0(1 + \cdot00367t_2),$$

and so on.

The coefficient of expansion of air (generally written α) may also be found by the method indicated in the following example:—

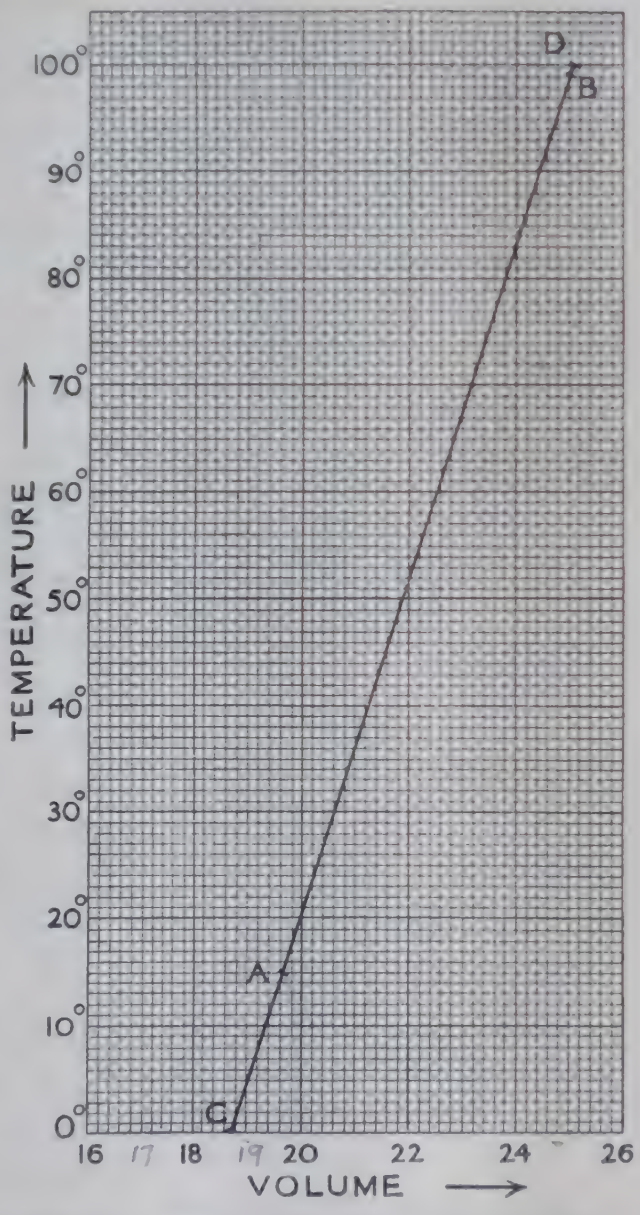

Fig. 234.—Relation between the Volume and Temperature of a Quantity of Gas

A flask was found by the method of Exercise 96 to have a capacity of 245 cub. cm. When fitted with a delivery tube and pinchcock it weighed 90 grm. It was placed up to the neck in a bath at 100° C., and after some time the pinchcock was closed.

The flask was then put into a vessel of water at 20° C. with its mouth downwards and the pinchcock opened. Water rushed in. The pinchcock was then closed, and the flask taken out, dried, and weighed. The weight was 142 grm.,

showing that 52 cub. cm. was occupied by water, and therefore that the air it contained occupied 193 cub. cm. at 20° C. But this 193 cub. cm. occupied 245 cub. cm. at 100° C. Hence we have—

$$\frac{\text{Volume of air at } 100^\circ}{\text{Volume of air at } 20^\circ} = \frac{245}{193}.$$

$$\therefore \frac{1 + 100\,\alpha}{1 + 20\,\alpha} = \frac{245}{193},$$

$$\text{whence } \alpha = \cdot 0036.$$

The coefficients of expansion of other gases are about the same as that of air.

223. RELATION BETWEEN THE TEMPERATURE AND PRESSURE OF A GAS.

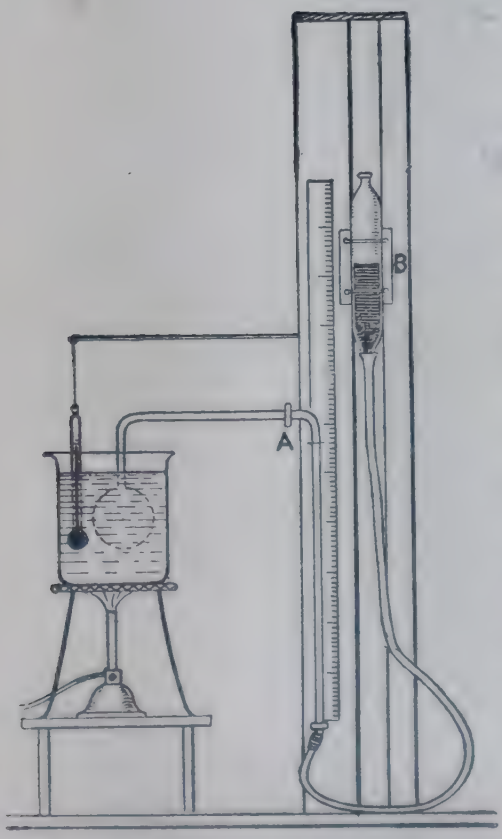

Fig. 235.—Apparatus to show the Relation between the Pressure and Temperature of a Quantity of Gas

Ex. 362.—If a mass of gas be enclosed in a vessel so that it cannot expand, the effect of heating it is to cause it to exercise a greater pressure on the walls of the vessel. This pressure may be measured by a manometer.

Fig. 235 shows the arrangement. A large glass bulb or a flask contains air, and is connected by a fine tube to a mercury manometer mounted on a stand. The bottom of the manometer consists of flexible tubing, so that the open arm can be raised or lowered, and thus any desired pressure obtained.

(1) The bulb is placed in a water bath, so that its temperature may be varied as desired and read on a thermometer contained in the bath.

The volume of the air is not allowed to change when the temperature changes. By altering the pressure the surface of the mercury in the closed arm is always kept at the same point—a mark A near the bend of the fine tube.

The bulb being at the temperature of the room, the pressure is adjusted until the mercury stands at A.

Read the thermometer (t_1) and the difference (p_1) between the levels of the mercury at A and B.

Warm the bath about 20° C., and again record its temperature (t_2) and the difference of level (p_2) between B and A.

Proceed in this way with intervals of temperature of about 20° C. until the water boils.

Read the height (H) of the barometer, and add it to p_1, p_2 . . . Record thus:—

Temperature of Air.	Pressure on the Air	
	due to Mercury Column.	due to Mercury Column + Atmosphere.
$t_1°$	p_1	$p_1 + H$
$t_2°$	p_2	$p_2 + H$
...	..	...

(2) Make a graph of the numbers in the first and third columns, as in Exercise 361, setting out pressures along the horizontal axis and temperatures along the vertical axis. A line drawn fairly through the points obtained will be straight if the observations are accurately taken.

Find where the graph cuts the line of 0° C., and thus obtain p_0, the pressure at 0° C. Then as above—

$$p_2 = p_0 (1 + \alpha t_2)$$
$$p_3 = p_0 (1 + \alpha t_3),$$

and so on. Each of these equations enables a value of α to be found. Its value is the same as that found in Exercise 361, viz. ·00367 or $\frac{1}{273}$.

224. THE AIR THERMOMETER.—The apparatus shown in fig. 235 may be used as an air thermometer. The bulb is placed in the space whose temperature is to be measured, and the mercury is adjusted to the fixed mark. The pressure is then ascertained, and the temperature is then calculated from the pressure according to the results of the last experiment.

Example.—When the bulb of an air thermometer is at 0° C. the pressure of the enclosed air is that of 760 mm. of mercury. The bulb is placed in a vessel, and the pressure is found to be that of 1856 mm. of mercury. What is the temperature of the vessel?

$$\begin{aligned} p_1 &= p_0 (1 + \alpha t_1) \\ 1856 &= 760 (1 + \cdot00367\, t_1) \\ 2\cdot442 &= 1 + \cdot00367\, t_1 \\ \cdot00367\, t_1 &= 1\cdot442 \\ t_1 &= 393°. \end{aligned}$$

225. ABSOLUTE ZERO OF TEMPERATURE.—It has been shown above that 1 cub. cm. of air increases in volume by $\frac{1}{273}$ cub. cm. when its temperature is raised 1° C. Similarly, when its temperature is lowered 1° C. it contracts $\frac{1}{273}$ cub. cm. If it continued to contract at this rate, at −273° C. its volume would be nothing. This temperature −273° C. is called the absolute zero of temperature.

Ex. 363.—(1) A quantity of air occupies a volume of 200 cub. cm. at 0° C. Calculate its volume at 100° C., at 58° C., at 273° C., at −58° C., and at −273° C., assuming that the pressure always remains the same.

(2) A quantity of air exercises a pressure equivalent to that of 1038 mm. of mercury at 100° C. Calculate its pressure at 0° C., −100° C., and 546° C., assuming that the volume remains unchanged.

Liquids

226. ABSOLUTE AND APPARENT EXPANSION.—In the preceding experiments on air, the expansion of the vessel that contained the air was neglected because it was small. In determining the coefficients of expansion of liquids, the expansion of the vessel cannot be neglected.

The effect of expansion of the vessel that contains a liquid was shown in Experiment 309.

There are thus two coefficients of expansion of a liquid—the coefficient of absolute expansion which shows the amount it actually expands, and the coefficient of apparent expansion which shows the excess of the liquid expansion over that of the containing vessel.

Fig. 236.—Bulb Tube for Comparing the Expansions of Liquids

227. APPARENT EXPANSION OF LIQUIDS.—The apparent expansion of different liquids may be compared by the method of Exercise 306.

Bulbs of suitable size and graduated tubes which may be joined to them by indiarubber (fig. 236) are sold by instrument makers.

Ex. 364.—Fill the bulb with the liquid, and push in the tube until the liquid stands a short distance up the graduated stem. Place the bulb in a cold-water bath beside a mercury thermometer, and take the readings of each liquid on its scale.

Slowly warm the bath, comparing the readings of the mercury thermometer and the liquid thermometer every 10 degrees.

Repeat the operation with some other liquid in the bulb tube.

Make a graph showing for each liquid on the same sheet of paper the amount of expansion with increase of temperature.

These are apparent expansions. Compare them.

Ex. 365.—To find the coefficient of apparent expansion of a liquid in glass.

Mercury or glycerine is suitable.

Weigh a specific-gravity bottle.

Fill the bottle with mercury at the temperature of the air (suppose 18° C.), and (without handling the bottle unnecessarily) weigh again.

The mass of *mercury only* is m_1 grm. (suppose 213·2 grm.).

Place the bottle in a water bath up nearly to the top of its neck, and warm the bath to $t_2°$ (suppose 100° C.).

Allow the apparatus to cool. When cool weigh again. The weight of the *mercury only* is m_2 grm. (suppose 210·6 grm.).

Then taking the figures given as above, if x is the coefficient required,

$$\frac{1 + xt_2}{1 + xt_1} = \frac{m_1}{m_2}; \text{ whence } \frac{1 + 100x}{1 + 18x} = \frac{213{\cdot}2}{210{\cdot}6}, \text{ and } x = {\cdot}00015.$$

A bottle or tube used as in this exercise is usually called a **weight thermometer.**

228. THE ABSOLUTE COEFFICIENT OF EXPANSION OF LIQUIDS.—In Exercise 174 the densities of two *different*

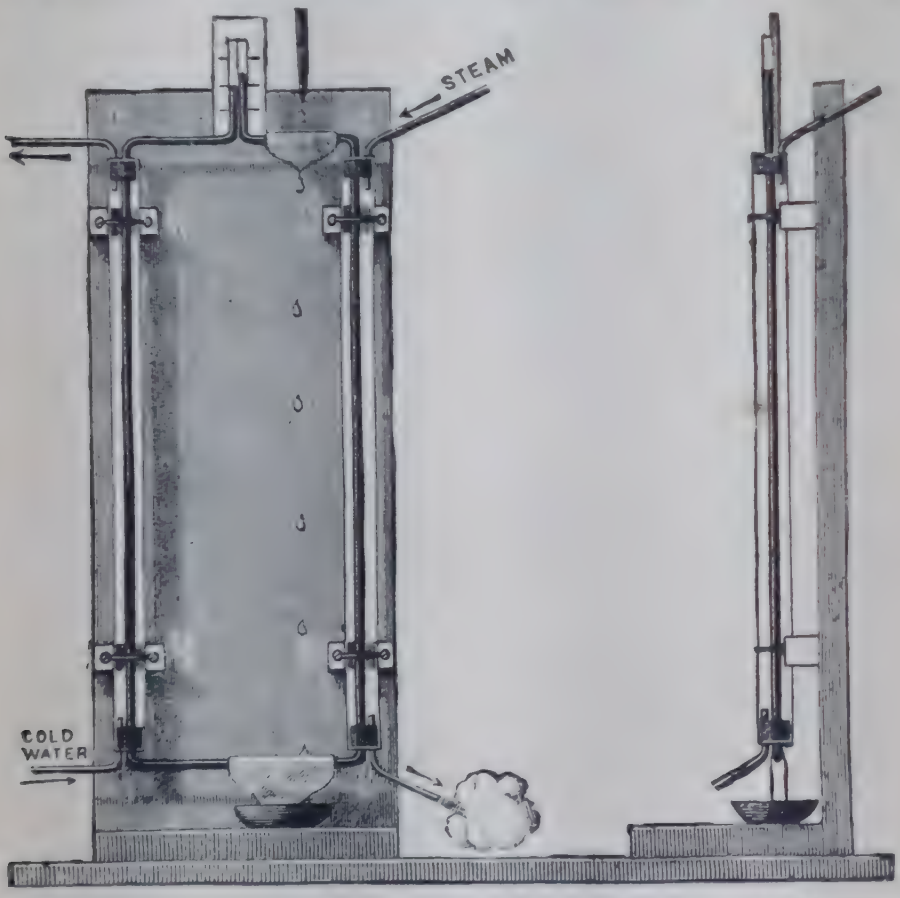

Fig. 237.—Apparatus for Determining the Absolute Expansion of a Liquid

liquids were compared by placing them one in each arm of a U tube and noting the difference of level of their surfaces. This difference was due to the difference in densities of the liquids. Now since the same quantity of a liquid occupies more space when it is hot than when it is cold, the hot liquid must be less dense than the cold

liquid. Hence if we have a U tube with only one liquid in it, but with one arm hot and the other cold, there will be a difference of level between their surfaces as in Art. 104, which will enable us to compare their densities.

This principle is used to measure the absolute expansion of liquids.

A convenient form of the apparatus is shown in fig. 237.

The U tube is the small tube, the top of each arm being bent round so that the two upper ends lie side by side. A millimetre scale placed behind these open ends enables the difference of level to be accurately read.

When the apparatus is all at the same temperature the surfaces of the liquid in these two tubes are exactly at the same height.

Each arm of the U tube is surrounded by a wider tube or jacket. If we pass steam through one jacket and cold water through the other for several minutes the surface of the liquid on the warm side rises to a higher level than that on the cold side.

As it is only the vertical arms which we consider, it is necessary to keep the rest of the apparatus cool. This is effected by allowing the cold water supply also to drip on to blotting paper placed on the horizontal arms, near the warm tube.

The following observations obtained in an experiment on mercury shows how the coefficient is obtained:—

Height of cold column of mercury	45·3 cm.
Difference of levels	·7 cm.
Temperatures of columns	100° C. and 15° C.

Then if y is the coefficient, we have as before—

$$\frac{V_1}{V_2} = \frac{1 + 15y}{1 + 100y} = \frac{45{\cdot}3}{46}$$

$$\therefore 3840y = {\cdot}7$$

$$y = {\cdot}00018.$$

The difference between this value and that obtained in Exercise 365 gives the coefficient of expansion of the glass, which is about ·00003.

Each different liquid has a different value for the coefficient of expansion.

229. EXPANSION OF WATER. — The changes of volume undergone by water as its temperature changes are of especial interest.

Ex. 366. Take a small bulb of 2 or 3 cub. cm. capacity, such as that shown in fig. 236. Attach a short piece of indiarubber tube to its neck; put in the bulb some mercury equal to one-seventh of the volume of the bulb; fill up the bulb with distilled water that has been recently boiled, and push down into the water a graduated tube of fine bore, such as an old thermometer tube. The water must rise up the tube, and no air must be enclosed.

Place the bulb in a water bath of temperature about 15° C. Read the level of the top of the water column and the temperature on a thermometer placed in the bath. These two readings are to be taken for several temperatures down to about 0° C. Record them in two columns. The temperature intervals should not be more than 2° or (when below 10°) more than 1°. To obtain the temperatures make a mixture of ice and salt in a separate vessel, and drop small quantities of this freezing mixture into the bath as required.

The reason for adding the mercury in that particular amount is that the mercury contracts just as much as the bulb does, so that the part of the bulb filled with water does not change in volume. The effect observed is that due to the contraction of the water only.

Make a graph of the observations, plotting temperatures along the base line, and heights of liquid column along the vertical line.

The correct curve is given in fig. 238. It shows that water as

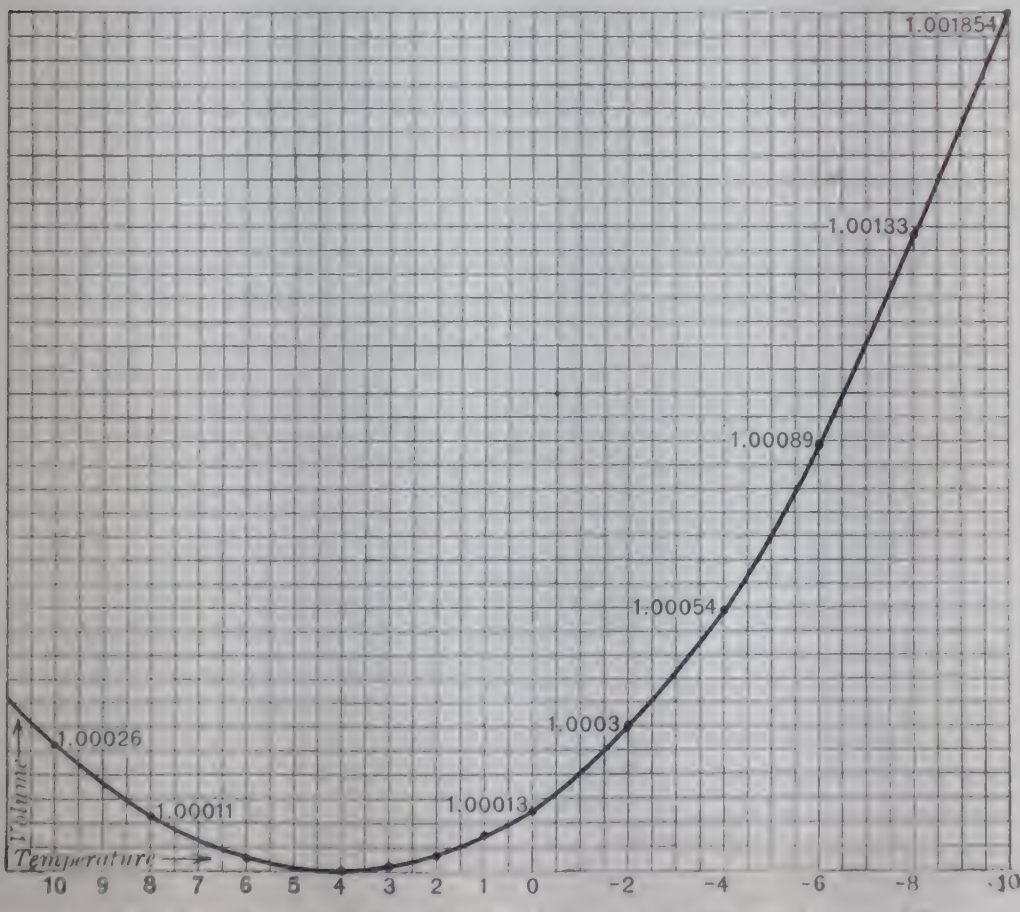

Fig. 238.—Curve showing the Changes of Volume of Water round about 4° C.

it is cooled continues to contract until its temperature reaches 4° C., after which further cooling causes it to expand.

The temperature of maximum density of water may be observed by an apparatus such as that shown in fig. 239.

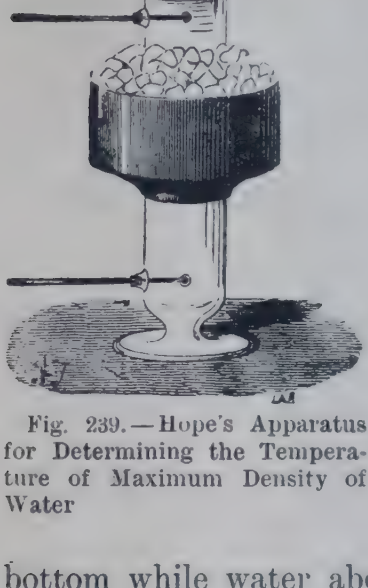

Fig. 239.—Hope's Apparatus for Determining the Temperature of Maximum Density of Water

Ex. 367.—A cylinder having two thermometers projecting from the side, one near the top and the other near the bottom, is filled with water at the temperature of the room. Round the middle of this cylinder is a trough which is filled with a mixture of ice and salt or some other freezing mixture. This cools the central belt of water, which contracts, becomes bulk for bulk heavier, and therefore sinks to the bottom. This process continues until the lower thermometer marks 4°, the upper thermometer so far remaining almost stationary. When, however, the lower thermometer marks 4° it falls no lower; but the upper one is now affected, and rapidly falls to 0°. The colder water has now gone to the top, showing that it is lighter bulk for bulk, or less dense.

Since the water at 4° sinks to the bottom while water above or below that temperature rises to the top, 4° C. must be the temperature of maximum density of water. In consequence of this fact the water at the bottom of lakes is very seldom below 4°. When the surface water is chilled to that temperature it sinks, and the lighter rises to the surface to be chilled and to sink in its turn. So that before the surface can freeze, the whole mass must be reduced to 4°, and if the depth be considerable this takes much time, so that the process of freezing is thereby retarded.

Solids

230. COEFFICIENT OF EXPANSION OF SOLIDS.—The amount of lengthening of a rod of a metre in length or less is very small when the rise in temperature is not more than 100° C.

Various methods of measuring this small quantity are in use. They are of two kinds:

(*a*) By using a lever to magnify the motion in a known ratio.

(*b*) By actually measuring the increase of length itself by a vernier, a wedge, a micrometer screw, a spherometer, or a microscope.

In all the methods one end of the bar is firmly fixed, and the

lengthening is measured at the other end, which is free, i.e. not fixed.

The measurement made is usually the alteration in length that takes place when a bar is heated from the temperature of the laboratory to that of steam.

Ex. 368.—*First Method.* A convenient apparatus is shown in fig. 240, but need not be exactly of that form.

A tube S is firmly fixed near one end A. The other end passes at B through

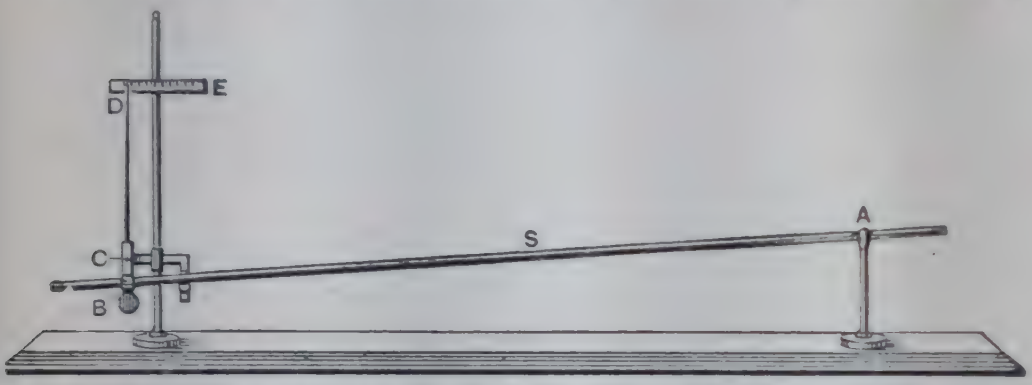

Fig. 240.—Measurement of Solid Expansion by the Lever Method. (Comerton's apparatus.)

a lever BD, whose fulcrum is a pin at C. The arm BC is short; the arm CD is long. The point D has a millimetre scale behind it.

Suppose the length AB is one metre, and that CD = 10BC.

(1) At the temperature of the laboratory (suppose 15°) read the position of the index D on scale E (suppose 2).

(2) Pass steam (at 100° C.) through the tube S. When it has passed for some time read the position of D on the scale E (suppose 17). The elongation of the bar is therefore $\frac{17 - 2}{10}$ = 1·5 mm. Then—

A bar of length		For a rise of temperature		Has increased in length by
100 cm.	.....	85°	.....	·15 cm.
∴ 1 cm.	.....	1°	.....	$\frac{\cdot 15}{100 \times 85}$ = ·000018 cm.

The quantity ·000018 is the linear coefficient of expansion of the bar. The determination of the length of the bar at 0° C. is unnecessary in this case, as the change is so small.

A **coefficient of linear expansion** is thus a small fraction expressing the ratio between two quantities, viz.:

(*a*) The *change* in length of a rod composed of the given substance when its temperature is altered 1°.

(*b*) The *whole* length of that rod at 0°.

The coefficient is different for each different solid.

Ex. 369.—*Second Method.* In another form of apparatus (fig. 241) the ba A is surrounded by a glass tube B, which is slightly shorter than the bar. Bot are placed vertical, standing on a slab of glass or slate. The upper end of th bar projects a very short distance through hole in a flat plate on which stands a spher meter with its central screw touching the to of the bar.

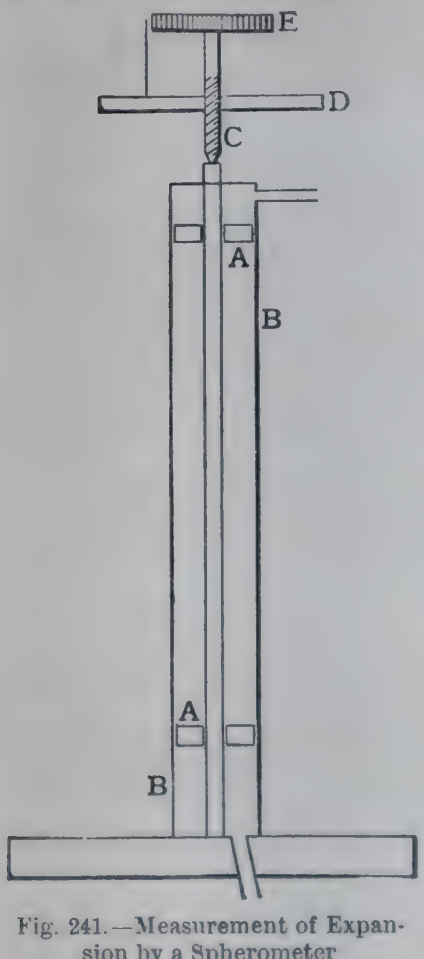

Fig. 241.—Measurement of Expansion by a Spherometer

The whole apparatus being at the tem perature of the laboratory ($t_1°$), the spher meter is adjusted as in Exercises 39 and 4 and its reading taken.

Steam is then passed for several minut through the tube B, and when the bar is 100° C., the spherometer is again adjuste and its reading taken.

The difference between the readings of th spherometer shows the increase of length the bar when heated from $t_1°$ C. to 100° The calculation of the coefficient is exactly in Exercise 368.

In another convenient form apparatus one end of the bar is fixe and the other end abuts against a thi wedge graduated like the cone gaug shown in fig. 10. This wedge fills u a crack left between the end of th bar and an arm fixed on to the stan of the apparatus.

When the bar is cold the wedge inserted in the crack and measures i width.

The bar is then heated by steam passed through its jacket, in the preceding experiment, and the width of the crack is aga measured by the wedge.

Example.—A bar of copper was 50 cm in length. The readings of the wedg which was read three times, were:—

At 14° C.: 4·64, 4·60, 4·58; mean 4·61.
At 98° C.: 5·27, 5·35, 5·31; mean 5·31.

Thus a bar 500 mm. long, heated from 14° C. to 98° C., lengthened by ·7 mm and approximately the coefficient of expansion is $\frac{\cdot 7}{500 \times 84} = \cdot 000017$.

231. RELATION BETWEEN THE LINEAR AND VOLUM COEFFICIENT OF A SOLID.—Suppose we have a cube wh

edge measures exactly 1 cm., and whose volume is 1 cub. cm. at 0° C. (fig. 242). Then if this cube be heated to 1° C. its edge measures $(1 + x)$ cm., and its volume is $(1 + x)^3$ cub. cm.

$$\text{And } (1 + x)^3 = 1 + 3x + 3x^2 + x^3.$$

But x itself is very small, and $3x^2$ and x^3 are much smaller. Practically, therefore, they are neglected in considering expansions, and it is taken that the volume of the unit cube mentioned above is $(1 + 3x)$ cub. cm. The increase of volume is $3x$ cub. cm. This $3x$ is the coefficient of cubical expansion of the solid, and is expressed by a number three times as great as that expressing the linear coefficient.

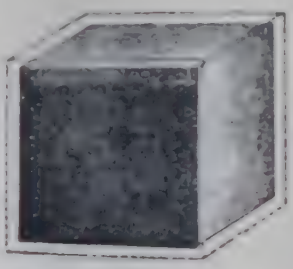

Fig 242.—Linear and Cubical Expansion

For coefficients of expansion, see table on p. 409.

Ex. 370.—(1) A metre scale AB has a needle passed through it at a point C, which is 2·5 cm. from A. The end A is placed against one end of a metal rod at 15° C., and the end B against a scale. When the metal rod, whose length was 100 cm., was heated to 100° C., the end B of the lever moved through a distance of 46 mm. Find the coefficient of linear expansion of the metal.

(2) A thin flat bar of iron is firmly screwed on top of a bar of copper of exactly the same size, so as to make a bar twice as thick as either separately. When heated this bar bends. Why? In which direction does it bend when heated?

(3) A glass bottle holds exactly 20 lb. of mercury at 0° C. How much will be expelled when the temperature is raised to 80° C. if the coefficient of apparent expansion of mercury in glass is ·000154?

(4) A density bottle is filled with glycerine at 15° C., and its weight is then 168 grm. After being heated to 100° C. its weight is 163·6 grm. The bottle itself weighs 5·6 grm. What is the apparent coefficient of expansion of glycerine?

(5) What is the change of length of 75 m. of telegraph wire (coefficient of expansion ·0000123) when its temperature is raised from 0° C. to 30° C.?

PART V.—SOUND

CHAPTER XXVIII

Sound

232. SOUND PRODUCED BY VIBRATION.—It was demonstrated in Exercise 261 that a rod vibrates isochronously, and such vibrating rods were used in Chapter XVII to measure short intervals of time. It was also shown that the shorter the rod the quicker the vibrations.

Ex. 371.—Clamp a thin strip of steel, such as a piece of clock spring or a knitting needle, firmly at one end, and set it in vibration. Shorten it until a continuous low sound is heard (not the sharper sound produced at the instant of starting). Continue to shorten the rod by successive steps, and observe the effect of altering the length of the rod on the sound given out. Note also the appearance of the end of the rod.

Such regular sounds as those given out by the rod are called musical **notes**. The long rod produces a *low* note, the shorter rod a *higher* note.

When a rod is vibrating slowly the oscillations can be seen, but not heard; as the vibrations quicken they can be both seen and heard; when they are very quick they can be heard but not seen.

The number of vibrations made in a second is in sound called the **frequency** of the vibration.

A tuning fork is a steel bar bent into a U shape with a stem or handle at the bend. Such forks are frequently used for the production of musical notes. They should be carefully used, and struck either by a padded wooden hammer like a piano hammer, or on a padded block.

Ex. 372.—Fix the stem of a tuning fork firmly in a block of wood, or clamp it in a vice. Set the fork in vibration by drawing a resined violin bow over one prong, or by tapping it smartly with a proper hammer. The movement of the prongs gives them a shadowy appearance. Bring up against one prong of the

fork a pith ball suspended from a stand (fig. 243). The pith ball is knocked away. In a similar manner a match balanced on the top of the finger is knocked off.

Bring up against one prong the edge of a slip of paper held in the fingers. The jarring may be felt.

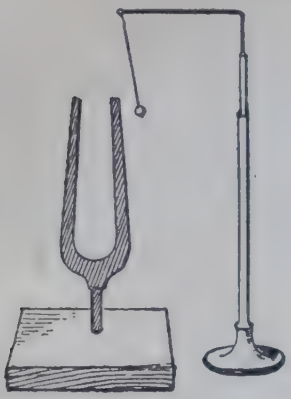

Fig. 243.—Pith Ball knocked away by a vibrating Tuning Fork

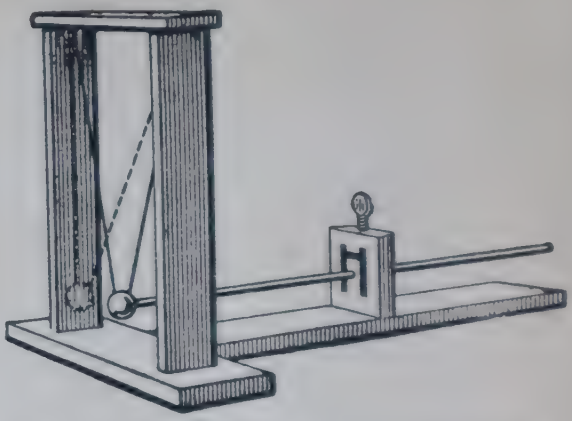

Fig. 244.—The Lengthening and Shortening of a vibrating Rod

Ex. 373.—Longitudinal vibrations of a rod. A long rod or tube of metal or glass is firmly clamped exactly in the middle. A hard ball is suspended so as to lightly touch one end (fig. 244).

Draw a damp cloth over that half of the rod which is farthest from the ball: a sound is produced, and the ball is knocked away as by a blow. In fact it received a blow. The two halves of the rod lengthened and shortened very rapidly, and when the rod lengthened a small quick blow was given to the ball.

Fig. 245.—A Vibrating String

Ex. 374.—Vibrations of a string. Take a piece of thin string about a metre long. Plant your heel firmly on one end, and hold the other in your left hand. Pull this end, and thus stretch the string moderately tightly. Pluck the string near the middle with one finger of the right hand. A sound will be heard, and on looking down along the string it will be seen for a short time to present the appearance shown in fig. 245. The spindle disappears, and the sound ceases at the same time.

This experiment is more under control if conducted in the manner indicated in fig. 246, where the string or wire is fastened to a board or a table at one end, passes over two bridges and a pulley, and is stretched by a weight attached to the other end. Such an instrument is called a *sonometer*.

With a small weight the string is not tightly stretched, the vibrations are plainly seen, and the sound heard is a low note.

With a large weight the vibrations are very rapid, and the sound heard is a high note.

Fig. 246.—A Sonometer

The difference between a high note and a low note is called a difference in **pitch**.

Thus the pitch of the note given out by a string depends upon the tension of the string.

Ex. 375.—Vibrations of a bell. These may be shown by a bell jar (fig. 247)

Fig. 247. The Vibrations of a Bell

or large evaporating basin supported with its mouth upwards, and kept in position by pressing down in the middle with a wooden rod.

Start the vibration by a tap on the rim, or by drawing a resined bow across it. Hang a pith ball against one part of the rim, and press a sharp point or the blade of a knife against another part. In some positions the pith ball is knocked away.

Half fill the vessel with water. On bowing the edge, wavelets are produced. If a little ether be poured on the water small drops spring up.

Ex. 376.—Vibrations of a plate. Strike a tuning fork, and hold it by the stem. The sound is distinctly heard, but is not loud.

While the fork is still vibrating place the stem in contact with a table or the panel of a door. A much louder sound is heard.

This suggests that the flat surface has been set in vibration. If the fingers be placed on the front of a piano and a low note be struck, the vibration of the wood may be felt.

Ex. 377.—Take a thin plate of glass or metal. Brass is best. It may be round or square, but should be quite flat.

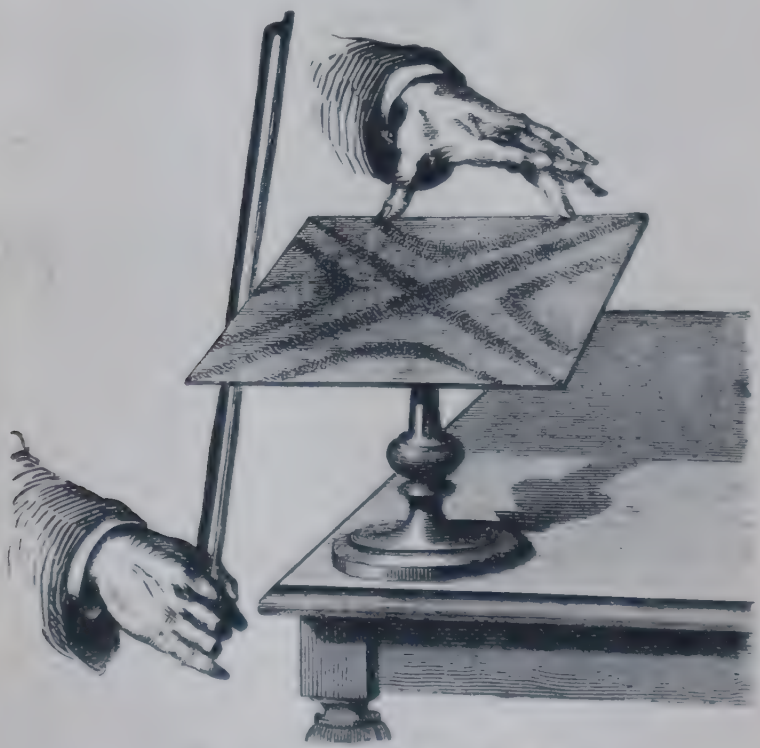

Fig. 248.—The Vibrations of a Plate

Fix it firmly at its middle point. This may be done either by screwing it o a stand, as shown in fig. 248, or by a clamp of the form shown in fig. 249, or les effectively by putting it on a small disk of cork and pressing on the upper side.

Strew a thin layer of fine dry sand over the plate.

Draw a resined violin bow carefully over the edge of the plate. If the plat be square, bow at the middle of one side. After a few attempts a musical not

will be heard. At the same time the sand will spring up and down, and begin to move towards some parts of the plate and away from others.

By pressing with the point of a nail or knife on some point of the plate, or as in fig. 248, and bowing in different places, the motions of the sand are made regular; it collects along certain lines and leaves other parts bare, thus forming patterns.

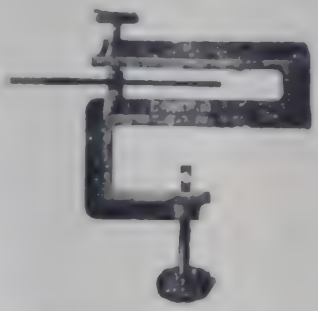

Fig. 249.—A Clamp for Chladni's Plates

These patterns are called Chladni's figures.

In the case of circular plates the sand in simple cases collects along radial lines.

The vibrations of different parts of the plate are shown by the behaviour of the sand. It is tossed away from parts where the motion of the plate is greatest, and collects at the parts where that motion is least, showing that the plate vibrates in parts or sections.

If lycopodium powder be also sprinkled on the plate, it forms little whirlwinds above the parts where the plate is vibrating most vigorously.

Ex. 378.—Vibration of a column of air. In whistles, flutes, and organ pipes the vibrating column of air is the source of sound.

Fix vertically a wooden organ pipe that has a glass front (fig. 250), and either by organ bellows or ordinary laboratory foot bellows blow air into it. Down the pipe lower a small tambourine on which are a few grains of sand. The tambourine must be considerably smaller than the pipe. When the pipe is sounding the sand is disturbed, and the rattling of the grains on the membrane may be heard, and their movement seen.

Fig. 250.—Vibration of a Column of Air

The vibration of the air in sounding tubes may also be shown by the method of Exercise 377. If a short wooden whistle be passed through an indiarubber stopper closing a large glass tube lying horizontally and having fine silica powder strewed along it, on blowing the whistle the powder collects into heaps.

That liquids vibrate in the same manner as air is shown by the fact that a whistle

placed in a tub of water may be blown by forcing water through it.

The foregoing experiments illustrate the chief means by which sound is produced, and it is seen that its origin is a vibrating body, which may be a solid, a liquid, or a gas.

Many of the sounds heard in daily experience—such as the noise made by a train or cart in motion—are very complicated, but such noises are often accompanied by larger vibrations that can be felt. The player of a wind instrument, such as a bassoon, can feel the trembling of the reed: if you put your fingers on your throat when talking, or on a cat which is purring, the vibrations can be felt.

It was shown in Exercise 261 that slow vibrations do not produce sound. Very quick vibrations also do not produce sound.

Sound is a sensation produced by the vibration of a mechanism inside our ears. The vibrations of bodies are communicated by them to the air, and by the air to this mechanism in the ear. But if the vibrations are slower than about 16 in a second, or quicker than about 30000 in a second, most persons cannot hear any sound.

233. TRANSMISSION OF SOUND VIBRATIONS. — Common experience shows that sound vibrations can pass along through air, as nearly every sound that we hear under ordinary circumstances reaches us through the air.

Many other substances—solids, liquids, and gases—can also transmit sound vibrations.

Solids

Ex. 379.—Set a tuning fork in vibration, and place the stem of the fork on the table or against a panel of a door, or any large thin piece of wood. Note the greatly increased loudness of the sound. The board is set in vibration by the fork. It is called a sounding board, and is employed in the piano and in the violin.

In order to test the capacity of substances for transmitting sound vibrations, place them between the stem of a vibrating fork and a sounding board, in contact with both fork and board. Test in this way:—

(1) Rods of wood, metal, glass, ebonite, shellac, resin.

(2) A candle, a cork, an indiarubber stopper, a thick layer of felt or flannel, a cushion, a piece of wax, a piece of putty.

(3) A piece of indiarubber tubing, having a piece of glass rod or wooden rod in each end.

(4) The same indiarubber tube quite filled with water up to the solid plugs.

(5) For such substances as wire, string, cotton, fasten one end to the sounding board and the other to a small piece of wood. The stem of the fork is pressed

against this small piece of wood. Try these substances when stretched tightly, and when hanging slack.

According to the loudness of the sound heard from the sounding board, classify the interposed substances into good, fair, and bad conductors of sound vibrations. The difference between (3) and (4) indicates the effect of the water.

Liquids

Ex. 380.—(1) Ring a small bell in a vessel of water.

(2) Place a liquid in a beaker standing on a sounding board. Just into the liquid dip a wooden rod, and apply the stem of a vibrating tuning fork to the other end of the rod.

(3) Let a stream of water from a tap fall on a sounding board. Apply a vibrating fork to the board and the ear to the water pipe.

Gases

The fact that gases conduct sound waves may be shown by an apparatus such as that of fig. 251.

An electric bell is suspended inside a bell jar by means of thin threads from an indiarubber stopper. Wires are brought from the terminals of the bell through the stopper and connected to a circuit containing a battery and a key (Chap. XL).

The bell jar is then placed on the plate of an air pump and the air pumped out. As the exhaustion proceeds, the sound heard when the bell is rung becomes fainter. When the vacuum is good scarcely any sound is heard. *A perfect vacuum does not conduct the vibrations at all.*

If a gas other than air is then admitted into the jar the sound of the bell is again heard.

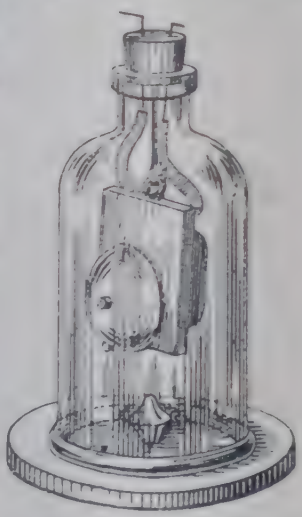

Fig. 251.—Experiment to show that a Vacuum does not conduct Sound Vibrations

All these experiments show that the vibrations that produce sound can travel through very many substances. Metals and wood are very good conductors. The tickings of a watch which is in contact with one end of a long rod can be plainly heard by an ear placed at the other end of the rod. If a piano be placed in one room and a sounding board in a second room, and a long rod connect the two, the sound of the piano is heard almost equally well in both rooms.

Felt, indiarubber, and putty are bad conductors of sound.

The experiments also show that substances which can be made to produce sound vibrations also conduct them.

The two effects are the same—the substance is set in vibration. Substances which cannot be set in vibration, of course, cannot either produce sound vibrations or transmit them. These substances are called **inelastic**. All gases and liquids and many solids are **elastic**. They can themselves vibrate, and generally transmit those vibrations which produce the sensation of sound.

234. MODE OF PROPAGATION OF A SOUND WAVE.—In Exercise 373 when the rod was giving out a sound the ball at the end of the rod was moved. If a long glass tube be in a similar manner held by the middle, and a damp cloth be rubbed quickly over one-half of the tube, the other half is sometimes broken into rings (fig. 252).

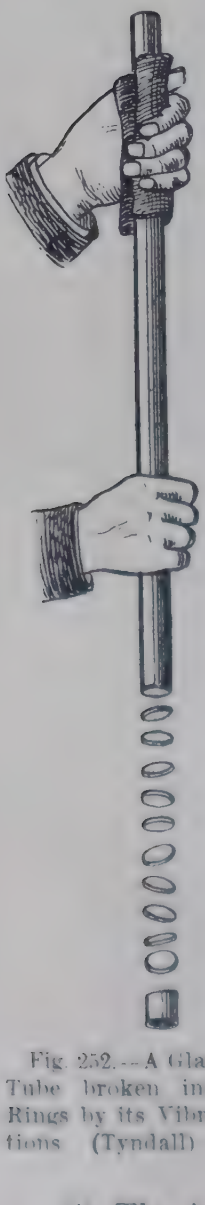

Fig. 252.—A Glass Tube broken into Rings by its Vibrations (Tyndall)

These experiments show that the particles of the tube are in motion to and fro with a regular swing.

The motion of the sand in Experiment 378 shows that when sound is passing through air the particles move to and fro.

An apparatus such as that shown in fig. 253 illustrates what is taking place. A number of wooden balls are suspended from a wire, each pair of balls being separated by a spiral wire spring. On striking the first ball the spring attached to it is compressed, and in the recovery from this compression pushes the second ball forward and the first ball backward; the second ball, in its turn, by means of the spring, communicates the motion to the third; and so on along the whole row of balls, so that the last ball can be made to knock against a plate and give out the energy originally communicated to the first. Thus a pulse travels along the spring. Within certain limits, the rate of propagation of the pulse depends on the stiffness of the springs. When they are stiff the pulse travels fast, when the springs are feeble the pulse travels slowly.

The following are the characteristic features of the motion:—

(i) The impulse given to the first ball is transmitted along the whole line.

(ii) At one portion of the line the balls are approaching and the springs are closing up; at another part of the line the springs are opening out. Call these portions respectively the *condensations* and *rarefactions*. If successive shocks be given to the first ball, the condensations and rarefactions travel alternately along the line, and such a regular set of movements is called a wave motion or a wave.

(iii) Each ball and each part of the spring vibrates to and fro on each side of its position of rest.

(iv) The direction of the swinging motion of the balls and spring is in the line along which the wave travels.

(v) The velocity with which each ball moves is quite different from the velocity with which the wave travels.

A careful consideration of these points will give a fair idea of the mode of propagation of sound in air. The balls represent particles of air, the springs represent its elasticity, and we have seen that air is easily compressed and is perfectly elastic.

When a number of the particles of air receive a knock of such a character as to produce simple rectilineal vibrations, these particles pass the knocks on to those lying near them, and these again to the adjacent particles, and so on.

This state of affairs constitutes a **sound wave.**

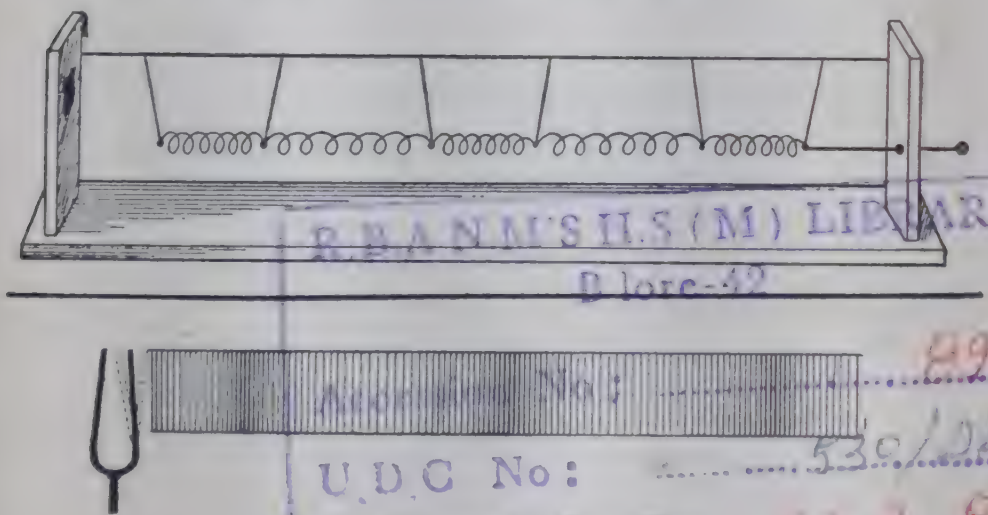

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Fig. 253.—Character of a Sound Wave

Thus the lower half of fig. 253 illustrates the condition of the air in a tube along which a series of sound waves, produced by a tuning fork, is maintained. In the darker portions the particles are crowded together: this represents the condensed part of the

wave. In the lighter portions—the rarefactions—the particles are farther apart.

Think only of the right-hand prong of the fork. When this prong moves to the right, it squeezes up the air in front of it, the motion being so quick that the air cannot get away. This portion of compressed air instantly begins to expand and squeezes the next layer, and this the next, and so the compression is passed on along the tube. When the prong moves to the left a rarefaction is produced; and this is in like manner passed on through the tube. Thus a series of condensations and rarefactions travels along the tube. Each separate particle of air merely swings to and fro through a very small distance, being at one time in the middle of a condensation, then in the middle of the succeeding rarefaction, then again in the middle of the next condensation, and so on.

Terms.—A wave length is the *distance* between the centre of one condensation and the centre of the next. It is also the distance through which the disturbance or pulse has travelled onward through the substance while each particle has performed one complete vibration.

The **period** of the vibrations is the time in which the pulse travels a wave length or a particle makes a complete vibration.

Amplitude of vibration has been explained in Art. 158.

Evidence of the existence of these condensations and rarefactions in the substance is given by the breaking of the glass tube (fig. 252), which is torn asunder by the particles moving in opposite directions. It may also be shown by causing two sound waves to meet in such a way that the condensations caused by one come in the same place as the rarefactions produced by the other. When this is done no sound is heard. The two sound waves thus imposed on one another cancel each other and produce silence. This effect may be got from a tuning fork.

Ex. 381.—(1) Sound a tuning fork with the stem held in the hand. Only a feeble sound is heard. Slip a small cylinder of paper over one prong without touching it. Is the sound louder? You have prevented the waves proceeding from one prong from interfering with those coming from the other.

(2) Hold a sounding tuning fork with its prongs near the ear. Turn it round into various positions, and note the change in the loudness of the sound. In one position almost complete silence is obtained. This position is when the rarefactions of the wave caused by one prong reach the ear at the same time as the condensations from the other prong.

235. REFLECTION OF SOUND WAVES.—In experiments with the spring (fig. 253), it is observed that the motion is turned back or reflected, just as a water wave is reflected by a wall.

When a person wishes to catch a very faint sound, besides turning his ear towards it, he often curves his hand in a forward direction round his ear.

A fan or a piece of wood held at a suitable oblique angle has the effect of enabling such sounds to be better heard.

The explanation of this fact is that the surface reflects into the ear a part of the sound wave that would have gone by the ear.

Persons who are partially deaf employ a special apparatus for this purpose. Fig. 254 shows such an *ear trumpet*, which is used to reflect waves into the ear.

Fig. 254.—An Ear Trumpet

Similarly, when it is desired that one's voice should be heard beyond the ordinary distance, speaking tubes and speaking trumpets are used.

A *speaking tube* is often put in a large building from one room to another. It is a small pipe having a mouthpiece at each end. A person speaking into one end is heard quite plainly by a person whose ear is placed to the other end.

Fig. 255.—Reflection in a tube

Ex. 382.—Take a long glass tube made up of several pieces joined together by short pieces of indiarubber tubing. Place a watch in a funnel at one end, and the ear against a funnel placed in the other end. The ticking of the watch may be heard much farther than through the open air.

Fig. 255 illustrates the action of the tube, which reflects part of the sound waves onward from one side to the other of the tube.

Ex. 383.—Place two large tubes A, B equally inclined to a wall or hard surface (fig. 256). At B place a watch, and cover the watch and the mouth of the tube with a cloth. The ticking of the watch may be heard by an ear placed at A. By changing the angles A*On* and B*On*, it is found that the loudest sound is heard when these two angles are equal, thus showing that not only is the sound wave reflected by the wall, but that the reflection obeys approximately the law of light reflection (Art. 259).

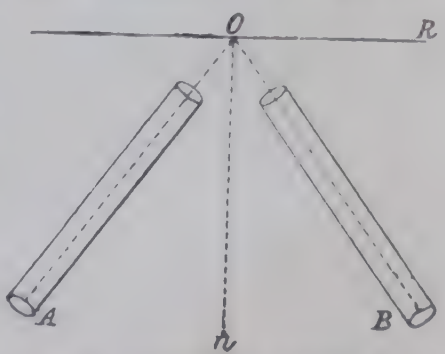

Fig. 256.—Reflection of Sound Waves by a wall

That this is the case, too, may be seen from the following ex-

periment. Let two concave mirrors be placed facing each other several yards apart (fig. 257). In the focus of one mirror (Art. 268) place a watch, and in the focus of the other place one end of a tube, of which the other end is held to the ear. The ticking of the watch will be plainly heard.

Whispering galleries are examples of this reflection; the sound

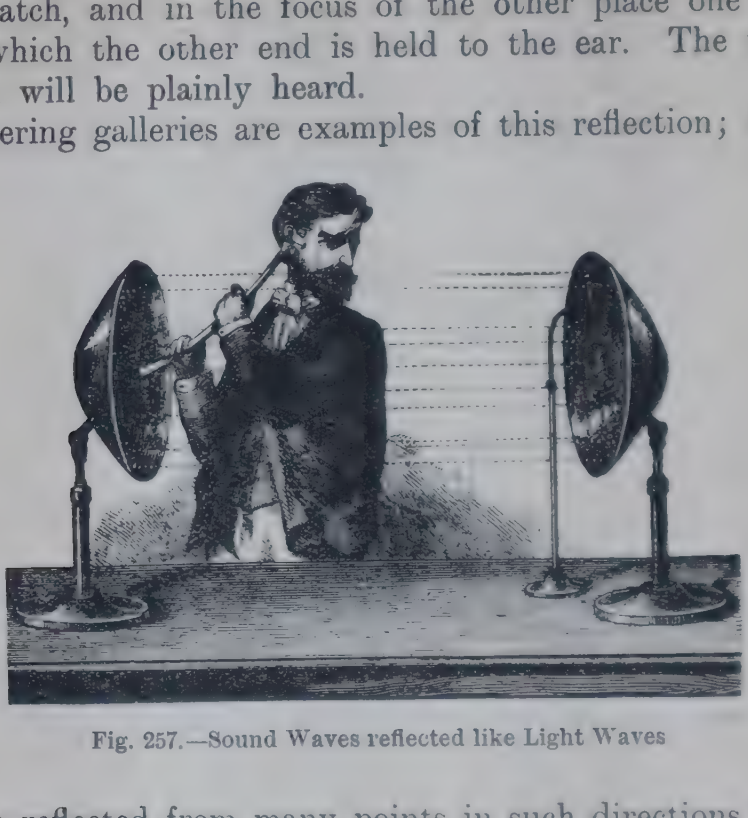

Fig. 257.—Sound Waves reflected like Light Waves

waves are reflected from many points in such directions that they converge to one point.

Echoes.—If a person standing in front of a high cliff or wall make a short sharp sound, he hears it repeated, provided that he stand far enough away from the wall. This second sound is called an *echo*, and is produced by the wave which has travelled to the wall, and has been there reflected to the ear. The echo cannot be heard as a distinct sound unless the observer stand sufficiently far off to allow the interval between the two sounds to be long enough for the ear to regard them as distinct impressions. If he stand near the obstacle the effect on the ear is to prolong the first sound.

If he retreat farther and farther away from the cliff, the interval of time between the sound and its echo will grow longer. A fair idea of the velocity of sound can be got in this way.

The rolling of a clap of thunder among mountains is mainly due to the waves which are reflected to the observer successively from hillsides more and more distant. So, too, the ring of the treble which is heard when a person walks between parallel walls is due to a long succession of small echoes which are produced by the suc-

cessive reflections of the sound waves from one wall to the other, in a manner similar to the reflection of the light in fig. 285.

Reflection of sound waves causing echoes may take place from the surfaces of liquids and gases. The rolling of thunder is in part due to the reflection of sound from masses of air which, by reason of difference in density, act as obstacles to the passage of the wave through them.

236. NOISE AND MUSICAL SOUND.—Most of the sounds developed by such means as those employed in Exercises 371 to 378 are musical sounds. The sound given out by a tuning fork may be taken as the type of a musical sound.

When a book falls flat on the floor the sound heard is a noise. If several successive notes on a piano be struck at the same time, the sound heard is a noise, whereas a single note or a properly selected combination of notes gives a musical sound. The difference between a noise and a musical sound is thus a matter of the effect on the ear.

The difference in this effect is due to difference in the character of the vibrations. In a noise there are many vibrations of different periods, which have no relation to one another; in a musical sound there is either a single vibration, or a combination of vibrations whose periods are related to one another.

237. FREQUENCY AND PITCH.—In Exercise 371 it was shown that the pitch of the note given out by a vibrating rod was higher when the vibrations were quicker and lower when the vibrations were slower. This is expressed by saying that the pitch of the note depends upon the frequency of the vibration.

The above general relationship may also be shown by drawing a piece of thin cardboard or the thumb nail over hard corded materials, such as corduroy, Brussels carpet, tapestry, or corded silk. Doing this at different rates, higher or lower notes are produced by quicker or slower motions.

In Savart's apparatus, a piece of cardboard is so fixed that the teeth of a revolving wheel strike against the card. If the wheel has x teeth, and revolves y times per second, then the frequency of the note given out by the card is xy. In some such manner frequency is measured.

Ex. 384.—Comparison of the frequency of the notes given by two tuning forks.

This may be done by the method of Exercise 262. The tuning forks may be

horizontal, and the plate may be pulled along underneath them, or the apparatus shown in fig. 258 may be used.

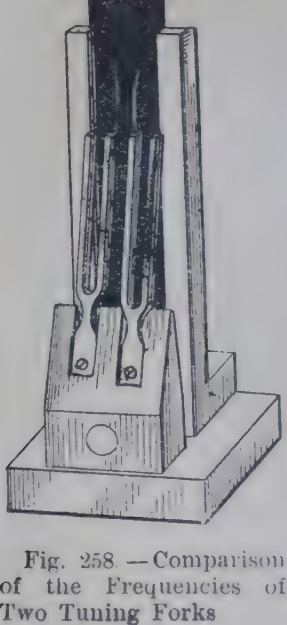

Fig. 258.—Comparison of the Frequencies of Two Tuning Forks

Attach to one prong of each fork a thin bristle. This may be done by wax. The movable plate is best made of glass which has been thinly covered with smoke by holding it above the flame of a candle or over burning camphor. Fix the stems of the forks firmly to a block of wood in such a position that the bristles just touch the plate.

Set the forks vibrating so that they are sounding loudly, and then if the plate is vertical let it fall, or if it is horizontal it may be moved along quickly by hand. Careful adjustment is required to get clear curves.

Choosing portions of the curves which are clear, count the number of waves in a measured length. Then these numbers are in the ratio of the frequencies of the forks.

The musical interval or the difference between the pitch of the notes of the forks must be judged by the ear.

Such intervals correspond to simple numerical relations between the vibrations.

Unison.—Notes that are in unison are produced by the same number of vibrations per second.

Octave.—The octave above any given note is produced by double the number of vibrations per second that correspond to that note; the octave below, by half that number.

Fifth.—The interval of a fifth is that between two notes whose rates of vibration are as 3 to 2.

Measurement of Frequency.—The frequency of vibration corresponding to any given note may be obtained directly by the siren. This instrument is shown in figs. 259 and 260. At the bottom is a pipe, by which air may be blown from bellows below into a chamber, the top of which is pierced with a number of holes bored through it slantwise at equal distances round a circle. On the top of this chamber is a circular plate also pierced with the same number of holes arranged round a circle of the same size. These holes slope in the opposite direction to those in the plate beneath. This upper plate has, passing through its centre, a spindle which is pivoted at the centre of the lower plate, and passes through holes in a framework above. The spindle also carries an endless screw capable of turning toothed wheels which are set up in this framework, and which carry pointers outside to act as counters. Arrange-

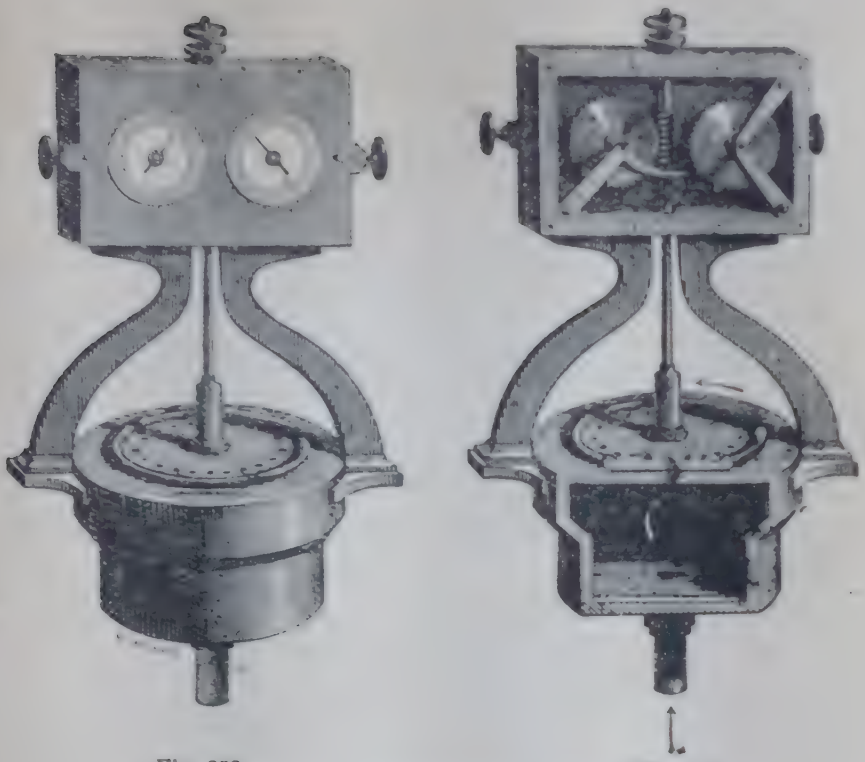

Fig. 259 Fig. 260

The Siren, by which Frequency is Measured

ments are made by which these wheels can be thrown in or out of action at pleasure.

The counters being read and thrown out of gear, air is forced through the pipe into the central chamber. There it rushes through the small holes, and as each jet of air strikes on the *side* of the corresponding hole in the upper plate the pressure causes that plate to revolve. As it revolves, whenever the passage is clear, little puffs of air issue from the holes; the puffs set up vibrations in the air, and when they succeed each other sufficiently rapidly a musical note results. The faster the wheel revolves the higher is the note produced. The speed of the wheel is adjusted by steady blowing and gentle friction of the axle until the note is that of which the rate of vibration is required. The counters are then thrown into gear, the note is kept steady for several seconds, when the counters are thrown out of gear and again read. The difference between the two readings of the counters gives the number of revolutions; this number multiplied by the number of holes in the movable disk gives the number of puffs, i.e. of vibrations. It only remains to divide by the number of seconds, and the number of vibrations per second is obtained.

238. VELOCITY OF SOUND.—That a sound wave takes an appreciable time in travelling through air is evident from common facts. Thus the flash of a gun is seen some time before the sound is heard by a person a few hundred yards distant. A clap of thunder is heard a greater or less time after the flash of lightning is seen, according to the distance of the observer from the centre of the discharge.

Moreover, it is evident that all ordinary sounds travel at the same rate, since a peal of bells may be heard two miles off in correct order, and a tune played by a band reaches the ear as it is played.

The velocity of sound in air has been determined very accurately by French and Dutch experimenters. Two elevated stations were chosen several miles apart. From these stations guns were fired and observers noted the times at which the flash was seen and the sound heard. The interval between these times was the time taken by the sound wave to travel the distance, since the time taken by the light in passing from one place to the other was too small to be taken into account. It was found that it took, at the ordinary temperature of the air, just about $4\frac{3}{4}$ sec. to travel 1 mile, which is at the rate of 340 metres in 1 sec.

239. CIRCUMSTANCES INFLUENCING THE VELOCITY OF SOUND IN AIR.

(*a*) **Temperature.**—It is found that sound travels faster through warm air than through cold air. The velocity depends on the ratio between the elasticity and the density of air, and it has been shown that these change with the temperature.

The velocity of sound in air increases with rise of temperature. Taking the velocity at 0° as 332 m. per second, the velocity at $t°$ C. $= 332\sqrt{1 + \cdot00366t}$ m. per second. In warm air each wave length is a little greater than in cold air.

(*b*) **Pressure.**—Change of pressure has no effect on the velocity of sound in air, owing to the fact that such change alters the elasticity and density in the same ratio; thus keeping the fraction $\frac{\text{elasticity}}{\text{density}}$ unchanged in value. The fact that sound travels slower at great heights than at the sea level is due to the lower temperature prevailing high up in the atmosphere, not to the diminished pressure there.

(*c*) **Wind.**—When the air is carried bodily forward in the same direction as the sound wave, the actual velocity of propagation

the wave is then equal to the velocity in still air plus the velocity of the wind. When the wind is in the opposite direction the velocity of the wave is similarly diminished.

240. CONNECTION BETWEEN WAVE LENGTH, VELOCITY, AND NUMBER OF VIBRATIONS.—Consider the action of a tuning fork (fig. 253). Each outward movement of the prong produces a condensation of the air, each retreat inward a rarefaction. These condensations and rarefactions continue to travel outward from the fork through the air. A condensation and a rarefaction together form one complete wave. Therefore the number of waves started by the fork in one second is the same as the number of complete vibrations made by the prongs in one second, however fast or slowly the fork may vibrate.

But the wave produced by a slowly-vibrating body reaches a distant point just as quickly as that produced by a quickly-vibrating body, since the velocity of sound is the same for a high note as for a low note. Hence it follows that *the velocity of sound divided by the number of vibrations per second made by the vibrating body gives the wave length of the note produced.*

Example.—Suppose, on a day when the velocity of sound is 1125 ft. per second, two trumpets making respectively 250 and 450 vibrations per second, and which are placed side by side, are simultaneously sounded for one second. Two series of waves proceed outwards in all directions, and reach a listener who is, suppose 1125 ft. distant, exactly at the same time—one second from the start. The trumpet producing the lower note has sent out 250 waves, and just as it sends out the last the first has reached the listener. The air between him and the trumpet is carved out into 250 exactly similar parts; the length of each part must therefore be $4\frac{1}{2}$ ft. Each of these parts is a wave length. The trumpet producing the higher note has sent out 450 waves, and has moulded the air between the two points into 450 waves, the length of each of which is therefore $2\frac{1}{2}$ ft.

Hence if v = velocity of sound,
n = number of vibrations corresponding to any note,
and l = length of the wave corresponding to that note;
$v = nl$.

241. MEASUREMENT OF THE VELOCITY OF SOUND IN AIR.—The reflection of sound waves takes place in a very simple manner in tubes and organ pipes. In such a tube the pulses are reflected at each end of the pipe, whether the pipe be open or closed. When a pulse is reflected from the external air a condensation is reflected as a rarefaction, and a rarefaction as a condensation

When, however, a pulse is reflected from an unyielding surface such as the substance of the pipe itself, a condensation is reflected as a condensation, and a rarefaction is reflected as a rarefaction. The reason of this difference is that when, e.g., a condensed portion of air is in contact with the closed end of the tube it cannot expand, owing to the rigidity of the tube; whereas, when such a portion of air is merely in contact with the external air, expansion takes place and a rarefaction is produced. So far as the fundamental wave is concerned, the undulations are thus of the following character:—

In a pipe stopped at one end a condensation, starting from the mouth, passes along the pipe to the closed end, is there reflected and travels back along the tube (fig. 261) to the mouth, where it is reflected as a rarefaction. This travels along the pipe and back to the mouth, where it is reflected as a condensation. Thus, after *two double journeys* of the pulse the state of affairs is the same as at the beginning. Such a complete round of change marks the performance of a complete vibration, during which the pulse has travelled a wave length. Hence the wave length of the note given out by a stopped pipe is four times the length of the pipe.

Fig. 261

On this principle is based a method of finding the velocity of sound.

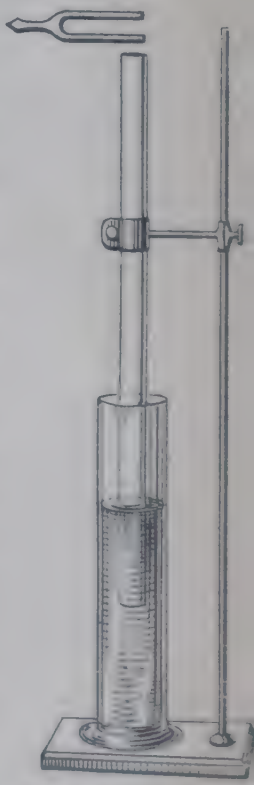

Fig. 262.—Velocity of Sound determined by Resonance Tube

Ex. 385.—In a vessel containing water place a glass tube about 40 cm. long and 3 or 4 cm. in diameter, as shown in fig. 262. Clamp the tube loosely, so that it can be raised or lowered so as to alter the length of the column of air it contains. This column of air is an organ pipe closed at the bottom. The water reflects the sound wave just as a solid body would do.

Sound a tuning fork whose vibration frequency (n) is known, and hold it over the open end of the tube. Raise and lower the tube and note the effect.

At a certain length the air column loudly resounds to the note of the fork, giving out the same note as the fork. This phenomenon is called *resonance*. It takes place when the state of affairs is that described above. When the reflected are very different from those, no resonance takes place. If you find two lengths that give resonance take the shorter of the two.

Having found approximately the proper position of the tube, proceed sy

tematically to raise the tube and to lower it through short distances, noting the increase or decrease of loudness of the sound. When the position of greatest resonance has been found measure the length of the air column (l); then $4l$ is the wave length of the note. The frequency is n, and therefore $4nl$ is the velocity with which the sound wave travels.

Example.—A fork whose frequency was marked as 256 gave the greatest resonance when the distance from the surface of the water in the tube to the top of the tube was 32·5 cm.; then the wave length of the note was 130 cm., and the velocity of sound was $130 \times 256 = 33280$ cm. per second.

It was stated in Art. 239 that the velocity of sound in air had been found to be affected by the temperature. Verify this statement.

Ex. 386.—Repeat the last exercise with the tube in water, as hot as can be maintained, taking care that the tube has had time to get hot also. Find the velocity of sound in exactly the same manner as in Exercise 385. It will be found to be considerably greater than when the air was cold.

242. VELOCITY IN WATER.—The velocity of sound in water was determined by Colladon in 1826. Two boats were

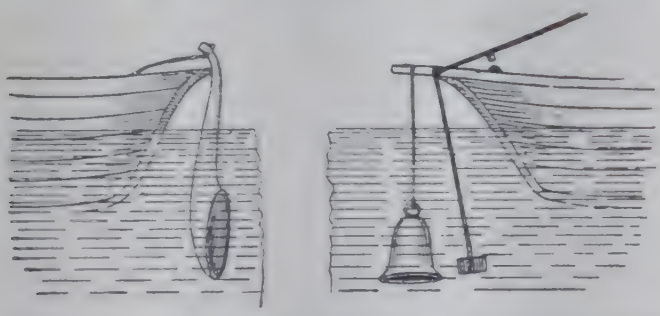

Fig. 263.—Determination of the Velocity of Sound in Water

moored 9 miles apart on the Lake of Geneva. Suspended from one boat at some depth in the water was a bell (fig. 263); and the observer in this boat, by moving a lever, simultaneously struck the bell and lit some gunpowder on the boat's deck. The observer in the other boat was provided with a chronometer and a hearing tube. The lower end of the tube was covered with a metal membrane, and turned towards the distant bell, while the upper end was placed against the observer's ear. The time taken by the sound in travelling was taken as the interval between the observer seeing the flash of the gunpowder and hearing the sound of the bell. The result arrived at was that the velocity of sound in water was 4708 ft., or 1420 m., per second.

Ex. 387.—(1) A person whose watch is in exact agreement with a large

public clock, notes that his watch is 10 sec. past one when he hears the clock strike one. His distance from the clock is 3400 m. Calculate the velocity of sound.

(2) A clap of thunder is heard 6 sec. after the lightning flash is seen. How far is the observer from the seat of the discharge?

(3) Explain the ringing noise heard when a person walks between two walls.

(4) If a person's vocal chords, when singing a certain note, vibrate 1000 times a second, what is the wave length of the sound?

(5) A tuning fork makes 384 vibrations per sec. What is the length of closed tube that would resound most loudly to the fork?

PART VI.—LIGHT

CHAPTER XXIX

Propagation of Light

243. TERMS.—Light is that agency or process outside ourselves by which we see. Light can pass from one place to another through "empty space", air, and certain material objects. It is by means of light that comes from an object and enters the eye that we see the object. The light has a special effect on the nerves of the retina at the back of the living eye, by which it produces the sensation of sight.

A burning lamp gives out light, which issues from it and spreads in all directions through a room. Such a body is said to be a **source of light**, and is called **luminous** or **self-luminous**. It is itself visible, and furnishes the means by which other objects become visible.

Non-luminous bodies are those which only become visible when light falls on them from a luminous body.

If we turn a lamp down everything in the room looks less bright. The quantity of light given out by the lamp has been diminished. This quantity when rendered definite is the **intensity** or **illuminating power** of the light.

Every point of every bright object sends out light in every direction. Sometimes we desire to think only of the light that is proceeding from one very small point to another very small point, as you might stretch a piece of cotton between the points. This line is called a **ray of light**. If the path of the light between the points is larger in section the term **pencil** or **beam of light** is used.

The light that falls upon a body is said to be incident upon it, and we call it the **incident** light, and speak of **incident rays**.

Fig. 278 shows the path of a conical pencil or beam of light.

If a multitude of people stand round a candle and look at the *point* of it each eye receives a conical beam. Each person sees the object by means of rays that have travelled along a different path. So that when we speak of the path of the rays by which in any case an object is seen, the path depends not only on the position of the object but also on the position of the eye.

244. BRIGHTNESS OF NON-LUMINOUS OBJECTS.—In a room where there is a lamp burning brightly, and no other source of light, fix your attention on a sheet of white paper on the wall. The paper looks bright. Turn down the lamp a little. What is the effect on your eye? Turn the lamp lower. Again what effect? You have altered the **intensity of illumination** of the sheet of paper. By turning the lamp down you diminished the quantity of light that it gave out. Therefore you also lessened the quantity that fell upon the paper, and consequently the amount that reached your eye.

The other objects in the room vary in brightness. A silver coin on the table looks brighter than a copper coin, because more light reaches the eye from the one than from the other. So with other objects; if much light passes from them to the eye, they look bright; if little thus passes, they appear dark.

245. TRANSPARENCY AND OPACITY.—A candle flame can be seen through a sheet of glass, but not through a sheet of iron; this means that light can pass through glass, but not through iron. Substances like glass, which allow light to pass through so that the objects from which the light comes can be seen through them, are called **transparent**; substances like iron, which do not allow light to pass through them, are termed **opaque**.

The term **translucent** is applied to substances, such as ground glass or greased paper, which allow light from bright objects to pass through, but through which ordinary objects themselves cannot be seen.

Ex. 388.—Hold various transparent objects, such as sheets of glass of various thicknesses and colours, glass vessels empty and full of water, between your eye and the light, and write down a list of the objects in the order of their transparencies.

Place the same objects between a light, and a white wall or paper screen. By comparing the relative darkness of the shadows on the screen, write the list in the order of their opacities.

Compare the two lists.

Since even a single sheet of glass casts a shadow on the screen

it is plain that glass is not perfectly transparent. Thus we speak of **degrees of transparency** according to the proportion of the incident light that a substance allows to pass through.

No ordinary solid or liquid is perfectly transparent. A thick layer of glass or of water is less transparent than a thin layer. At the bottom of the ocean perfect darkness prevails. Probably also no substance is perfectly opaque; very thin layers of gold, silver, and other metals allow light to pass through them.

Pure air has a high degree of transparency.

Empty space, such as that between the sun and the earth, appears to be perfectly transparent. Light seems to pass through it without undergoing any diminution of intensity.

246. REFLECTION, TRANSMISSION, AND ABSORPTION.—The light which reaches our eyes from non-luminous bodies, and by means of which such bodies become visible, has been turned back or *reflected* by these bodies. Such light must originally have come from some luminous body, and falling upon the surface of the non-luminous body, have been turned back, just as a wave of water is turned back when it advances towards and strikes against a stone wall.

A thin muslin curtain at a window turns back much of the light that falls on it, and thus persons who are on the same side of it as the light see the curtain very plainly, but see comparatively little of the objects on the other side. The foam of the sea looks much whiter and brighter than the water, powdered salt than rock salt, powdered ice than block ice—all because of the large amount of light sent to the eye from the small particles into which the substances are broken. When viewed from the side opposite to that on which the source of light is situated, these substances look dark.

Transparent substances reflect some of the light that falls on them; otherwise we could not see them. But most of the light passes into them, and as we have seen that they cast shadows, some of the light that enters them does not get through. This light is said to be quenched or **absorbed**. That which does pass through is said to be **transmitted**, and the process is called **transmission**.

Ex. 389.—Hold up a piece of white paper (*a*) so that the eye is between the paper and the light; (*b*) so that the paper is between the eye and the light.

In which position does it appear brighter? and why?

Make a grease spot on the paper, and repeat the observations.

Wet a portion of a white towel or piece of white calico with water, and repeat the observations.

When position (*a*) was used the object was seen by reflected light; in position (*b*) by transmitted light. The experiments show that the more light there is reflected the less is transmitted: the more there is transmitted the less is reflected.

Ex. 390.—Hold two sheets of white paper one in front of the other between the eye and a light—

(*a*) With the sheets in contact.

(*b*) With a space of several centimetres between them.

What difference is observed in the brightness? Remembering what has just been said, how do you explain the difference?

Dark-looking opaque bodies do not transmit light; they do not reflect much or they would appear brighter: they must therefore absorb most of the light that falls on them.

247. SOURCES OF LIGHT. — For us the sun is the main source of light. The light of lamps is given out by very hot solid particles in the flame. Light is also obtained by electrical action, as in the arc and incandescent lamps. Various chemical actions make the substances so hot that they give out light.

Ex. 391.—(1) Heat a piece of iron in a Bunsen flame for a short time. It gets hot, but remains dark. Continue the heating, it gets first "red hot" and then "white hot". Up to a certain point the hotter it is the more light it gives out.

(2) Hold other substances—such as the end of a carbon rod, copper wire, platinum wire, a piece of egg shell, an incandescent gas mantle, a little common salt—in a Bunsen flame. Note in each case the colour of the flame, and estimate the intensity of the lights comparatively.

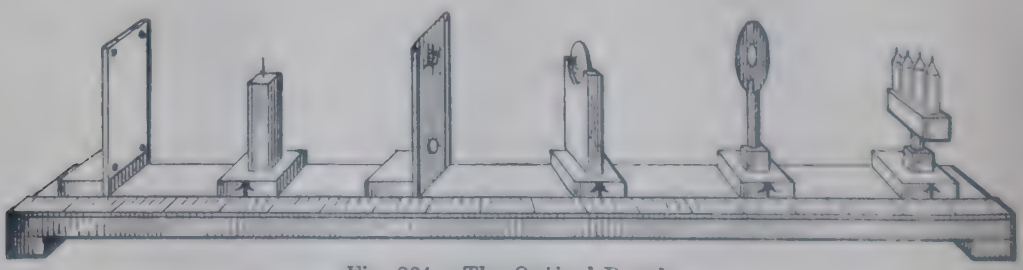

Fig. 264.—The Optical Bench

248. THE OPTICAL BENCH. — For many experiments on light some form of optical bench is necessary. A simple form is shown in fig. 264. It consists of a flat smooth board, preferably 2 m. long, with a millimetre scale along one edge.

The screens, lamps, mirrors, lenses, &c., that are used in the experiments are carried on stands that can be moved along beside the scale.

The base of each stand may be a block of wood, on which a mark is made exactly underneath the centre of the lens, candle, &c., that it supports.

The whole arrangement is designed to enable the different pieces of apparatus used in any experiment to be placed in one straight line and their distance apart to be measured.

As an elementary substitute a tape measure may be pinned to a table.

Sources of Light.—The sources of light required are a large globular light, a small bright light, and a line of light.

A large source may be obtained by using a table lamp with a large translucent globe.

A line of light may be obtained by turning a flat gas flame edgewise, or by using an incandescent lamp with part of the filament screened off, or by placing a screen with a linear slit cut out of it in front of a bright light.

A small point of light is best obtained from an arc light A portion of an incandescent gas mantle seen through a small aperture is a fair substitute.

A broad light may be obtained from a gas flame turned broadways.

These different sources may all be obtained from one of the petrol lamps or paraffin lamps that are made to burn with an incandescent mantle. With a translucent globe a large evenly illuminated source is obtained. If two chimneys are obtained, one may be painted black, except just round the part opposite the brightest portion of the mantle. On this part at equal intervals round the circumference four areas may be left unpainted—a small circle, a larger circle, a very narrow slit, a wider slit. When this blackened chimney is in use any one of these four sources may be used by rotating the lamp.

Screens are wanted on which to receive shadows and images. These may be made of wood, on the front face of which plain paper, blotting paper, or squared paper may be affixed by drawing pins. One screen should have a good-sized hole through it. The size of this aperture may be varied to suit different experiments by placing over it a piece of paper in which there is a hole of suitable size.

Several of the other objects required are shown in fig. 264.

The stands on which the objects are placed must be such that the centres of the objects that they support are at the same height

above the bench. This requirement may be fulfilled in various ways.

For many experiments a darkened room is required. Sufficient darkness may generally be obtained by the use of a closely-fitting black blind over the windows; or often by merely screening the surface on which an image is received from the general light of the room.

249. LIGHT TRAVELS IN STRAIGHT LINES.—Numerous experiences show that under ordinary conditions light does not bend round objects on which it falls.

The paths of rays of sunlight streaming by a dark cloud may often be traced by the reflection of light from motes present in the sunbeam. The straightness of the edges of the bright area may be plainly seen.

Ex. 392.—Place two screens, each pierced with a small circular hole, as in fig. 265, parallel to each other, between the eye and a candle: the candle can only

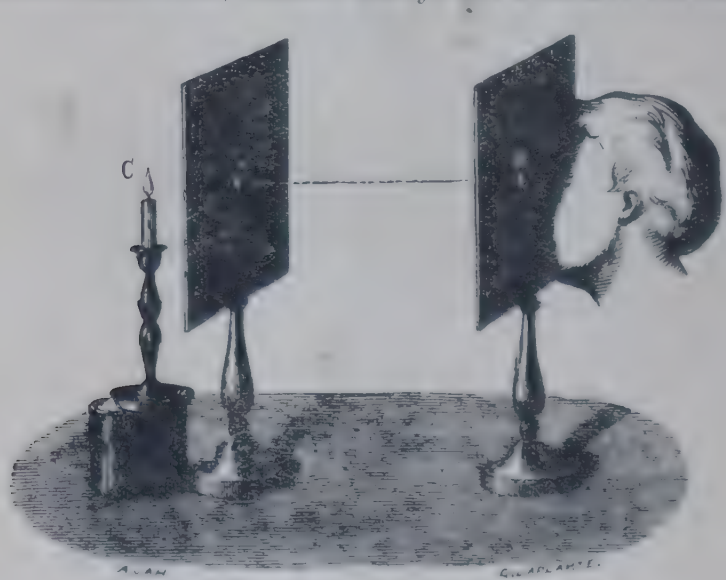

Fig. 265.—Light Travels in Straight Lines

be seen when the eye, the two holes, and the candle are all in the same straight line.

Replace the eye by a third screen placed parallel to the others. A patch of light is seen on the screen. Incline the third screen at an angle of about 60° to the incident light. Note the shape of the patch of light. Why is it not circular in this case?

250. FORMATION OF IMAGES THROUGH SMALL APERTURES.—In walking among trees when the sun is shining one may often see small patches of light on the ground where the

light of the sun has got through. The appearance is shown in fig. 266. It will be noticed that if the sun be high in the sky many of these patches are circular; if the sun be lower they are generally of an elliptical shape. Compare with the last experiment.

Fig. 266.—Formation of Images through Small Apertures

Ex. 393.—Cut two or three small holes of different sizes and shapes in a piece of opaque paper, and hold the paper so that the light of the sun falls perpendicularly upon it. Two or three feet behind these small apertures place a white screen parallel to the pierced piece of paper. Note the shapes of the patches of light on the screen.

Vary the distance between the apertures and the screen, and note the corresponding changes in the shapes and sizes of the patches of light. What is the effect of the shape of the aperture (*a*) when the screen is close to the hole? (*b*) when they are far apart?

Ex. 394.—Draw a small circle, about 6 to 10 mm. in diameter, on a piece of cardboard, and prick a hole about 1 mm. in diameter in the middle of the circle. Set up this perforated screen on an optical bench in a dark room.

On one side of it place a lighted candle, and on the other side a receiving screen, which may conveniently be covered with squared paper (fig. 267). The aperture and the candle flame should be at the same height above the table, the two screens parallel to each other, and the base of each piece of apparatus against

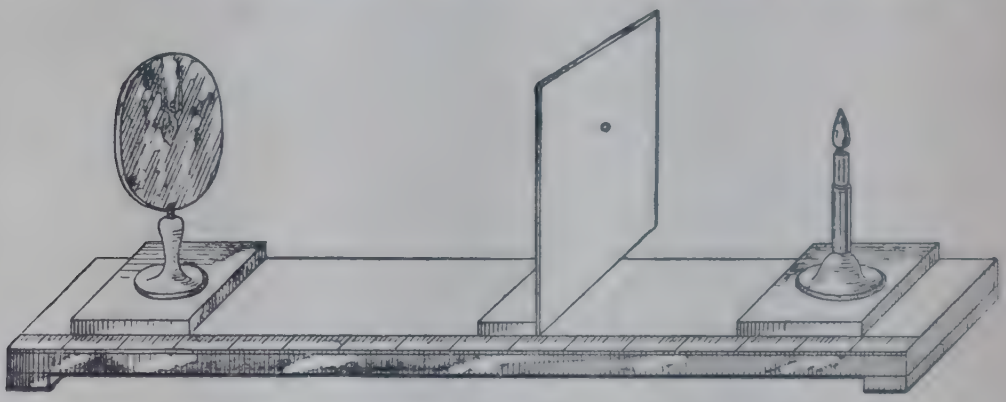

Fig. 267.—Formation of an Image through a Small Aperture

the measuring scale. The bright patch on the receiving screen is seen to be an image of the candle. An **image** is an appearance resembling a real object.

Note that the image is inverted. Adjust the apparatus till the image is quite clear; it may be necessary to enlarge the aperture slightly.

Move the receiving screen farther away from the light, and for each position measure—

(*a*) The distance between the screens.

(*b*) The length and breadth of the image between any two convenient points. If the screen is not in squares, measure with dividers.

Note the changes in brightness of the image as the distance changes. Obtain several images, and record the measurements thus:—

Experiment.	Distance of Aperture from Image, d.	Size of Image.			Relation between l and d, i.e. $l \div d$.	Relation between a and d^2, i.e. $a \div d^2$.
		Length, l.	Breadth, b.	Area, $l \times b = a$.		
1	...	...	...	...	...	...
2	...	...	...	...	...	...

Ex. 395.—Using the same apparatus as in Exercise 394, prick another small hole in the circle. Move the receiving screen to different distances, and note the appearances on the receiving screen.

Prick another hole in the circle and note the effect.

Add holes similarly one by one until there is a considerable number, and observe the effect on the screen.

Finally, cut out the small circle, and again observe.

251. THE PINHOLE CAMERA.—Fig. 268 is a darkened box having one end closed by a sheet of translucent glass, which acts as the receiving screen, and a small aperture in the other end. The distance between the screen and aperture is capable of adjustment to allow of images of objects of different sizes and at different distances to be included on the screen. The picture of a whole landscape may be thus obtained. If when the image is clear a photographic plate is put in place of the screen, a photograph is obtained. In using this camera the screen must be shielded from all light except that which passes through the pinhole.

Fig. 268.—The Pinhole Camera

These experiments show that when light passes from a bright object through a small aperture, and falls on a screen some distance away from the aperture, an inverted image of the bright object is formed on the screen. Provided that the aperture be small, it is of no consequence what shape it is.

With large apertures no image is obtained, and the reason was found to be that when a large aperture is used each small part of it acts as a small aperture and makes an image. These images then overlap and give a general blurred effect, yielding not an image but a general illumination of the surface on which they are formed.

252. SIZE AND BRIGHTNESS OF THE IMAGE.—Experiment 394 showed that the image is smaller and more distinct when the aperture and screen are close together, and that increase of size of the image is accompanied by loss of brightness.

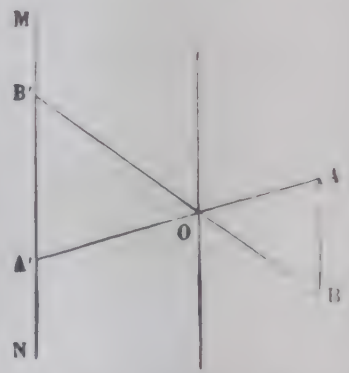

Fig. 269.—Relation between Size of Object and Size of Image

The cause of the inversion and of the change in size as the screen is moved may be seen from the explanatory diagram (fig. 269), where AB represents the candle, B′A′ its inverted image. Light from A proceeds outwards in all directions. The rays that pass to O continue onward through the aperture in

a straight line to A′, which is therefore the image of A. So some of the light from B proceeds through O to B′, and from any point between A and B light proceeds to a point between A′ and B′. The screen reflects the light that falls upon it to the eye, which therefore sees the object as though it were at A′B′. It is manifest from the figure that the farther the screen is from O the farther apart will be the points A′B′, that is, the longer will be the image, and its breadth will be of course similarly increased.

The measurements of similar triangles in Chapter II showed that the height of the image A′B′ is proportional to its distance from the point O. The values of the ratio $l \div d$ in Exercise 394 should therefore all be the same.

As the breadth is increased at the same time as the length and in the same ratio, it follows that the area of the image is proportional to the square of its distance from the aperture O.

The values of the ratio $a \div d^2$ obtained in Exercise 394 should therefore be constant.

As to the brightness of the image: supposing the distance between the candle and the aperture to remain the same, the quantity of light that passes through the hole remains the same.

When the receiving screen is farther away this quantity of light has to spread itself over a larger area, and this area is accordingly less brightly illuminated. Since the illuminated area increases as the square of the distance from the aperture, the brightness of the image or intensity of illumination is inversely proportional to the square of the distance from the aperture.

Ex. 396.—(1) A candle is placed inside a box in a dark room. Three very small round holes are cut in one end of the box, and a sheet of paper is held a small distance away from this end. Describe and explain the appearance seen on the paper.

(2) In Exercise 393 why were the patches of light on the distant screen circular?

(3) A candle flame 5 cm. high is placed at a distance of 20 cm. from a pinhole. At what distances must the screen be successively placed to give images whose heights are 5 cm., 8 cm., 10 cm.?

Find three numbers in the ratio of their respective brightness.

(4) By means of a pinhole camera an image is obtained of a tree. How could you calculate the height of the tree?

253. SHADOWS.—Everyone knows that shadows are produced by opaque objects stopping the light that falls upon them.

Ex. 397.—Place a lamp a few feet distant from a wall, and hold a piece of cardboard, a book, or similar object between the light and the wall, so that the

light falls perpendicularly upon it. Vary the position of the opaque object, and observe the size of the shadow and the character of its edges. When you can put a point exactly at the edge and say definitely, "Here the shadow begins", it is said to be sharply defined. Note how the sharpness of the *definition* varies with change of distance.

Repeat the observations with a thin rod, such as a pencil; a pointed object, such as the leg of a pair of compasses; the blade of a knife; a needle; a very thin wire or thread or hair. Look closely at the shadow, and describe for each the nature of the shadow at various distances from the screen, i.e. its intensity, the sharpness of the definition, and any other feature that you observe.

Ex. 398.—(1) Hold a rod between the lamp and

a plate of clear glass,
a plate of translucent glass,
the surface of a bowl of clean water;

observing the receiving surface with the eye near the rod.

Little or no shadow is seen on these transparent screens.

Remembering what was said in Art. 246, what is the cause of this?

(2) Make the water used above muddy. Can a shadow of the rod be seen upon it now that it is no longer transparent?

(3) Stand in front of a window with the sun behind you, and look for your shadow on the window.

These preliminary observations show that a shadow can only be seen upon an opaque screen, and that the sharpness of its definition

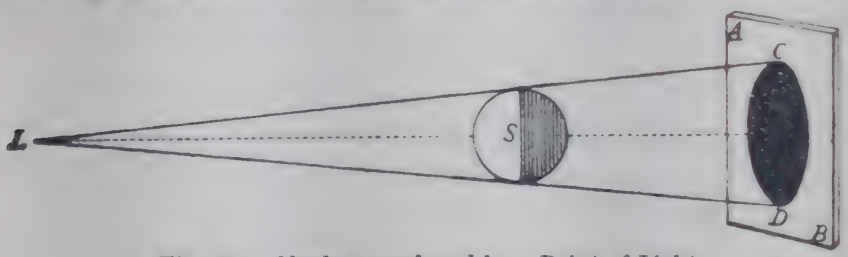

Fig. 270.—Shadow produced by a Point of Light

depends on its distance from the source of light and on its size relatively to the size of that source.

Fig. 270 shows the shadow thrown when light proceeding from a point is stopped by a sphere. The shadow region between the sphere and the screen is part of a cone, of which the upper part is filled with light. The part of the cone from which the light is thus shut off does not look dark any more than the other part looks light, for a cone of darkness looked at from the outside can no more be seen than can a cone of light.

Place opaque objects in that space. They look darker. If no light reached them indirectly they could not be seen at all. Hence, just as we can only tell that rays of light are passing through a certain space by the illumination of an opaque body placed there,

so we can only tell that such rays are not passing there by the more or less complete invisibility of an opaque body in the space.

Ex. 399.—(1) Arrange in a straight line on an optical bench in the following order—a small bright source of light (p. 267), a small opaque circular disk to cast a shadow, and a screen on which to receive the shadow.

Note the sharp definition of the outline of the shadow.

Place the eye in the shadow; no light reaches it from the source. Measure the distance (d) of the shadow from the source, and also the diameter of the shadow.

Place the receiving screen at several different distances from the disk, and repeat the observations and measurements. Do not move either the light or the disk. Record thus:—

	Distance of Shadow from Point of Light (d).	Area of Shadow (a).	Value of $a \div d^2$.
1	...	...	...
2	...	...	...
...	...	...	...

The area of the shadow is proportional to the square of its distance from the point of light.

(2) Remove the disk, and obtain the shadows of thin objects, such as a needle, a piece of cotton, a hair. Note the definition at different distances, and compare with the results obtained in Exercise 397. Why is the definition now sharper?

The character of the shadow obtained when the opaque body is smaller than the source of light is shown in fig. 271.

Fig. 271.—Shadow produced by Light coming from a large Source

Ex. 400.—Repeat Exercise 399, using a globe lamp in place of the small light.

Place the receiving screen near the opaque body.

Note that the shadow is not so intense near the edges as in the centre.

Remove the screen gradually farther and farther away from the opaque body, observing the character of the shadow at each stage.

Draw the transverse sections of the shadow thus obtained on the screen, and compare them with the longitudinal section shown in fig. 271. Endeavour to find the point o shown in that figure.

Place the eye in various positions behind the opaque disk. In some positions you see part of the globe, and in others none of it. How does this account for the character of the shadow on the screen?

Umbra and Penumbra.—Referring to fig. 271 the lines *beo* and *aro* are drawn touching the two circles and meeting at *o*. The part *oer* is a section of a cone from which the light of *s* is completely shut out, and within which no part of *s* can be seen. This is the region of total shadow, the *umbra*.

Outside the region *n'ern* the whole of *s* can be seen and none of its light is cut off. Within the region *n'ern* but outside the cone *oer*, part of *s* can be seen, and a body placed there is only partially obscured. This region is the *penumbra*.

On a screen placed between *t* and *o* the shadow is of this character:—In the centre is a circle quite dark; round this circle are rings which grow less dark as they grow larger, the outside ring. which just touches the lines *en'* and *rn*, being almost as bright as the part completely outside the shadow. These rings fade very gradually into one another, so that the outside boundary of the shadow cannot be exactly fixed.

The size and shape of the umbra and penumbra depend on the relation between the sizes of *s* and *t*. If *s* and *t* be of the same size the umbra is cylindrical in shape, as is indicated by the lines *dh'*, *ch*. As *s* is made smaller the region of the penumbra gradually grows less, and when *s* is a point this region ceases to exist; the umbra being then of the form shown in fig. 270. The closer a body in the penumbra is to the boundaries *n'e* or *nr* (fig. 271), the more light falls upon it; the nearer it approaches to the region *eor* the less light falls upon it.

254. ECLIPSES. (*a*) **Of the Moon.**—As the moon revolves round the earth it sometimes passes through the umbra of the earth's shadow. Since the moon only shines by the light which it reflects from the sun, it is, while in the umbra of the earth's shadow, quite dark, and we have an eclipse of the moon. Sometimes the path of the moon is such that part of it is in the umbra and part in the penumbra; it is then partially eclipsed.

To obtain the effects with the arrangement of Exercise 400 (fig. 271), in which the large sphere represents the sun and the opaque body the earth, pass a small disk to represent the moon across the shadow so as to pass through the umbra.

(*b*) **Of the Sun.**—These are produced by the moon passing between the earth and the sun, so that the earth passes through the

shadow of the moon. To obtain the effects, using the arrangement of fig. 271, in which in this case the opaque body represents the moon, pass a disk representing the earth through the shadow at such a distance that the apex *o* of the umbra just falls upon it, forming a small circular patch. Within this area the light is completely hidden, and the eclipse is total. Outside this area only a portion of the light globe is hidden, and the eclipse is partial.

Ex. 401.—Miscellaneous Exercises. (1) Clouds floating in the atmosphere are sometimes very bright and sometimes very dark. How is this difference to be accounted for?

(2) You have to place a number of vertical sticks in a straight line. How would you test by the eye whether they were or were not in the required positions?

(3) You cannot see the shadow of a hair held at a foot distance from a wall against which the sun is shining; whereas you see the shadow when the hair is held close to the wall. What is the reason?

(4) The progress of an eclipse of the sun is watched on a pinhole camera. Give drawings of what is observed on the screen at several different stages, and explain them.

(5) Explain with a diagram the mode of production of the round patches of light often seen on the ground under trees.

(6) In the partition separating two rooms A and B there is a small window. From a candle in A, light passes through the window on to the opposite wall of B. Show how by measurements made in room B you could calculate exactly where the candle was. (Refer to Exercise 399.)

(7) Compare the heights of two trees by measuring the lengths of their shadows. Find also the length of the shadow cast by a metre scale, placed vertically, and hence deduce the actual height of the trees in metres.

Why must the shadows all be measured at the same time as nearly as possible?

CHAPTER XXX

Photometry

255. Photometry is the measurement of the illuminating power or luminosity of different sources of light; and of the degree of illumination (the brightness) of surfaces. The measurements are made by *photometers*.

The unit of luminosity is that of a candle of certain specified make called a *standard candle*. Other lights are compared with this standard, and are said to be of so many **candle-power**.

The comparison is made by causing the lights to illuminate a surface to an equal degree, and then comparing their distances from that surface.

We have first to find out in what way the distance enters into the measurement.

256. LAW OF INVERSE SQUARES.

Ex. 402.—(1) Arrange a dull white screen, a rod just in front of it, a single candle, and (in place of the gas jet) a pair of candles, along an optical bench, as in fig. 272. The candles must be exactly alike.

Fig. 272.—The Shadow Photometer (Rumford's)

Place the pair of candles close together, and farther away from the screen than the single candle, and a little out of line, so that two shadows of the rod appear close together on the screen. Arrange for them just to meet, with no black or white line between them.

Compare the two shadows. Move the pair of candles towards or away from the screen until the two shadows appear equally dark.[1]

Satisfy yourself that the shadow cast by each of the lights is illuminated by the other light.

Measure the distances d and d' of the lights from the shadows.

Replace the pair of candles by a set of three close together, side by side as shown in fig. 273. Make the same adjustments and measurements as before, keeping the single candle always in the same position.

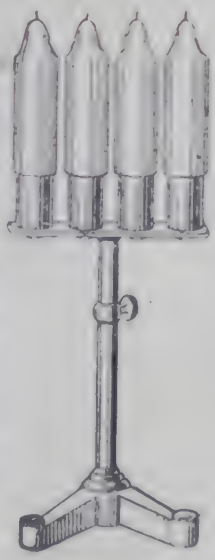

Fig. 273

[1] It has been shown that light is reflected from nearly all substances. If light is thus reflected on to the screen of Rumford's photometer, on to the grease spot, or on to the paraffin slabs from the walls of the room, the bench, one's dress, &c., or reaches the photometer from a window, or in any other way except straight from the lights themselves, the result may be an inaccurate measurement. A dark room, in which everything is a dull black, removes this cause of error. If that is not available, screen the photometers from stray light as far as possible by black cloth or black paper. The paraffin-wax photometer is most easily screened

If convenient, repeat the measurements with a set of four candles. Record thus:—

Number of candles	1	2	3	4
Distance from shadow	d	d	d	d
Square of distance from shadow ...	d^2	d^2	d^2	d^2
Ratios	$1 \div d^2$	$2 \div d^2$	$3 \div d^2$	$4 \div d^2$

Compare the ratios in the fourth line.

If the experiment is carefully done the ratios will be approximately equal.

(2) Repeat the measurements with a different value of d.

The mean of experiments shows that the ratio between the number of candles and the squares of their respective distances from the screen is constant.

Hence if we have two sources of light whose candle-powers are x and a, and these two sources placed at distances y and b from a screen, illuminate the screen to the same degree, then

$$\frac{x}{a} = \frac{y^2}{b^2}.$$

This rule is called the **Law of Inverse Squares**.

The law of inverse squares follows from geometrical considerations.

Fig. 274.—Geometrical Illustration of the Law of Inverse Squares

Suppose a small flame to be placed at the apex of a hollow pyramid having a square base (fig. 274). From the apex rays of light stream out in all directions. If a square piece of card be placed *halfway up* the pyramid, so as to just fit in it, no light can fall on the base—it is all stopped by the card. The area of the base of the pyramid is just four times that of the card, which, however, intercepts all the light that would otherwise reach the base. The card is therefore four times as brightly illuminated as was the base, which is twice as far off. One candle placed at the apex would illuminate this little square just as brightly as four candles placed there would illuminate the base. (Compare Exercise 399.)

257. PHOTOMETERS.—All photometers compare the intensities of different sources of light by finding the distances at which the lights *equally illuminate* a screen placed to receive their rays, and then calculating the ratio of their intensities from the law that their illuminating powers are proportional to the squares of their distances from the screen. The simplest photometers are Rumford's and Bunsen's.

The apparatus shown in fig. 272 is Rumford's, or the shadow photometer.

Ex. 403.—By means of the shadow photometer find the candle-power of a lamp or gas jet. The method and calculations are as described above. Make several determinations using different distances. Each pair of distances gives a value for the candle-power. For final value take their mean.

Example.—Suppose in fig. 272 the candle is 60 cm. from the screen, and the gas jet 150 cm. from it, then the illuminating power of the jet is $\frac{15^2}{6^2} = 6\frac{1}{4}$ candle-power.

Bunsen's, or the grease-spot photometer, consists of a spot of grease on a sheet of white paper. It may be made by dropping a spot on the paper from a stearine candle, and removing it when hard. The paper may then be pasted over a hole in a piece of stiff cardboard.

Instead of a grease spot, a tightly stretched piece of tissue paper may be sandwiched between two perforated pieces of cardboard.

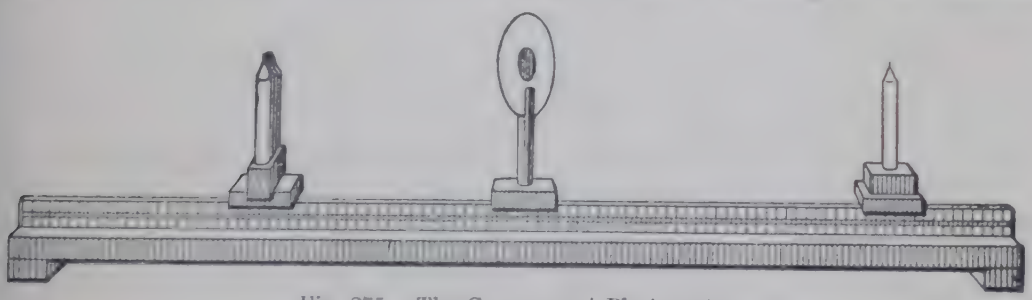

Fig. 275.—The Grease-spot Photometer

The translucent patch thus prepared is placed on the optical bench, and the two lights to be compared are placed one on each side of it (fig. 275). The lights are adjusted until a position is obtained in which the translucent patch looks neither darker nor brighter than the rest of the screen.

Ex. 404.—Verify the law of inverse squares by means of the grease-spot photometer, placing one candle on one side, and one, two, three, &c., successively on the other side of the spot.

Take readings by viewing the grease spot from each side: they will probably

be a little different: take their mean as the true distance. Always hold the hea in the same position when viewing the spot.

The distances measured are from the screen to the lights.

Ex. 405.—By means of the grease-spot photometer find the candle-pow of the light used in Exercise 403. Make several determinations, using differe distances, and take their mean. Compare with the previous result.

The Paraffin-wax Photometer.—A better arrangement tha the grease spot on paper is shown in fig. 276. Two rectangul

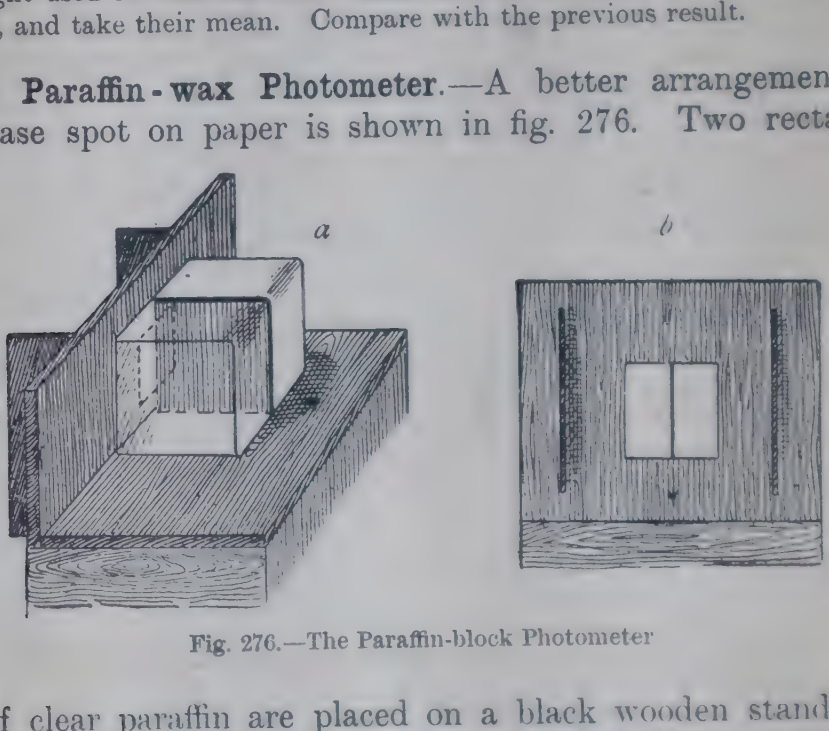

Fig. 276.—The Paraffin-block Photometer

slabs of clear paraffin are placed on a black wooden stand with layer of tinfoil between them, as shown in fig. 276, *a*. A windo in the front of the stand allows the central part of the blocks be seen (fig. 276, *b*). The photometer is placed on the optical ben with the plane of the foil perpendicular to the line joining t lights. Each paraffin slab is thus illuminated by one light on no stray light must fall on the slabs. The lights are placed that the two slabs when viewed from the front through t window look equally bright. (Screen the eyes from the light The distances of the lights from the tinfoil are measured, and t calculation is the same as for the grease-spot photometer.

Ex. 406. - Miscellaneous Exercises. (1) With any one of the photome set up in correct final position, show the effect of stray light by holding a mi or a sheet of white glazed paper in various positions so as to reflect light on to photometer.

(2) Remembering that in Rumford's photometer each shadow is illumin by one of the lights, describe clearly how it is that the comparison is made between shadows but between the illuminations produced by the lights.

(3) Give a simple proof of the law that light emanating from a point di ishes in intensity as the square of the distance from the luminous point increa

(4) Of two gas flames one gives out twenty-five times as much light as

other. If you are testing their illuminating powers by Rumford's photometer, and you place the smaller flame at a distance of 60 cm. from the screen, at what distance from the screen must the larger flame be placed in order that the shadows of an opaque object cast by the two flames may be equally illuminated?

(5) If you hold a sheet of blotting paper, in the middle of which a grease spot has been made, first behind and then in front of a flame, a difference in the appearance of the spot will be noticed. How do you explain this difference? How would you use such a piece of blotting paper to compare the illuminating powers of two flames?

(6) A gas jet 16 ft. and a candle 4 ft. from a screen illuminate it equally. Compare their illuminating powers.

(7) You have a candle (A), an ordinary lamp (B), and a powerful light (C). Using a paraffin-wax photometer, you obtain equal illumination of the slabs at the following distances:—

For A and B: A 60 cm. : B 300 cm.
For B and C: B 1·2 m. : C 4 m.

What is the candle-power of the light C?

CHAPTER XXXI

Reflection from Plane Surfaces

258. A general idea of the reflection of light has been given in Art. 246. In a room lit by a lamp or from a window some light gets under tables and behind articles of furniture. In the open air light similarly permeates everywhere. This general diffusion of light is produced by reflection.

Ex. 407.—Take a sheet of white paper into a dark corner of a room: let light fall upon it from the lamp or window; turn the paper into different positions, and note the differing degrees of illumination of the wall. Try the effect of other reflecting surfaces of different characters and colours. What is the effect produced by a dull black surface? a piece of polished metal? a mirror?

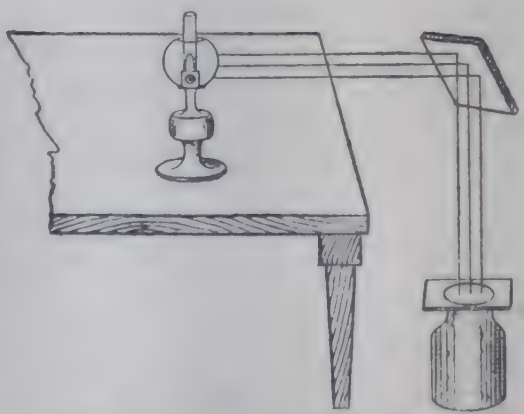

Fig. 277.—Regular and Diffused Reflection

Ex. 408.—In a room in which there is a bright light, such as an incandescent mantle, burning on a table, and no other source of light, place on the floor beside the table a beaker of water to which one drop of milk has been added. If the room is a "dark room", the liquid will only dimly be seen. If the room has

reflecting walls, screen off the stray light from the beaker of liquid with black cloth or paper.

Hold a small mirror opposite the lamp and above the beaker, and tilt it so as to reflect the light on to the liquid (fig. 277), allowing none to strike the floor directly.

What effect is produced on the liquid itself and on the floor near it?

259. LIGHT IS INVISIBLE.—If, in a perfectly dark room, a small hole be made in a shutter on which the sun is shining, the

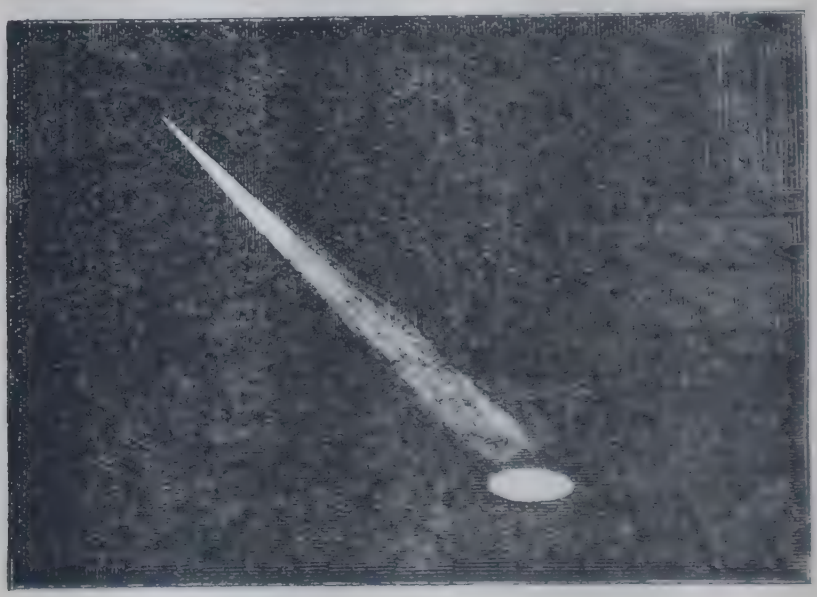

Fig. 278.—**Path of a Conical Sunbeam**

path of the light can be traced, as in fig. 278. But on looking closely at it, multitudes of small specks are seen floating about. From each of these particles of dust some light is reflected to the eye, and hence the whole region looks bright. If smoke be blown into the region it looks brighter still, because the smoke contains many small solid particles. *If especial care be taken that none of these particles be present, the bright appearance is no longer seen in the air*, and there is only the patch of light on the floor.

Sometimes the rays of the sun can *apparently* be seen shooting down from behind a cloud. The appearance is due to the reflection of the light to the eye from small particles of dust or water.

The surfaces of most objects in reflecting light diffuse it or scatter it in all directions; their surfaces are rough, having on them multitudes of small projections, which behave in the same manner as the small particles of milk. Smooth or polished surfaces do not

scatter much of the light that falls on them. The reflected pencil takes one path in a definite direction. This is regular reflection.

Exercise 408 illustrates the processes of (*a*) **diffused reflection** or **diffusion** from the small particles; (*b*) **regular reflection** from the mirror.

260. **Laws of Reflection.**—We have now to investigate the laws of regular reflection. Strips of mirror glass 12 or 15 cm. long and 4 or 5 cm. wide are sufficient for most of the experiments.

Ex. 409.—(1) Hold a short needle perpendicular to a plane mirror in a well-lit room, and look along its length (*a*) with both eyes, (*b*) with each eye separately. With both eyes open and adjusted for distant vision there appear to be two actual parallel needles and two converging images.

(2) Hold the needle rather close to and near one end of the mirror, and view it very obliquely with your back towards the light. Whether you use one eye or two you see at least three images, one bright and two "ghosts".

Let the point of the needle come in contact with the glass. One of the ghosts touches the needle itself. See the thickness of the glass.

(3) Repeat the last experiment with a candle in a dark room: several images are seen.

These preliminary experiments show that the character of the images seen in a mirror depends not only on the position of the object but also on the position of the eye; that the principal image is formed by the silvered surface at the back of the glass; and that the front surface of the glass also acts as a reflector and gives an image.

In making experiments with mirrors care must be taken to avoid mistakes arising from these facts.

Normal.—A line perpendicular to a mirror at any point is called the normal at that point. All angles are measured from the normal.

Ex. 410.—Arrange an apparatus on the model of fig. 279, which consists of a small plane mirror, a protractor, a normal to the mirror, a bright object at the end of an arm which shows its angular position, and a sighting hole at the end of another arm. It may be made with a sheet of paper on a drawing board, a mirror whose horizontal width is small, a semicircular protractor with its straight side accurately flush with and at the middle of the side of the board, some tall needles, and three threads.

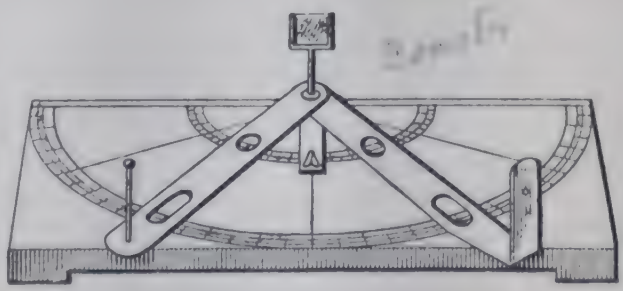

Fig. 279.—Apparatus to Demonstrate the Laws of Reflection

Referring to the plan in fig. 279*a*, M is the mirror, and MN the normal. Place the bright object at B and the eye at A, so that the image is seen along the line AM in the mirror above the normal line MN. From the protractor read off the angles BMN and AMN. The angle BMN is the **angle of incidence**, AMN is the **angle of reflection.**

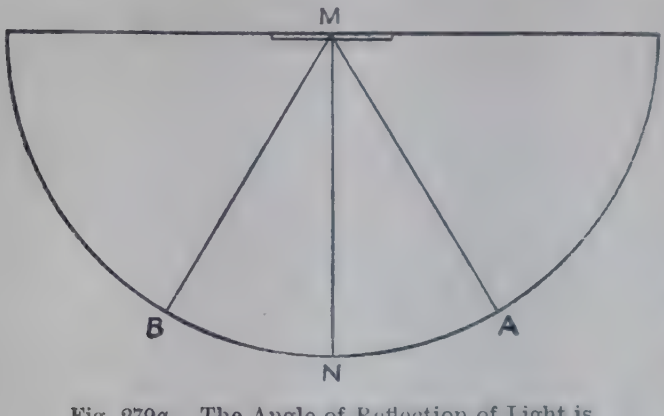

Fig. 279*a*.—The Angle of Reflection of Light is equal to the Angle of Incidence

Move B farther round the semicircle, and again read the pair of angles.

Take several measurements with B in different positions on the semicircle, write down the angles of incidence in one column and the angles of reflection in another.

Examine the figures obtained. Does there appear to be a relation between the angle of incidence and the angle of reflection?

To lessen the effect of mistakes compare the sum of all the angles of incidence with the sum of all the angles of reflection.

Ex. 411.—(1) Fasten a strip of plane mirror on the front surface of a block of wood so that the mirror is in a vertical plane.

On a sheet of paper preferably fastened to a drawing board, rule a long straight line AF (fig. 280), and place the mirror with its reflecting surface (the back) along that line.

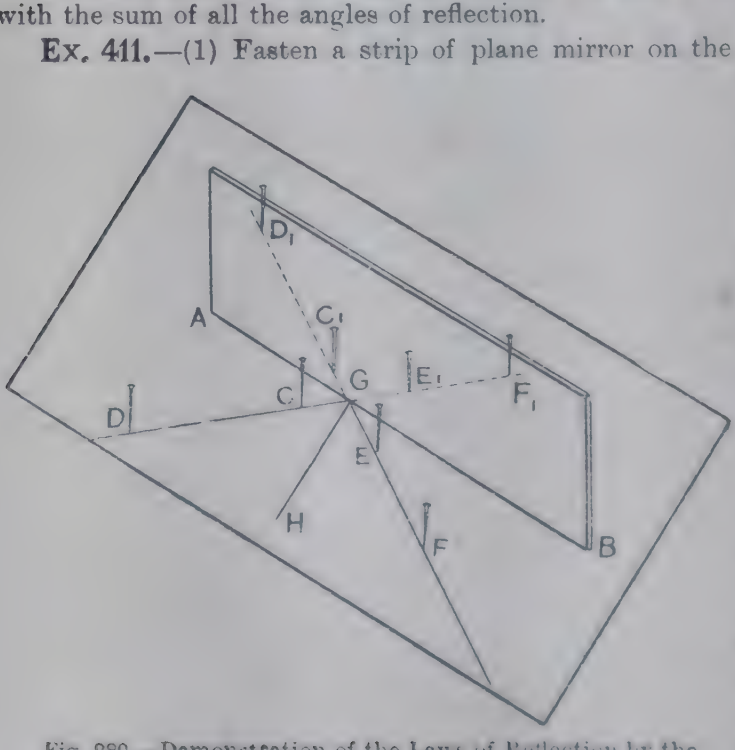

Fig. 280.—Demonstration of the Laws of Reflection by the Pin Method

Stick a pin C near the mirror and another at D some distance away, so that the line DC is oblique to the mirror. Look along the line DC with one eye and keeping the head in the same position stick in two more pins E and F in such positions that their images E_1F_1 in the mirror appear to be in the prolongation of the line DC.

Look along the line FE, and see that the images G_1D_1 are in the prolongation of that line.

When the lines have been got quite accurately, remove the mirror and the

pins from the paper. As you remove each pin in turn, mark its position accurately with a fine ×, the centre of which passes through the centre of the hole.

Draw the lines DEG and FEG. If the work has been carefully done these lines meet on the line AB.

At G draw a line GH perpendicular to AB. This is the normal to the mirror. Measure with a protractor the angles DGH and FGH, and record the values. For the given positions of the pins and the eye these two angles are the angles of incidence and reflection.

(2) Repeat the whole operation with the pins in different positions so as to give different values of the angles of incidence and reflection. An inspection of the values shows that these angles are equal to each other.

Ex. 412.—Place one end of a knitting needle in contact with a plane mirror, and turn the needle into various positions. However it be turned, there is always one position of the eye in which the needle exactly covers its image. Find this position. If the image were a real needle one flat surface would pass through the object, the image, and the angle, i.e. they would be in the same plane.

Hold the needle away from the mirror at various inclinations on a level with the eye and horizontal. Do the object and image appear to be in the same plane?

Ex. 413.—By tying on a rectangular mirror a piece of cardboard of slightly less width than the mirror obtain a mirror about 3 mm. wide.

Place this narrow mirror lengthwise on a flat board. Near one end of the mirror and 8 or 10 cm. in front of it place on the board a small bright object—a bead or a needle.

On what plane must the eye be placed in order to see the image?

Are the eye, the image, and the object in the plane of the board?

The foregoing experiments show that when a ray of light is regularly reflected from a plane polished surface—

(i) *The angle of incidence is equal to the angle of reflection.*

(ii) *The incident ray, the reflected ray, and the normal are all in the same plane.*

261. IMAGES. — When a pencil of light rays proceeds from an object, and is reflected from a polished surface to an eye, the effect on the eye is the same as if the bundle of rays had come straight from the object. The eye gets the same impression as if it were turned towards the object. But the judgment that is formed is that the direction of the object is that in which the light rays are travelling at the time that they reach the eye. The object appears like what it really is, but removed to a new position. The imaginary object in this new position is called an image. The image formed by the pinhole camera (Art. 251) was received on a screen. Such images are called **real**. The image produced by a plane mirror cannot be received on a screen: such images are called **virtual**.

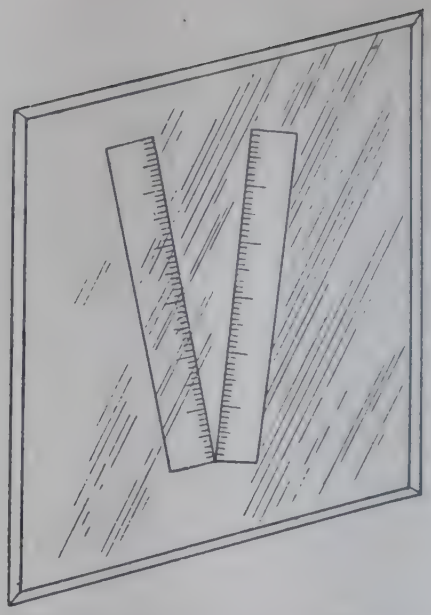

Fig. 281.—Equality of the Distances of Image and Object from a Plane Reflecting Surface

We have now to consider the position of the image formed by a plane mirror.

Ex. 414.—Hold a measuring scale with one end of its graduated edge in contact with a mirror, or (better) a piece of bright sheet tin; and the other end a short distance away from the mirror (fig. 281).

Look at each graduation mark and at its image. Compare the distance of each mark from the reflecting surface of the mirror with the apparent distance of its image behind the mirror.

What appears to be the ratio between these distances?

Ex. 415.—Place a narrow strip of mirror on a sheet of paper, with its reflecting surface along a line AB (fig. 282).

Put tall pins, C and D, one in front of and one behind the mirror.

Place the eye on a level with the pins, so that the top of D can be seen over the mirror at the same time that the image of the lower part of C is seen in the mirror. Look at them from various positions. Is there a gap between the top of D and the image of C?

Move one of the pins until a position is found in which this gap disappears,

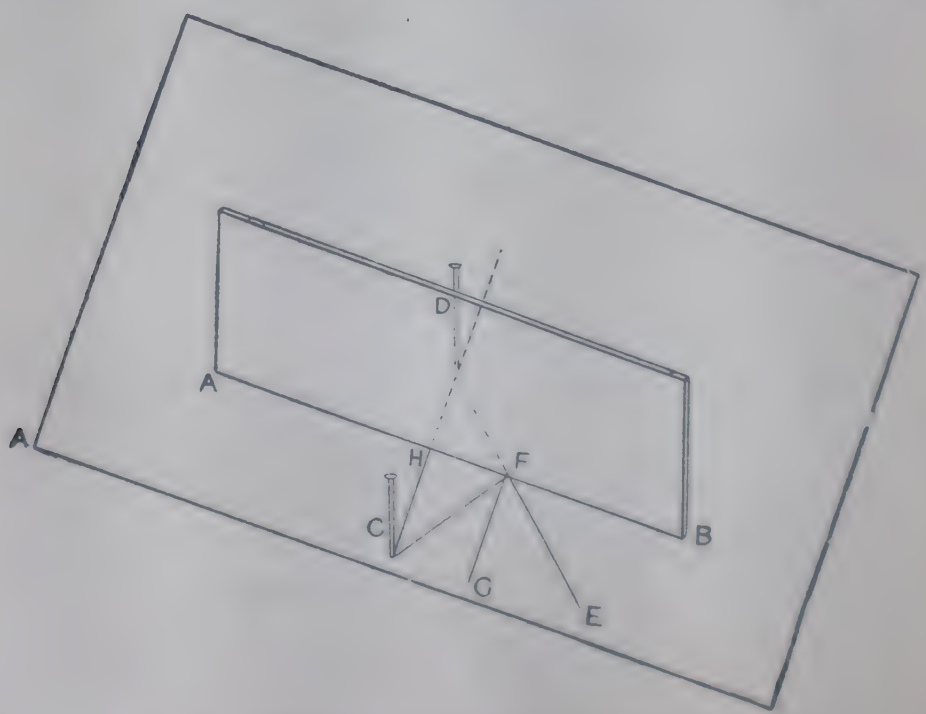

Fig. 282.—Position of Image found by the Parallax Method

and the top of D appears to be a prolongation of the image of C *wherever the eye may be.*

When the correct position is found choose some position E for the eye, and mark the line EF, which runs from any point E towards D.

Remove the mirror, and on the paper draw accurately the lines CD, EF, CF, and FG the normal to AB.

Measure CH and HD. If the work is accurate, they are equal. The image of C is the same distance behind the mirror as C is in front of it.

The line CFE is the path pursued by the light from C to the eye at E. Are the angles of incidence (CFG) and reflection (GFE) equal?

This method is called the parallax method.

Ex. 416.– (1) Using the parallax method as in the last exercise, place in front of the mirror an object whose two ends are not alike, such as a paper arrow AB (fig. 283). By pins determine the position of A′B′, the images of its two ends. Mark on the paper the line of the reflecting surface of the mirror and remove the mirror. Join A′B′, and thus reproduce the image.

(2) Measure the length of A′B′. The image is the same size as the object.

(3) Measure the inclinations of the lines AB and A′B′ to the mirror. They are the same.

(4) Draw, as in fig. 283, the actual paths of the rays by means of which an eye in some position, as O, sees the images of the two ends AB of the object.

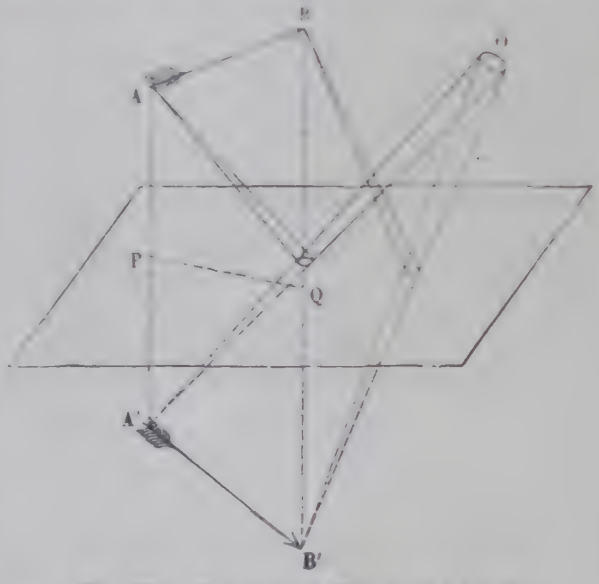

Fig. 283.—Relation of Object and Image

262. LATERAL INVERSION.—This phrase means change of side, and is illustrated by the last experiment. If a person stand in front of a tall plane mirror and hold out his right hand as though to shake hands with his image, the image advances *its left* hand. If the person has his hair parted on the left-hand side, the image will have the parting on *its right.*

If a directed envelope be held up in front of the mirror, the writing appears as it does when seen through the paper; the slope is reversed. The impression left on blotting paper is similar. All these are instances of lateral inversion.

The preceding group of experiments show that when an image is formed by a plane mirror, the image is as far behind the mirror as the object is in front, the image is the same size as the object, and is laterally inverted.

263. FIELD OF VIEW IN A MIRROR.

Ex. 417.—Suppose yourself to be standing in front of a tall mirror, so that you can just see your image full length. Determine, by drawing, the smallest height that the mirror can have and how far above the ground such a mirror must be placed.

264. EFFECT OF ROTATING A MIRROR.

Ex. 418.—Draw a large circle and two diameters AB, CD at right angles to each other (fig. 284). Along AB place a strip of mirror with its centre at E. Look along the line DE: its image is seen along CE.

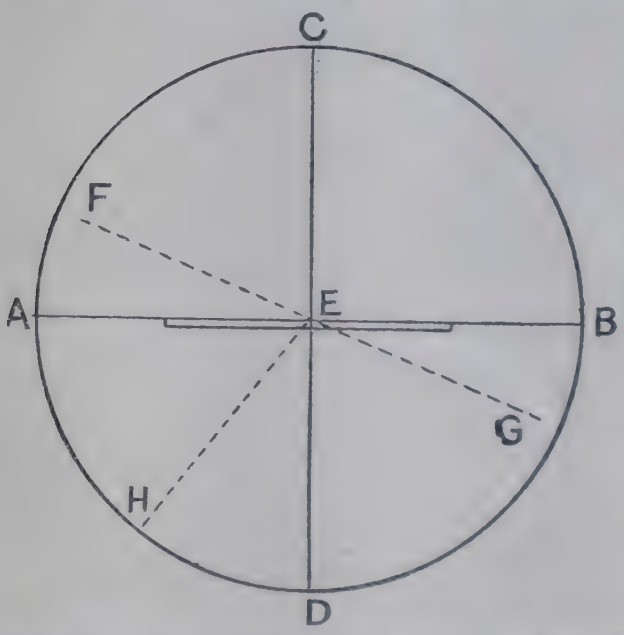

Fig. 284.—Displacement of Image by Rotating a Mirror

Turn the mirror into the line FG, and mark the line on the paper. The image of DE will now be seen along HE produced. Looking along HE, by pins or dots mark out this line also.

Remove the mirror; draw FG and HE accurately; measure the angles—

AEF, through which the mirror has been turned;
DEH, through which the image of DE has been turned.

What is the relation between these two angles?

Repeat the measurement with a different value of the angle AEF.

The experiment shows that when a mirror is rotated through a certain angle the direction of a reflected ray is changed by double that angle. For instance, if the mirror is turned from 20° to 30°, that is, through an angle of 10°, the reflected ray is turned from 40° to 60°, through an angle of 20°.

The reflected ray is sometimes said to have double the **angular velocity** of the mirror.

265. MULTIPLE IMAGES IN PARALLEL MIRRORS.

Ex. 419.—On a sheet of paper place two strips of mirror glass, A and B (fig. 285), parallel to each other a few centimetres apart, and mark their positions. Between them, and nearer to one mirror than the other, stick a tall pin into the paper.

Find a position for the eye in which several images of the pin are seen in mirror A. By the parallax method mark the positions of the first three or four of these images.

Do the same for mirror B.

Remove the mirrors.

Were the images a_1, a_2 . . . , b_1, b_2 in a straight line?

Each image in A acts in turn as an object for the mirror B, and each image in

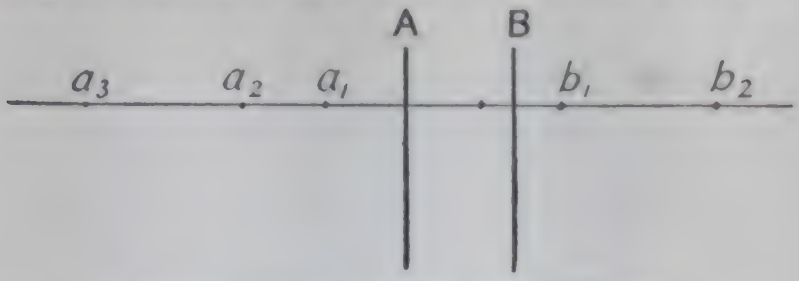

Fig. 285.—Multiple Images in Parallel Mirrors

B acts similarly towards mirror A. Measure the distances from b_1 to A and from a_3 to A. Are they equal?

In the same way, verify the law of distances for the other images.

266. INCLINED MIRRORS.

Ex. 420.—On a sheet of paper set out a large semicircular protractor, marking every 10 degrees.

Place two rectangular mirrors vertical with one end at the origin of the angles. They may be hinged by a piece of gummed paper.

Set the mirrors so as to enclose an angle of 90°, and place a candle, piece of chalk, or other bright object, between them, as in fig. 286.

Find a position for the eye in which all the images can be plainly seen.

Count the images (n), and note the angle between the mirrors (a°). (Among n include the object.)

Rotate one mirror round the hinge. Note the behaviour of the images.

When the angle between the mirrors is 72° again record a and n.

Repeat for angles of 60°, 45°, 30°.

For each observation find the value of $\frac{360°}{a°}$. Is the quotient equal to the number of images seen at that angle?

Example.—What would be the number of images formed with the mirrors at 20°? 10°? 5°?

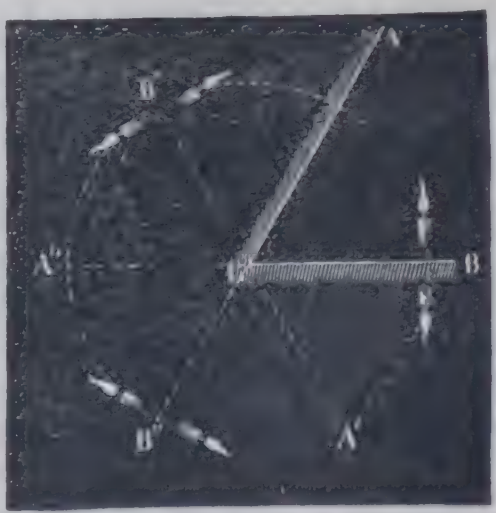

Fig. 286.—Images in Inclined Mirrors

Fig. 286 shows the appearance when the mirrors are placed at angles of 60°. The image in the sector ACB′ is the first image of the candle in the mirror AC; the image in the sector A′CB″ is the reflection in BC of the first image above mentioned; and this

secondary image has in turn a third image in the sector B″CA″. Starting with the first image in BC a similar set is obtained, the third of which coincides with the third obtained above.

Ex. 421.—Show by a drawing how diffused reflection from rough surfaces follows from the laws of regular reflection. Draw a wavy line. On it fix on several points, and mark them A, B, C, &c.

Mark an incident ray falling on A; find as nearly as possible the normal at A; and draw the reflected ray.

Repeat the operation at each of the other points, making the incident rays all parallel to each other.

Note the various directions of the reflected rays.

Ex. 422.—Miscellaneous Exercises. (1) Draw and describe a diagram showing the angle of incidence and the angle of reflection. What is the value of each of these angles when the sun is 25° above the horizon and you see his image reflected in still water?

(2) Place a candle beside a vessel full of water, and look at the reflection from the other side of the vessel. Note the image.

Produce slight ripples on the water. What is the appearance now?

(3) In the manner indicated in Exercise 421 explain and illustrate by a drawing the formation of the long band of light often seen on the surface of water on which the moon is shining.

(4) A candle is caused to shed light through two long narrow tubes, upon two flat mirrors so placed as to reflect the light to one and the same spot. Give a sketch showing accurately the paths of the two beams of light.

(5) Two plane mirrors are inclined to one another at an angle of 72°. Show how to determine the position and number of the images of an object placed between the mirrors.

CHAPTER XXXII

Spherical Mirrors

Fig. 287.—A Concave Mirror

267. Spherical mirrors are reflectors having the form of a small part of the surface of a sphere (fig. 287). The edge of the mirror is generally circular. Such mirrors are either **concave** or **convex**.

Referring to fig. 290, where a section of a spherical mirror is shown:—

The **centre of curvature** C of the spherical mirror is the centre of the sphere of whose surface the mirror is a part.

A straight line such as IC or OC drawn from

any point in a spherical mirror to the centre of curvature is a **normal** to the surface of the mirror at that point.

The length CI or CO is called the **radius of curvature**.

The **vertex** or centre of the spherical mirror is at O, the middle point of its surface.

The **principal axis** is the line OC passing through the centre of curvature C and the vertex O.

A **principal section** MN of the mirror is a section produced by any plane passing through the vertex O and the centre of curvature C.

268. THE CONCAVE MIRROR.—The focus and focal length of a concave mirror may be demonstrated thus:—

Ex. 423.—Place the mirror facing the sun. Place a small cardboard screen between the sun and the mirror, but slightly to one side, so as not to cut off the light from the mirror. Adjust the positions of the screen and the mirror until a patch of light appears on the screen. This patch of light is an image of the sun.

Move the screen straight towards or straight away from the mirror, and note the changes in the size of the image. In this way find the position of the screen when the image is smallest and most sharply defined.

When this position has been obtained the image is situated at the **focus** of the mirror, and the distance from the focus to the centre of the mirror is the **focal length** of the mirror. **Measure this distance.**

If the sun is not available use a bright light placed as far away as possible.

Fig. 288 illustrates what has taken place. The rays from the sun are parallel, and are represented by the parallel lines.

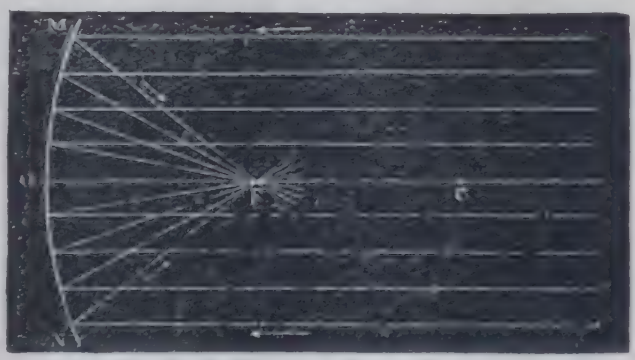

Fig. 288.—Parallel Rays converging to a Focus

Each of these rays on striking the mirror is reflected according to the laws of Art. 260, with the result that the reflected rays all pass through a region F, which is the focus of the mirror. In all the following figures the focus is marked F; and the focal length is written *f*.

269. CENTRE OF CURVATURE.

Ex. 424.—Take a screen with a small hole through its centre. Across the hole gum two threads (cross wires), or fasten a piece of wire gauze over it.

Place the screen on an optical bench. Just behind it place a globe lamp or other uniform source of light (fig. 289). On the other side of the screen place the concave mirror that was used in the previous exercise.

Adjust the position of the mirror until an image of the aperture appears on

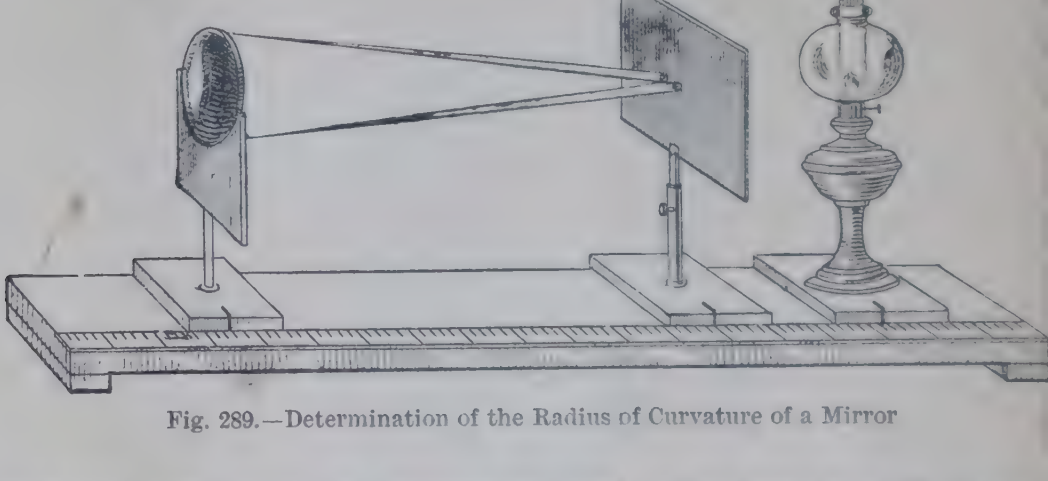

Fig. 289.—Determination of the Radius of Curvature of a Mirror

the screen, close beside the aperture itself and of the same size, with the cross wires sharply defined.

Measure the distance from the screen to the centre of the mirror. This distance r is the radius of curvature of the mirror, since the rays are normals.

Compare the values of f and r obtained in the last two experiments. There is a simple relation between them. What is it?

Radius of Curvature and Focal Length.—The relation between these two quantities f and r is shown in fig. 290.

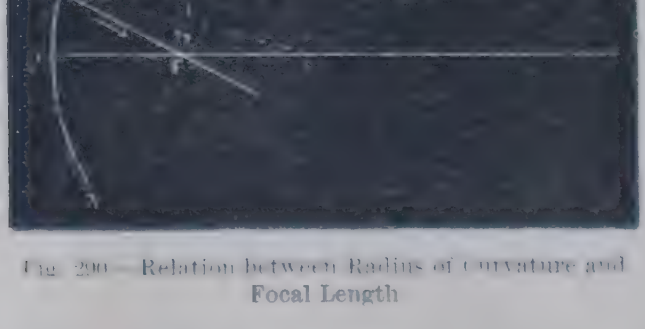

Fig. 290.—Relation between Radius of Curvature and Focal Length

Let RI represent a ray of light parallel to the principal axis. Then by the laws of Art. 260 the angle CIF is equal to the angle RIC, and the reflected ray passes through a point F on the principal axis. But the angle RIC is equal to the angle ICO. Therefore the angles FIC and FCI are equal, and therefore FC is equal to FI. But FI is *very nearly* equal to FO, therefore FO is very nearly equal to FC. If, then, the point I be close to the point O, F is regarded as being midway between O and C. The point F thus determined is the point to which incident rays parall

to the principal axis converge after reflection, that is the principal focus of the mirror.

Thus $2f$ is very nearly equal to r.

Ex. 425.—Find the radius of curvature r of the mirror by means of the spherometer by the method indicated in Art. 20. Compare the value with that obtained in Exercise 424.

270. IMAGES FORMED BY A CONCAVE MIRROR.

Ex. 426.—On an optical bench arrange a bright object, a screen, and a mirror in the manner indicated in fig. 291.

The object may be a candle an electric lamp having the filaments in one

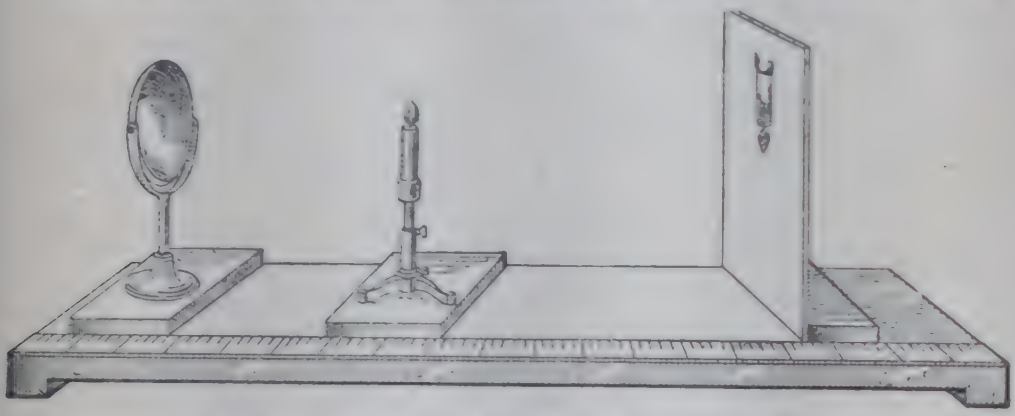

Fig. 291.—Formation of an Image by a Concave Mirror

plane, a gas flame placed so that the breadth of the flame is at right angles to the measured scale; or a window in a blackened lamp chimney (Art. 248).

The mirror must be placed a little obliquely to the line of the bench.

Call the distances—*mirror to object* **u**,

mirror to image **v**.

By trial find several values of u and v which give a clear image of the object on the screen, and other cases when no such image can be got. After this general experience proceed systematically thus:—

(1) Make u less than f, the focal length. Trial will show that in no position of the screen is an image formed on it. Look in the mirror itself. The image is there, virtual, erect, and larger than the object.

(2) Make u as nearly as possible equal to f. A patch of light will be seen on the screen or on the wall beyond it.

(3) Make u a little greater than f. An image is formed, far off, so that v is large.

Slowly increase u: note that the image approaches the mirror, and that it is real, inverted, and larger than the object. When the image comes on the measuring scale, measure u and v, and record them.

Slightly increase u, and again measure u and v. Continue in this way, and take several values of u and v.

The object and screen have been approaching each other. When the distances are such that the candle is in the way of the screen, push the screen back a little

away from the measuring scale, so that it will pass the object. Rotate the mirro on its upright, so that an image is formed on the near edge of the screen. Con tinue the observations as before.

(4) Note particularly the case when $u = v$. Is each of these numbers = f (Exercise 424.)

(5) Make u greater than v, and take three or four values in this stage the experiment.

(6) Employing a suitable object and screen, interchange them, and using th values of u and v obtained above, show that their positions are interchangeabl

Some of the measurements when v was very large and u small, or when u w very large and v small, will be unreliable, because a large difference in one mea but a small difference in the other, and the exact points were not easy to fix.

Treat the middle set of observations as is done in the following example, an make a graph of the values of u and v.

	1	2	3	4	5	6
Values of v	50	40	35	30	25	20
Values of u	21·5	23·5	26	28·8	35·6	55
Values of $\frac{1}{v}$	·020	·025	·029	·033	·04	·05
Values of $\frac{1}{u}$	·047	·043	·038	·035	·028	·018
Values of $\frac{1}{v} + \frac{1}{u}$	·067	·068	·067	·068	·068	·068

The values of v and u are set out in the graph (fig. 292). Fr the origin O draw the line OBA, making an angle of 45° with ea axis. Then the curve is seen to be symmetrical about the line i.e. if the paper be folded across this line the two arms of the cu coincide. While thus doubled, prick a hole through anywhere the curve: the co-ordinates of these points are conjugate foci. T point A on the curve is where $u = v = 29{\cdot}4$. The radius curvature of this mirror was therefore 29·4 cm., and its focal leng $\frac{1}{2}(29{\cdot}4) = 14{\cdot}7$ cm. The focal length of a concave mirror n be accurately obtained in this way.

Referring to the table above it is seen that the values $\frac{1}{u} + \frac{1}{v}$ are (within limits of experimental error) always the sa The mean of these values above is ·0676. The value found f from the graph was 14·7 and $\frac{1}{14{\cdot}7} = {\cdot}068$.

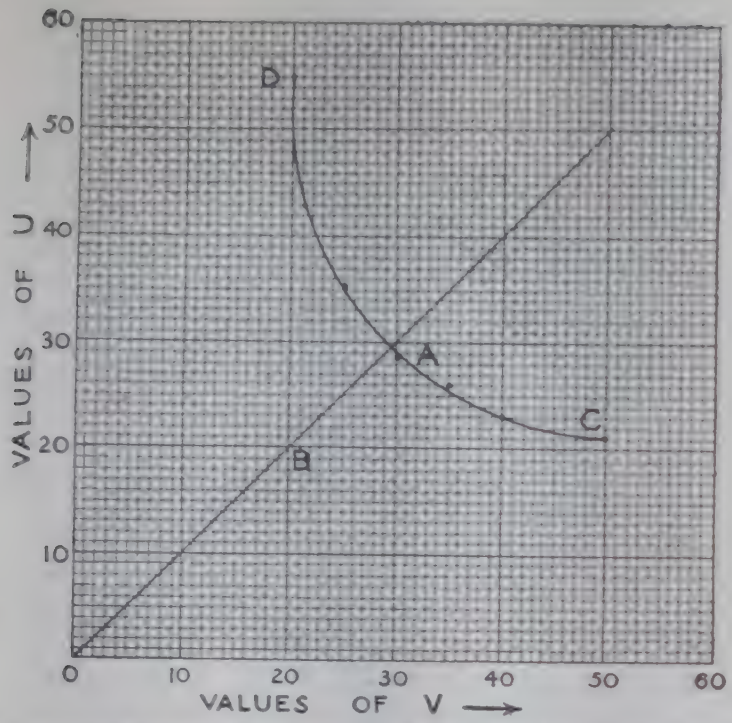

Fig. 292.—Relation between the Distances of Object and Image from the Mirror

Hence it is shown experimentally that the relation between u, f, v, and r is given by the equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{r}.$$

271. SIZE OF THE IMAGE.

Ex. 427.—Using the same arrangement as in the previous exercise, take several observations of u and v, measuring on each occasion the height of the image.

For each observation calculate the ratio $\frac{\text{Distance of object from mirror}}{\text{Distance of image from mirror}}$, and also $\frac{\text{Height of object}}{\text{Height of image}}$. Compare these ratios.

272. SOLUTIONS OF PROBLEMS GRAPHICALLY.—Since rays of light travel in straight lines, and the laws of their reflection are known, problems on reflection from spherical mirrors may often be solved by drawings made to scale.

Thus fig. 290 shows how it is that when rays that are parallel to the principal axis fall on a concave mirror, the reflected rays pass through the principal focus.

By the same method the point may be found to which the rays converge that start from any point on the principal axis.

273. CONJUGATE FOCI.

Ex. 428.—With centre C (fig. 293) describe an arc of a circle MON to represent the section of a concave mirror. Draw its principal axis OP. Bisect OC at F. On the principal axis fix a point P beyond C; draw PQ to the mirror; join QC, make the angle CQP′ = PQC, and thus obtain the point P′ on the principal axis.

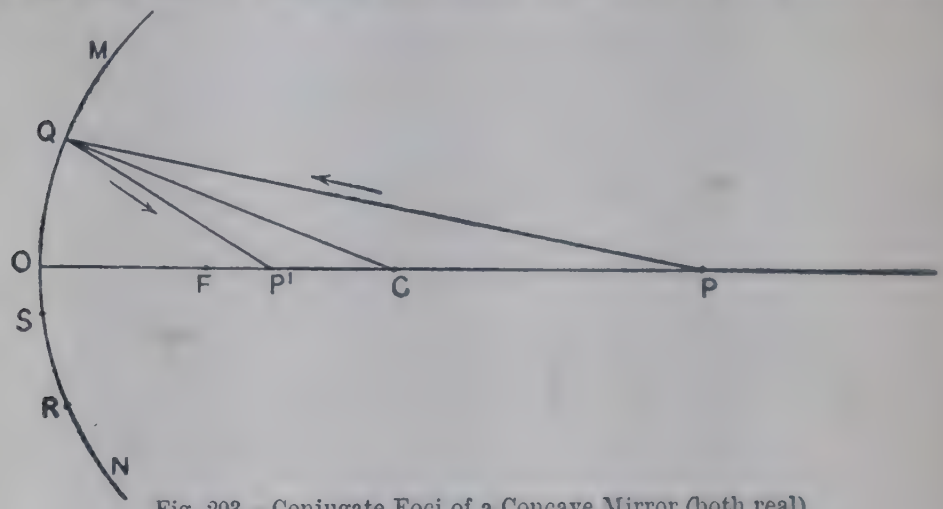

Fig. 293.—Conjugate Foci of a Concave Mirror (both real)

Then light proceeding along PQ after reflection passes through P′.

On the same figure repeat the construction for any rays PS and PR. For the also the reflected rays pass through P′.

Hence the whole beam that starts from P and falls on the mirror is reflected to P′. And conversely if the beam start from P′ it converges to P.

Points thus related are *conjugate foci*. Note that wherever P may be (beyond C), P′ falls between F and C.

Each pair of values of u and v obtained in Exercise 426 were conjugate foci.

Ex. 429.—Repeat the above construction with the point P between F and the apex O (fig. 294). The ray after reflection goes along QP″. Produce this line

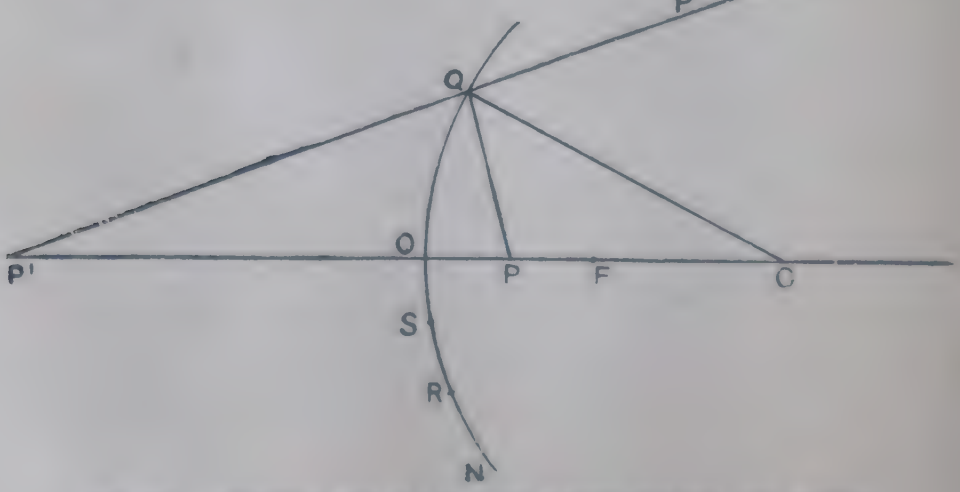

Fig. 294.—Conjugate Foci of a Concave Mirror (one real, one virtual)

(behind the mirror) to P′. The rays PS and PR in the same way proceed *virtually*, not really, to P′, which is the focus conjugate to P.

274. TO FIND BY DRAWING THE IMAGE OF AN OBJECT.—Rays proceed from each *point* of the object to the whole mirror, and are there reflected and brought to a focus at another point. These two points are conjugate foci: at one is a point of the object, at the other is the corresponding point of the image. To find the image of a simple rod we find the focus conjugate to each of its ends and join the two points thus found.

The method of finding the position of an image of a point is based on the following principle:—*Since the rays proceeding from a point* P (fig. 293) *are* all *reflected to another* point P′, *if the point be found where* two *rays from* P *cross after reflection, we know that all the rest will pass through that point, which must therefore be* P′, *the image of* P.

Ex. 430.—With radius CO draw a section MN of a concave mirror (fig. 295), and the principal axis OCS; mark the centre C and the focus F. Draw an arrow AB to represent an object on the side of C remote from O. From A draw two lines representing two rays:—

(1) AI parallel to the principal axis of the mirror. This ray after reflection passes through F along the line IFA′ (Art. 269).

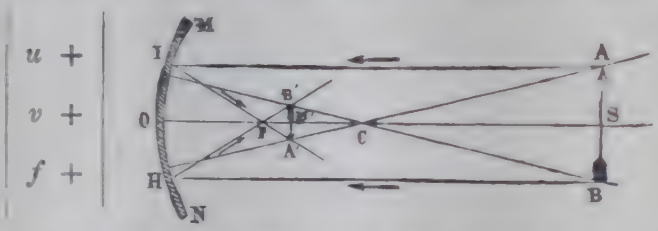

Fig. 295.—Real Image formed by a Concave Mirror

(2) ACA′ passing through C. This ray is reflected from the mirror back along the same line A′CA, because it is normal to the mirror.

Then the point A′ where these lines intersect is the image of A.

A similar construction gives B′ the image of B. Join A′B′; then A′B′ is the image required. The following cases arise:—

(i) When the object (fig. 295) is at a greater distance from the mirror than OC the image is between F and C. It is real, inverted, and smaller than the object.

(ii) When the object is between C and F the image is on the other side of C. It is real, inverted, and larger than the object. In fig. 295, A′B′ now represents the object and AB the image.

Ex. 431.—Repeat the above construction with the object between F and O (fig. 296). The image is then on the other side of O behind the mirror, as in fig. 296. In that figure AB is the object. The ray along AI is parallel to CO, and

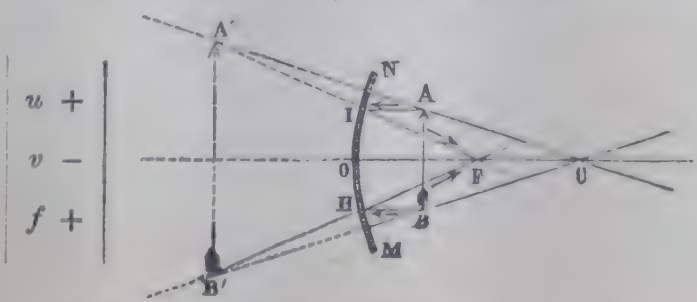

Fig. 296.—Virtual Image formed by a Concave Mirror

therefore after reflection passes through F. The ray AN along the line CN i reflected back to C. Where the *continuation of the lines of direction* of these tw rays meet, i.e. at A′, is the image of A.

A similar construction gives the position of B′, the image of B, and A′B′ is th image of AB. It is virtual, erect, and larger than the object.

It is clear from the similar triangles CAB and CA′B′ that th height of the object and image are in the ratio of their distance from the centre of curvature of the mirror.

275. THE CONVEX MIRROR.—A convex mirror is part o the surface of a sphere as viewed from the outside of the sphere The definitions of Art. 267 apply to convex mirrors.

Ex. 432.—Place a candle in front of a convex mirror. Can an image of th candle be obtained on a screen in front of the mirror?

Look into the mirror. At whatever distance the candle may be placed fro the mirror, the image is always behind the mirror—virtual, erect, diminishe Note the variation in size with distance from the mirror.

Ex. 433.—To find the focal length of a convex mirror.

In front of a convex mirror place a sheet of paper fastened to a drawing boa or block of wood which is placed in such a position that the horizontal sheet paper lies along the pri cipal axis and bisec the mirror, as shown plan in fig. 297. Pla a similar sheet of pap behind the mirror.

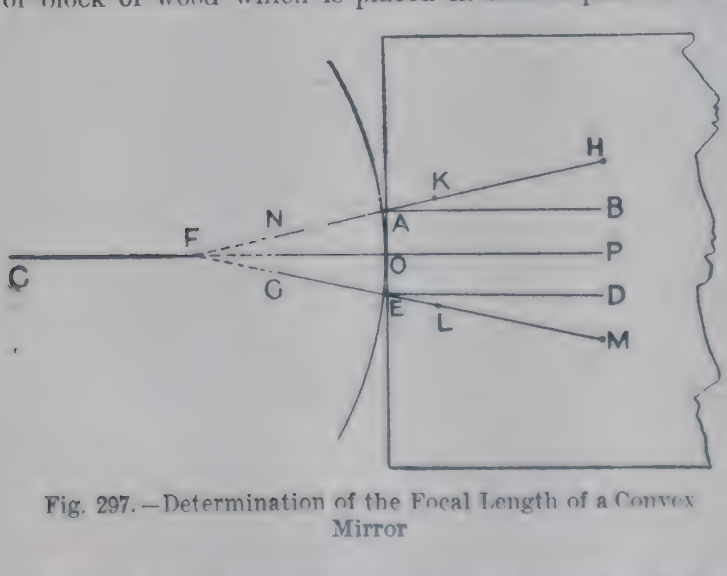

Fig. 297.—Determination of the Focal Length of a Convex Mirror

On the front she of paper rule a line along the principal ax and two other lines A ED parallel to it.

The images NA, of the lines in the mirr converge.

See that the refl tion of OP is in a li with OP. Place straight-edge on the paper, and looking along it into the mirror mark accurat KH the continuation of NA, and LM the continuation of GE.

Remove the mirror, and produce HK and LM to meet. They will be found meet on the principal axis. The point F thus found behind the mirror is t principal focus of the mirror, and FO is the focal length (f).

276. RADIUS OF CURVATURE.

Ex. 434.—Set up the apparatus exactly as for the last experiment. Dra line representing the principal axis. Its reflection is in the same straight line

Place a straight-edge inclined to the axis, as KH in fig. 297, and adjust it until the straight-edge and its reflection are in a straight line. Draw the line.

Repeat the operation on the other side of the axis, as at LM.

Then the three lines on the paper are all normals to the surface. Remove the mirror. Produce the lines till they meet at the centre of curvature, and measure the radius of curvature (r).

Compare the values of the radius of curvature and focal length thus found. As before, $r = 2f$.

Ex. 434*a*.—Make one or two measurements of u and v for a convex mirror by the parallax method (Exercise 415) using a candle as the object, and a knitting needle in place of the pin D in fig. 282.

277. TO SHOW GRAPHICALLY THE FORMATION OF IMAGES BY A CONVEX MIRROR.

Ex. 435.—Draw a section of a convex mirror (fig. 298). Mark its principal axis, its focus (F), and centre of curvature (C). Let AB represent the object.

u +
v −
f −

Fig. 298.—Image formed by a Convex Mirror

Draw AI representing a ray parallel to the principal axis. This ray is reflected so that the *continuation of its line of direction* passes through F. Draw AC representing a ray proceeding towards the centre of curvature. This ray is normal to the mirror, and returns from the mirror along the same line through A. The *continuation of its line of direction* cuts IF in A′. Then A′ is the image of A. A similar construction gives B′ the image of B.

Repeat the exercise with AB in various positions. As AB is moved from a position close to the mirror to a position infinitely distant from it, A′B′ moves from close to the mirror towards the focus F, growing continually smaller as it approaches F. There is only one case to consider here. The image is always virtual, erect, and smaller than the object.

The figure shows that the relation between the sizes (heights) of the object and image is the same as the relation between their distances from C the centre of curvature.

278. POSITIVE AND NEGATIVE VALUES OF u, v and f.

In applying the formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

to all the different cases determined in the previous exercises, it is necessary to take note of the direction in which the light is travelling and to call one direction + and the opposite −. *This*

tances measured in the direction from the mirror towards the object are $+$: *distances measured in the opposite direction are* $-$. Thus we have

u: always $+$;
f: $+$ for concave mirror,
$-$ for convex mirror;
v: $+$ for real image,
$-$ for virtual image.

The sizes of image and object are always positive.

Examples.—(1) A square placed 18 cm. in front of a concave mirror has a virtual image of four times its area. Find the radius of the mirror.

Here we have $u = 18$ cm. Since the image is twice the height of the object and is virtual, $v = -36$ cm.

$$\text{Then } \frac{1}{18} + \frac{1}{-36} = \frac{2}{r},$$
$$\text{i.e. } \frac{1}{18} - \frac{1}{36} = \frac{2}{r},$$
$$\text{and } r = 72 \text{ cm.}$$

(2) Find the position and size of the image of a rod 5 cm. high placed 16 cm. in front of a convex mirror whose focal length is 12 cm.

$$\text{Here we have } u = 16 \text{ cm.}; f = -12;$$
$$\therefore \frac{1}{16} + \frac{1}{v} = \frac{1}{-12}.$$
$$\therefore \frac{1}{v} = -\frac{1}{12} - \frac{1}{16} = -\frac{7}{48},$$
$$v = -6\tfrac{6}{7} \text{ cm.}$$

The negative sign shows that the image is virtual.

$$\text{Also } \frac{\text{Height of image}}{5} = \frac{6\frac{6}{7}}{16} = \frac{48}{112},$$
$$\therefore \text{Height of image} = 2{\cdot}1 \text{ cm.}$$

Ex. 436.—Miscellaneous Exercises. (1) Draw the section of a concave mirror; place an arrow in front of it, and show by a diagram how an image of the arrow is formed by the mirror.

(2) Show by diagrams how the above-mentioned image changes when the distance of the arrow from the mirror is changed.

(3) Give a diagram showing how the image of a rod is formed in a polished metal ball, and explain the construction of the figure.

(4) What is the difference between a virtual and a real image? Enumerate in separate lists all the cases of their formation by reflection.

(5) The focal length of a concave mirror is 5 in. What is the relative height of the image of a small object placed 15 in. from the mirror?

(6) The image of a candle placed 20 cm. in front of a concave mirror is seen distinctly on a screen placed 50 cm. from the mirror. What is the focal length of the mirror?

(7) A candle is placed 3 ft. from a concave mirror, and a distinct image is obtained on a screen placed 8 ft. from the candle. The candle is then moved

6 in. farther from the mirror. Where must the screen be placed to show a distinct image?

(8) The focal length of a concave mirror is 15 cm. Find the nature, position, and magnitude of an object placed (*a*) 20 cm. from the mirror, (*b*) 10 cm. from the mirror.

(9) The radius of a concave mirror is 6 ft., and a circular disk 1 in. in diameter is placed on the axis of the mirror, at a distance of 2 ft. from it. Determine the size and position of the image.

CHAPTER XXXIII

Refraction

279. BENDING OF LIGHT RAYS.—In Chapter XXIX it was seen that rays of light travel in straight lines in air and through space; and when they fall on certain substances they are reflected and proceed along another straight line. We have now to see what happens when the rays pass into a transparent substance such as water or glass.

Ex. 437.—(1) Hold a beaker nearly full of water between the eye and a thin vertical rod or string. Move the beaker slightly from side to side. Note the appearance of the rod seen through the water. Tilt the beaker a little: what appearance does the rod present? Draw these appearances.

(2) Place the beaker of water on the edge of a sheet of white paper. Look along the edge of the paper, and move the head slightly from side to side. Note the appearances of the edge of the paper seen through the beaker.

(3) Replace the beaker of water by a rectangular bottle containing water or by a block of glass, and make similar observations.

Ex. 438.—Place a lighted candle behind a rectangular bottle filled with water. Look at the flame through the water. Rotate the bottle into various positions. Note what changes appear to take place in the position of the candle flame.

If a part of the candle itself can be seen below the bottle, note its position with regard to the part seen through the bottle.

Ex. 439.—Hold a rod with one end dipping some distance into water, as in fig. 299, and

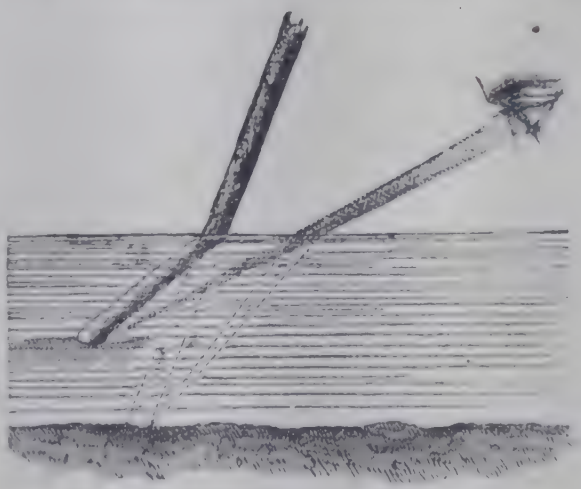

Fig. 299.—Appearance of a Rod immersed obliquely in Water

compare the immersed part with the remainder, viewing it from several different positions.

Change the position of the rod, and repeat the observations, noting the direction and apparent size of the immersed part.

Find a position in which the whole rod appears straight.

Ex. 440.—Hold a strip of wood, such as a flat broad measuring scale, end on and perpendicular to a mirror of thick glass silvered at the back, or a thick piece of ordinary glass having dark paper at the back.

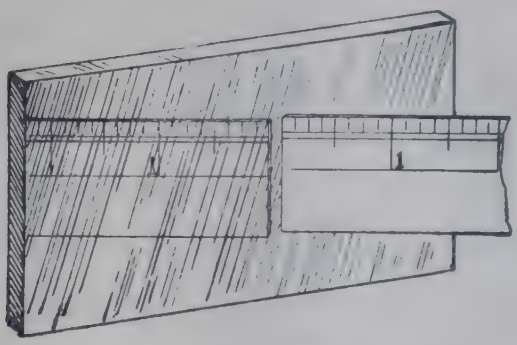

Fig. 300.—Image of a Scale in a Mirror viewed Obliquely

Placing your head about half a metre from the scale, and about 10 or 15 cm. from the plane of the mirror, look towards the gap between the scale and its image.

The image of the scale seems nearer to you than the scale itself.

View the gap with the right and left eye alternately. The scale appears to move.

These observations show that when an object is viewed in certain positions through glass or water the object appears to be displaced. (The complicated effects observed with the beaker are partly due to images formed by reflections from its curved surface.)

The next experiment shows the cause of this displacement.

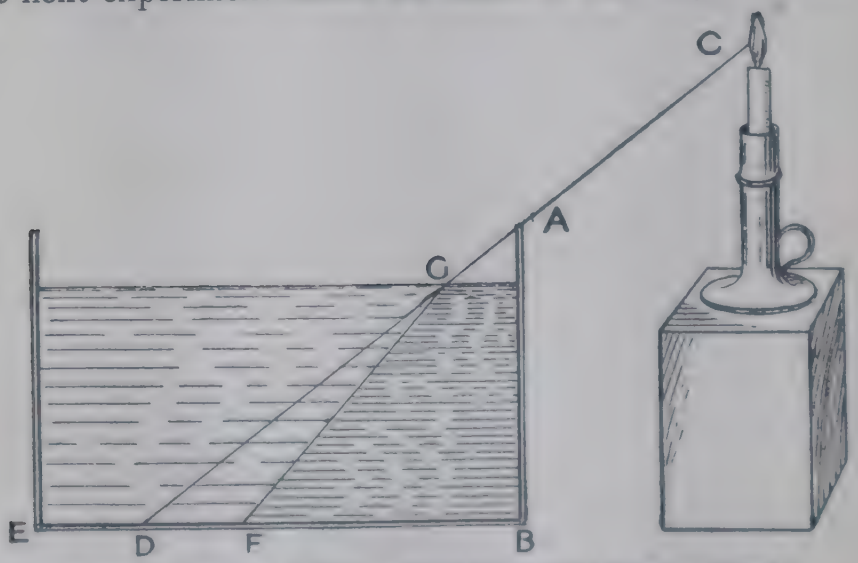

Fig. 301. Bending of Oblique Light Rays at the Surface of Water

Ex. 441.—Take a rectangular vessel that will hold water, such as a small oblong biscuit tin. If the vessel be of glass, paste a piece of black paper over one end AB (fig. 301). Fit a piece of white paper in the bottom of the vessel, and put a weight on each end of it

Let the light of the sun or of a candle stream obliquely over the opaque end of the vessel along a line CD, so that the part DE is light and BD in shadow. Gently pour water into the vessel. Note the movement of the line of shadow to a position F as the water is poured in.

The displacement of objects noticed in Exercises 437 to 440 is thus shown to be due to the fact that when light passes obliquely from air into water or glass its direction is changed. The rays no longer proceed forward in a straight line, but undergo change of direction or deviation. This process is called **refraction**, and is produced by most transparent substances.

Any material through which light passes is called a **medium**. Hence we may make the general statement that *when light passes in an oblique direction from one medium to another it is refracted.*

Refraction takes place in a regular way.

280. INDEX OF REFRACTION.

Ex. 442.—(1) On a sheet of glass, tin, or wood mark clearly the circumference of a large circle with two diameters at right angles to each other, AB and CD (fig. 302), and a radius EF making an angle of about 40° with ED.

Place the circle in a vessel of water in such a position that AB is along the surface of the water.

From above, look straight along the line EF, and mark on the circumference the point G, which appears to be exactly in the continuation of the line FE.

Remove the circle from the water, and measure the perpendicular distances GH and FK on to the line CED, which was the normal to the surface of the water.

Find the value of the ratio $\frac{GH}{FK}$.

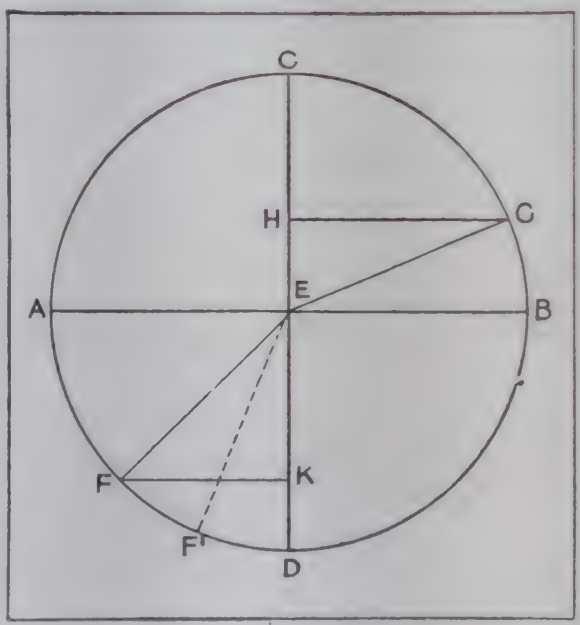

Fig. 302.—Construction for the Index of Refraction

(2) Repeat the experiment with a different radius EF′ (fig. 302), and thus find another value for the above ratio. Compare the values obtained.

(If the disk be of wood, pieces of thin string fastened by drawing pins may be used for the radii EF and EG.)

It is found that whatever radius EF of the circle is used, provided that GE comes above the surface of the water, the ratio $\frac{GH}{FK}$

is always the same for the same substance. It is called the *index of refraction*. For water its value is 1·33. Each substance has a different index of refraction.

Fig. 302 enables us to arrive at a more convenient expression for the index of refraction. The lines GE and EF are called the incident and refracted rays, and the angles GEH and FEK the angles of incidence and of refraction.

In Art. 26 was shown the meaning of the term sine of an angle.

The angle GEH in fig. 302 is the angle of incidence, and $\frac{GH}{GE}$ is its sine: the angle FEK is the angle of refraction, and $\frac{FK}{FE}$ is its sine.

Since FE = GE, $\therefore \frac{GH}{GE} \div \frac{FK}{FE} = \frac{GH}{FK}$.

Hence *the index of refraction is the ratio of the sine of the angle of incidence to the sine of the angle of refraction.*

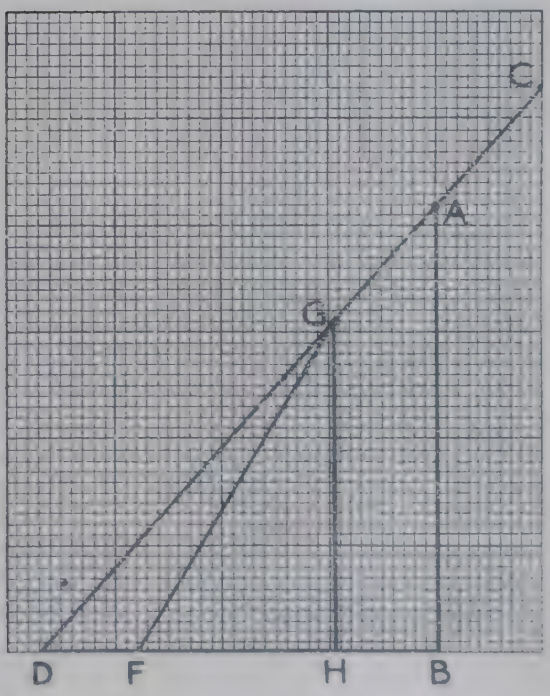

Fig. 303.—Measurement of the Index of Refraction of Water

Ex. 443.—Measure the index of refraction of water by the method of Exercise 441. Cover the bottom of the vessel with a white surface. Place the vessel so that the shadow line falls at D (fig. 301), and with dividers measure its distance from B.

Pour in water until the vessel is about three-fourths full. Measure the distance of the new shadow line F from B; the depth of the water; and the depth of the side AB of the vessel.

Draw the right-hand side of fig. 301 to scale as in fig. 303. Then G is the point where the ray CA strikes the surface of the water. AG is the incident ray, GF the refracted ray, and GH the depth of the water.

HGD is the angle of incidence, and its sine is $\frac{HD}{DG}$.

HFG is the angle of refraction, and its sine is $\frac{HF}{FG}$,

and the index of refraction is $\frac{HD}{DG} \div \frac{HF}{FG}$.

The lengths of all the lines are known, since the figure is drawn to scale.

Example.—In an experiment AB was 8·3 cm., BD 7·36 cm., BF 5·6 cm., and

GH 6·1 cm. These numbers on a scale of one-half give fig. 303; and by measuring the lines we obtain—

$$\frac{HD}{DG} \div \frac{HF}{FG} = \frac{5{\cdot}3}{8} \div \frac{3{\cdot}6}{7{\cdot}1} = \frac{{\cdot}662}{{\cdot}507} = 1{\cdot}3.$$

It is important to notice in which direction the light is deviated. The general rule is, as in the above experiments, that when the light passes from a rarer into a denser medium the deviation is towards the normal: when the light passes from the denser to the rarer medium it is deviated away from the normal.

Ex. 444.—On a sheet of paper rule two thin clear parallel lines about a centimetre apart.

On them place a slab of glass in the position shown in fig. 304, as viewed from the top.

Look horizontally along the lines AC, BD from the ends A, B; see that the lines appear straight.

Turn the block into the position shown in fig. 305, as viewed from the top. Again look horizontally along the lines AC, BD; note that the lines appear broken. Vary the angle which the faces of the block make with AC, BD, and note the changes in the apparent displacement of these lines.

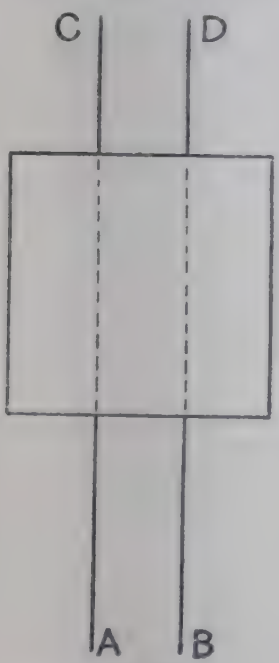

Fig. 304

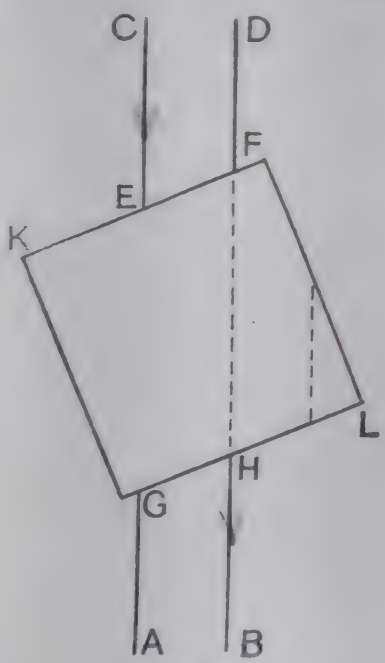

Fig. 305

Parallel Lines viewed through a Block of Glass Perpendicularly and Obliquely

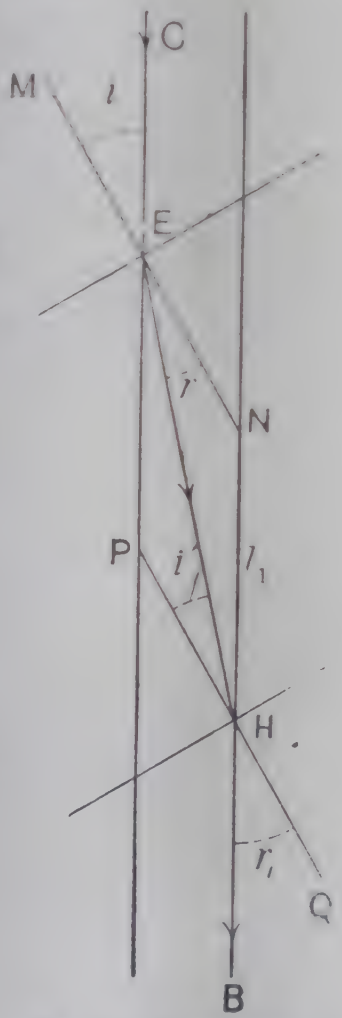

Fig. 306.—Measurement of the Index of Refraction of a Block of Glass

Find the position of the block in which CE (fig. 305) appears exactly in line with HB; then mark accurately on the paper the lines FK, LG.

Remove the slab, and drawing very accurately develop fig. 306, where CEHB

is the path of the ray that came to the eye. Draw the normals MEN, PHQ. Measure carefully with a protractor the angles of incidence (i) and refraction (r); obtain their sines either from a set of tables or from the methods of Art. 26, and thus determine the index of refraction of the glass.

Measure the angles i_1 and r_1 (fig. 306), and find the index of refraction of a ray passing from glass to air.

Note that HB is parallel to CE: the direction of the light is unchanged by its passage through the slab.

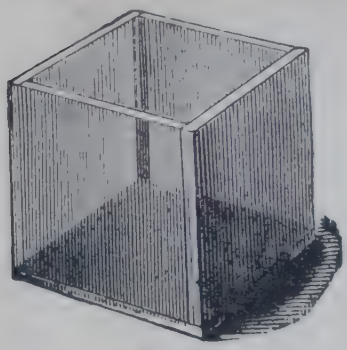

Fig. 307

281. The refractive index of liquids may be found in precisely the same manner as that of a slab or block of glass by placing the liquid in a hollow glass cube or bottle having parallel sides (such as those in which scents are sometimes sold).

282. APPARENT DISPLACEMENT OF OBJECTS BY REFRACTION.—It is clear that when light proceeding from an object to the eye has been refracted the object is not seen in its true position. It appears displaced because the eye is looking along the refracted ray, and really sees an image.

Fig. 308 Image of an Object seen obliquely through Glass

Fig. 308 shows how it is that an eye looking at an object S obliquely through a glass plate or a layer of any transparent material sees an image at S′ raised above the object. This illustrates why water is always deeper than it appears to be.

Ex. 445.—From the hint given by fig. 308 make a drawing showing the position in which a fish in a river sees the head of a person standing on the bank.

Ex. 446.—Place a coin in a basin, and the eye in such a position that the coin is just hidden by the side of the vessel. Pour water into the vessel: the coin comes into view. Make a drawing showing the paths of the rays of light, and where the eye sees the coin.

283. LAWS OF REFRACTION.—The main facts of refraction are summed up in the two following statements:—

1. *When a beam of light passes obliquely from one medium to another*

it undergoes refraction, and the incident ray, the refracted ray, and the normal at the point of incidence are in the same plane.

2. *For the same two media the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction: this ratio is the* ***refractive index.***

284. REFRACTION THROUGH A PRISM. — A prism in optics always means a triangular prism (fig. 309). The base CDEF is not used: the edge AB parallel to the base is the refracting edge.

Fig 309. An Optical Prism

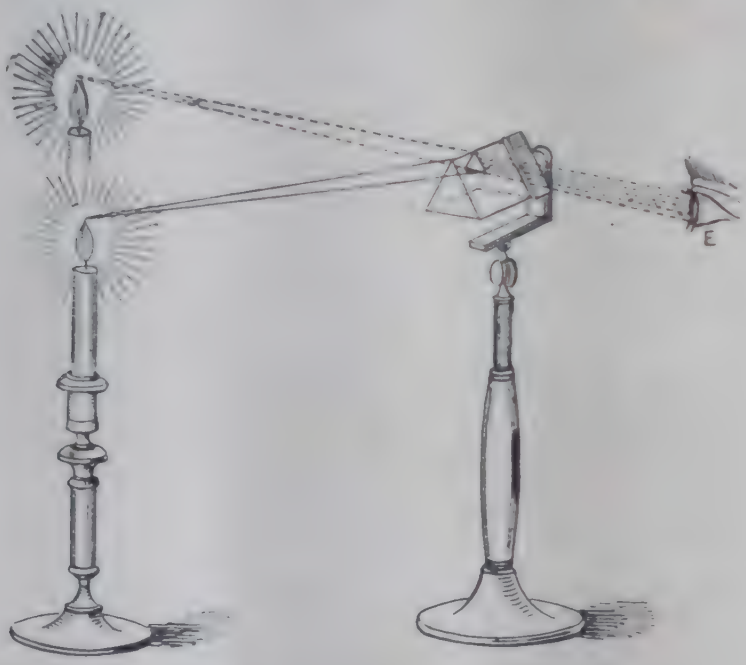

Fig. 310.—Image of a Candle viewed through a Prism

Ex. 447.—Hold a prism between the eye and a candle, with its refracting edge horizontal (fig. 310). The candle flame appears to be displaced because the light proceeding from it has undergone deviation. It entered one face of the prism along the lines shown continuous in the figure, and emerged from the other face to the eye along the broken lines. The eye sees the object along the continuation of these (broken) lines.

Rotate the prism slightly in both directions. Note the movement of the image. Does the deviation of the light depend on the position of the prism?

285. TO TRACE THE PATH OF A RAY OF LIGHT THROUGH A PRISM.

Ex. 448.—(1) Place a prism on a sheet of paper with its refracting edge vertical, and with a fine pencil trace its outline on the paper. On or close to this line place two vertical pins N and N′ (fig. 311).

Look, with one eye only, at the prism in the direction RN, and find a point R

(as far off from N as is convenient) such that a pin at R appears to be in a straight line with N and N′, when viewed along the line RN.

Look at the other face of the prism, and find a point R′ such that R′N′N appear to be in a straight line.

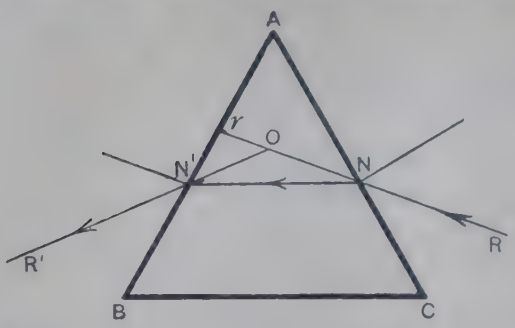

Fig. 311.—Path of Light Rays through a Prism

Then neglecting the coloured fringes, the line RNN′R appears to be straight whichever end it is viewed from.

Remove the prism. Draw the lines RN, NN′, N′R′ through the pinholes. Then this line is the path of a ray of light through the prism.

Produce RN to r and R′N′ to meet this line in O. Then light that started from R towards r has been bent round into the direction OR′. The angle rOR′ therefore measures the deviation. Measure the angle rOR′.

(2) Repeat the experiment, having first arranged that the pins N and N′ are at equal distances from the refracting edge A (fig. 311).

The deviation is now as small as it can be. The line NN′ is parallel to the base of the prism. This path of a ray is the path of *minimum deviation*.

Ex. 449.—Place two prisms ABC, MBC (fig. 312) with their bases in contact along a line DG drawn on a sheet of paper.

By means of pins trace out the line DEFG of a ray of light through one prism, and the ray DHKG through the other prism, BC, EF, and HK being parallel.

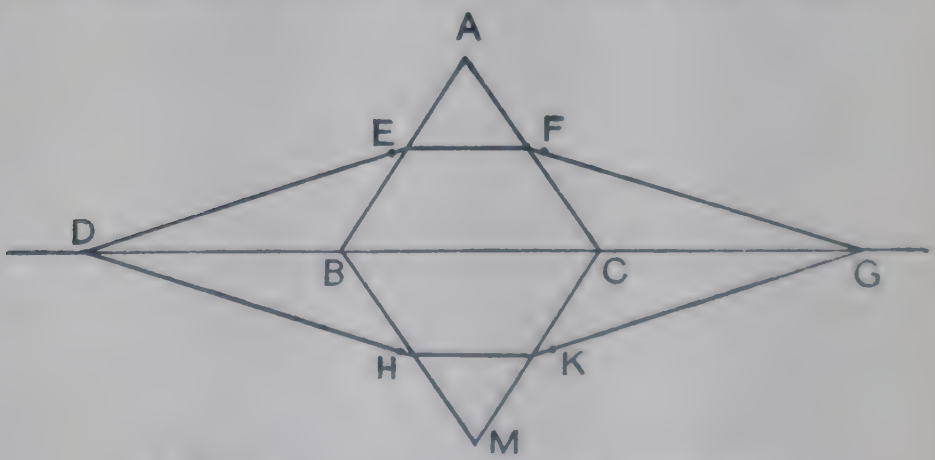

Fig. 312.—Convergence of Light Rays by a Double Prism

Note that the diverging beam of light DEH is converted into a converging beam FKG. If the prisms are exactly alike, G will be in the same position for both prisms.

286. MECHANICAL ILLUSTRATION OF REFRACTION.

Ex. 450.—Fix with drawing pins or glue on a smooth board a rectangular piece of plush or imitation sealskin or similar material.

On the board place a light dumb-bell, and tilt the board a little so that the dumb-bell slowly rolls down the plane on to the plush.

(1) Let the dumb-bell roll straight down the board so that it travels straight across the plush. The speed of its motion is diminished by the pile.

(2) Let the dumb-bell roll down the board obliquely, so that one end reaches the pile before the other. The dumb-bell swings round and changes the direction of its motion. On leaving the board it changes direction again, going on parallel to its original direction.

As this illustration suggests, the cause of refraction is that the light experiences more difficulty in passing through glass or water than through air, and therefore travels slower. An oblique pencil of rays is swung round into a different direction, because one side of it enters the denser medium before the other.

CHAPTER XXXIV

Lenses

287. In this chapter we have to deal with images formed by rays of light that have undergone refraction.

Ex. 451.—(1) Fill a flask with water, and hold it so that the light of the sun or from a lamp falls on the flask.

On the side of the flask opposite to the source of light place a screen. Move the screen nearer to and farther from the flask.

What has the flask of water done to the light rays that fell on it?

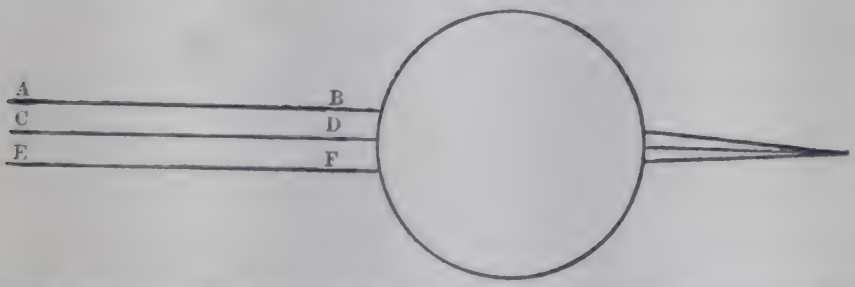

Fig. 313.—Convergence of Parallel Rays by passage through a Transparent Sphere

(2) Rule three thick parallel straight lines AB, CD, EF (fig. 313) on a piece of cardboard. Cut a hole in the cardboard just big enough to let the flask go through. Put the flask on the table, and support the cardboard firmly at the level of the centre of the flask.

Look through the flask from the other side. How do the lines appear?

Place a flat ruler with its edge in a continuation of the centre line, and when it is quite correct draw that line. Do this for each of the other lines (fig. 313).

In its passage through the sphere of water the light from the lines has been **refracted**, so that the refracted rays pass nearly through one point.

288. LENSES.—A lens is a portion of some refracting substance, generally glass, that has one or both of the opposite faces portions of surfaces of spheres, as was the case with the flask in Exercise 451.

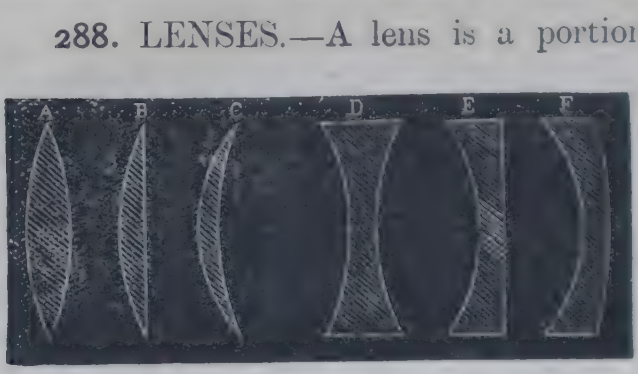

Fig. 314.—Forms of Lenses

Fig. 314 shows sections of a set of lenses.

Some of the experiments are best made with half lenses.

Ex. 452.—(1) Examine forms of lenses. Hold a straight-edge across the middle of each face and examine its reflection. Those which are thicker in the middle than at the edges (A, B, C above) are called **convex** lenses; those which are thinnest in the middle (D, E, F) **concave** lenses.

(2) Hold the lens near the eye and look at some small object or the page of a book on the table. Move the lens to the left and right. Note that with a convex lens the object looked at appears to move in the opposite direction to that in which the lens is moved; with a concave lens the object appears to move in the same direction as the lens.

If no such apparent movement of the object occurs, the glass is plane and not a lens at all.

The **optical centre** of a lens is the middle point of its substance. The thickness of lenses is neglected in elementary work and thus any ray passing through the centre of a lens is considered as passing straight on with its direction unchanged.

The **principal axis** of a lens is the line joining the centres of the two bounding surfaces. In what follows it is supposed that points of light are on this axis, and that objects are bisected by it.

The terms **real** and **virtual** have the same meaning for lenses as for mirrors (see Art. 261).

289. THE DOUBLE-CONVEX LENS.—Many of the experiments with a double-convex lens are similar to those made with a concave mirror.

Ex. 453.—Principal Focus. (1) Place the lens between the sun and a screen, and obtain on the screen a clear image of the sun's disk. When the screen is at such a distance from the lens that the image is the smallest possible, this distance is the **focal length** of the lens, and the image is at the **principal focus** of the lens.

Measure the focal length. We shall always call it f.

If the sun is not available a globe lamp or flame at a distance of several metres may be used instead.

(2) In the path of the light place a glass vessel, such as a bottle with parallel sides, which is filled with water in which a very small amount of soap has been dissolved. The cone of light formed by the lens will be then clearly seen. Move the lens about so that different parts of the cone become visible.

The experiment shows that the light that passed through the lens took the form of a cone of which the base was the lens and the apex the image, as in fig. 313.

Ex. 454.—Place a lamp having a translucent globe close behind a small hole in a screen, and place the lens on the other side of the screen and parallel to it. Make the distance from the hole to the lens = f. Then the patch of light formed on the wall of the room beyond will be of about the same size as the lens.

Thus it is shown that parallel rays falling on a convex lens converge to the principal focus; and rays starting from the focus emerge from the lens as a parallel beam.

290. FOCAL LENGTH AND POWER OF A CONVEX LENS.

Ex. 455.—Hold two or more lenses opposite a window so as to get a clear image of the bars of the window on an opposite wall.

Measure the distance from the lens to the wall. This will not be the focal length, but the relative magnitudes of the focal lengths will not be very different from the relative distances thus measured.

Place the lenses in the order thus obtained, and examine the convexity of their surfaces. What is the connection between the convexity and the focal length?

The greater the convexity of the surface of a lens the smaller is its focal length and the greater is its *power*.

What connection did you observe between the focal length and the size of the images formed by the lenses?

Ex. 456. — To find the focal length and power of a convex lens.

Referring to fig. 315, on a sheet of paper rule 3 or 5 parallel lines ss_1. Across these at right angles place a half lens; or cut a hole through the paper and support the lens halfway through the

Fig. 315. Convergence of Parallel Rays to a Focus by passage through a Convex Lens

hole. Let the lines fall on the middle part of the lens. Look at the parallel lin from the other side F of the lens, with the eye nearly on a level with the paper.

With a straight-edge draw on the paper the apparent continuations of th lines. It is found that they converge to the same point F.

Measure the distance of F from the centre of the lens. This is f, the foc length. Expressing f in metres, find the value of $\frac{1}{f}$. This is the power of th lens measured in **dioptres**.

View the system of lines from the side ss′ of the lens. The lines whi diverge from the point F appear to be parallel.

The lines are the paths of rays of light which travel along them to the lens.

Ex. 457.—Draw 3 or 5 convergent lines, as shown on the right-hand side fig. 316, and looking through the lens from the other side s in the same mann as in the previous e periment, mark th apparent continuatio and find their point of convergence. T rays of light in th passage through the l have been made m convergent. Meas the distance of s fr the centre of the l Is this distance grea or less than f? V the system of li

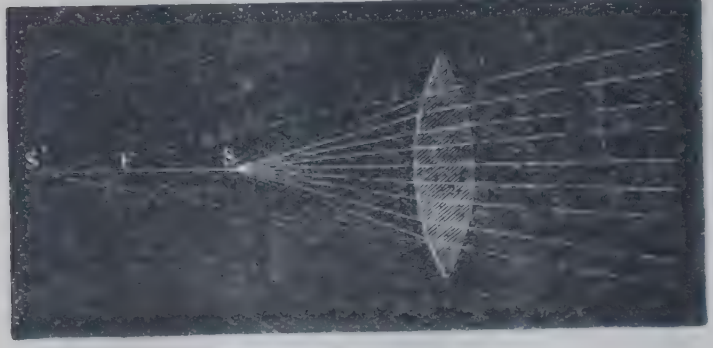

Fig. 316.—Virtual Focus

from the other side of the lens. Again they appear straight.

Ex. 458.—Draw 3 or 5 divergent lines, i.e. lines which are wider apart c to the lens than they are farther away from it; and in the same manner m out their apparent continuations on the other side of the lens. Probably th continuation lines do not converge to a point, but in any case their diverge is less than that of the lines whose continuations they appear to be.

291. CONJUGATE FOCI OF A LENS.

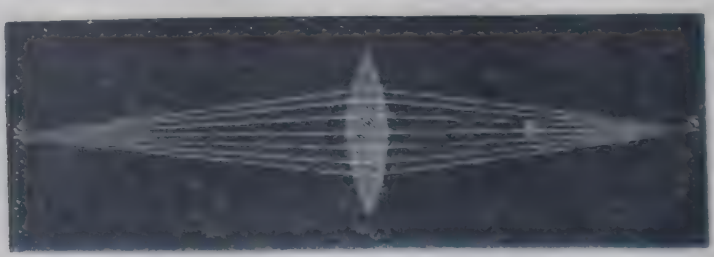

Fig. 317.—Conjugate Foci of a Convex Lens

Ex. 459.—Draw a long line on a sheet of paper, and on the line place a of a convex lens of short focal length, so that the line is its principal axis. M the position of the lens and of its foci F (fig. 217). Fix a point s′ on principal axis between F and the lens. From s′ draw 3 or 5 divergent lines to the lens

By the method of Exercise 456 find the point s to which the rays proceeding from s′ converge.

View the lines from the other side of the lens: the effect is the same.

Thus light proceeding from either one of the points s or s_1, after passing through the lens converges to the other point. Such a pair of points are called **conjugate foci**.

If an object be placed at s its image is formed at s_1: if the object be at s_1 its image is at s.

292. FORMATION OF IMAGES BY A LENS.

Ex. 460.—Place on the optical bench a bright object (see Exercise 426), a lens, and a screen, as in fig. 318. Keep the object fixed throughout the experiment.

Call the distances—*lens to object* **u**,
lens to image **v**.

(1) Find by trial which positions of the lens and screen give an image on the screen, and in which positions no image can be obtained. Note in each case

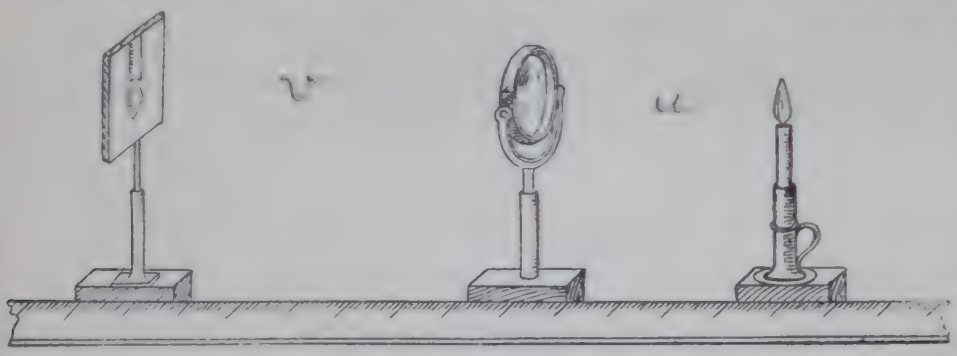

Fig. 318.—Image formed by a Convex Lens

whether the image is real or virtual, erect or inverted, magnified or diminished. Then proceed as follows:—

(2) Put the lens close to the object, and move the screen into various positions. If the distance *u* is too small, no image can be got on the screen, but a patch of light is seen on the wall of the room beyond the screen.

Remove the screen for a moment, and note the character of the image on the wall. Gradually increase the distance *u* during the remainder of the experiment, adjusting the screen as required.

(3) When the distance *u* is large enough to allow an image to be formed on the screen in its most distant measurable position, get a clear image, and read the values of *u* and *v*. Compare the state of affairs with fig. 320.

Slightly increase *u*, and take another pair of values of *u* and *v*.

Continue to increase *u* by short distances, and take readings of *u* and *v* until the screen is off the measured scale.

(4) Record the readings obtained; treat them as in the following example; and make a graph for *u*, *v*.

Example.—(The measurements are in centimetres).

Values of v	41·5	33·5	30	27	25	23·5	22	21
Values of u	18	20	22	24	26	28	30	32
Values of $\frac{1}{v} + \frac{1}{u}$ ...	·0797	·0798	·0788	·0787	·0785	·0783	·0788	·0789

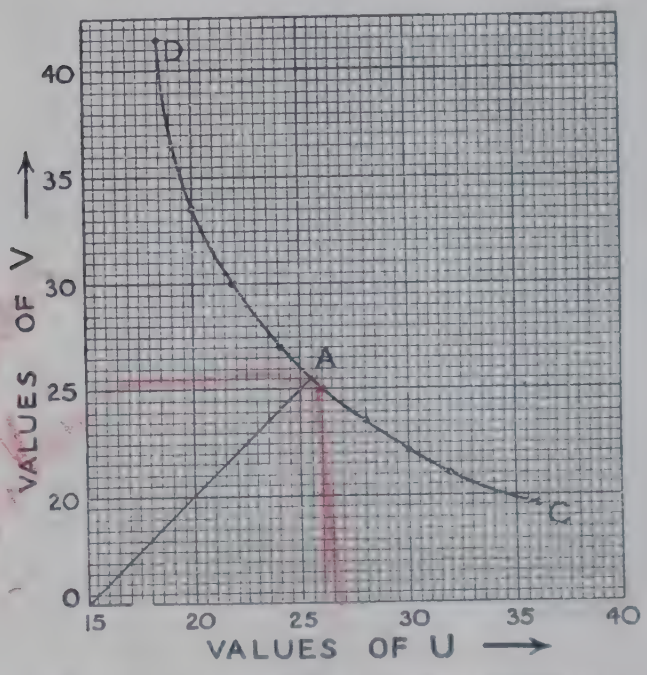

Fig 319. Curve showing the Relation between the Positions of Object and Image in a Convex Lens

The values of $\frac{1}{v} + \frac{1}{u}$ fo
each pair of readings is nearl
constant. The mean is ·078
the reciprocal of which num
ber is 12·67.

The values of v and
are set out in the graph (fi
319). The curve is exact
like that of fig. 292, a
what is said about that cur
applies to this. The cur
gives $u = v = 25·5 =$ t
radius of curvature; t
focal length is therefo
12·75, which is very near
the same as the 12·67 o
tained above by calculatio
Hence for this case—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{r}.$$

For the general rule
signs, see Art. 301.

293. SIZE OF IMAGE.

Ex. 461. Using the same apparatus as in the previous exercise, take a ser
of observations of u and v, measuring on each occasion the height of the imag
and thus obtain several values of the ratios $\frac{\text{Distance of object from lens}}{\text{Distance of image from lens}}$ a
$\frac{\text{Height of object}}{\text{Height of image}}$. Take the mean of each set, and compare the ratios.

The sizes of object and image are in the ratio of their distances from the le

294. GRAPHIC CONSTRUCTION SHOWING THE FO MATION OF IMAGES.

Ex. 462. Referring to fig. 320, draw a section of a lens A′ and its princi
axis. Draw AB representing the object, and mark F the principal focus. Fr
A draw two rays.

(1) The ray AA' parallel to the principal axis. This ray is refracted by the lens so as to pass through F.

(2) The ray AO through the optical centre of the lens. This ray passes on with its direction unchanged.

These two rays intersect at *a*, and at that point accordingly the image of A is

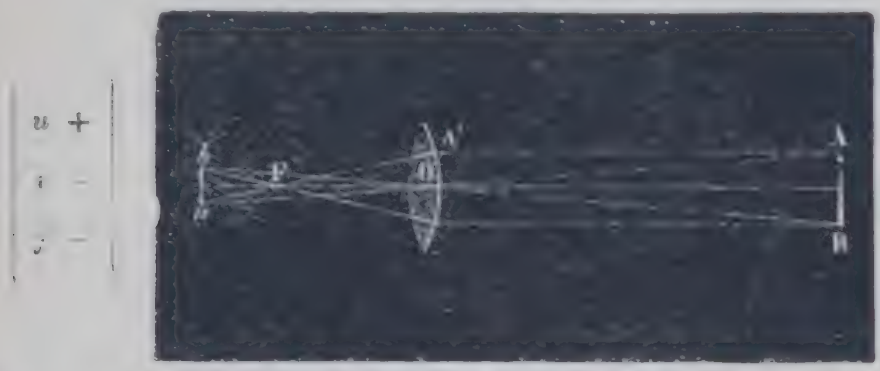

Fig. 320.—Graphic Construction for the Image of an Object beyond the Focal Length

formed. A similar construction gives the point *b* as the image of B. The line *ab* is, therefore, the image of the line AB. If *ab* were the object, AB would be its image.

Two cases arise, shown in figs. 320, 321. Make a drawing for each.

(*a*) When the object is at a distance from the lens greater than the focal length (fig. 320), the image is real and inverted. Show geometrically that the

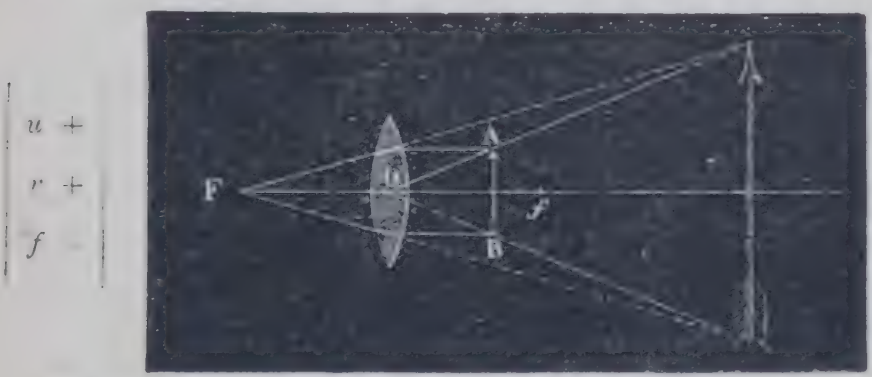

Fig. 321. Construction when the Object is within the Focal Length

relative sizes of object and image are in the ratio of their relative distances from the lens (Art. 293).

(*b*) When the object is within the focal length of the lens, as in fig. 321, if the paths of the two rays proceeding from A be drawn as before, it will be found that the rays do not intersect on the side of the lens remote from A. The *continuations of their lines of direction* intersect on the same side as A at a greater distance from the lens. The image is virtual, erect, and larger than the object.

295. THE CONCAVE LENS.—The concave lens leads to results like those produced by a convex mirror.

Ex. 463.—To find the focal length of a concave lens.

Rule parallel lines on a sheet of paper, and across them place a half le or a whole lens in the manner described in Exercise 456, and shown in sectio in fig. 322. Viewed from the point D, the parallel lines appear to conver

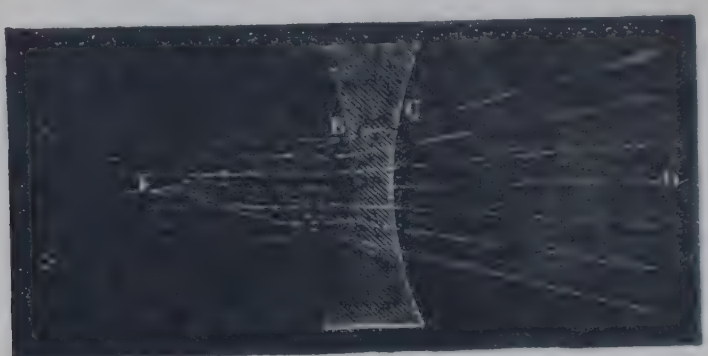

Fig. 322.—Virtual Focus of a Concave Lens

towards some point F, and to diverge towards the eye. By means of a straig edge mark the divergent lines on the side D.

Remove the lens, and produce the lines to meet at F. Then F is the pr cipal focus of the lens. Measure the focal length f.

Ex. 464.—Find (by drawing to scale on millimetre paper) the focal leng of a concave lens with which the following experiment was made. Two paral lines were viewed through it, and the distances between their images measur Close to the lens the lines were 1·25 cm. apart; at a distance of 3·6 cm. fr the lens they were 2·8 cm. apart.

Ex. 465.—To find conjugate foci of a concave lens.

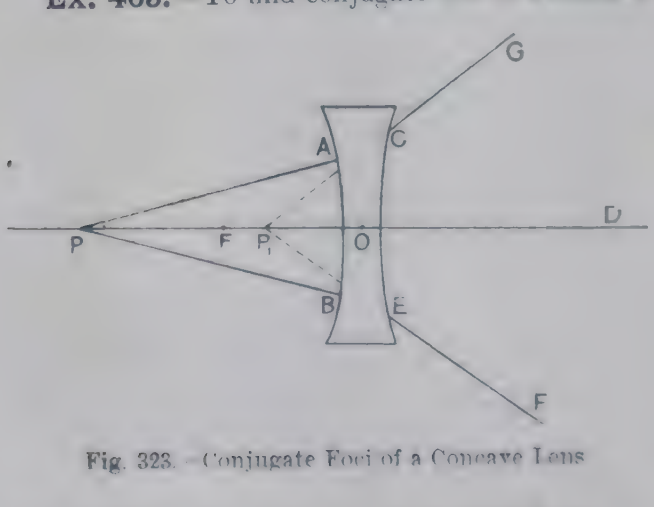

Fig. 323.—Conjugate Foci of a Concave Lens

Draw a line PD on a sh of paper to represent the pr cipal axis of the lens, pl the half lens across it in previous exercises, a trace its outline.

Mark F the principal fo in its proper position, so t FO = f (fig. 323). Fi point P on the principal a such that PO is greater t f. Draw two diverging li PA, PB to the lens; and before obtain GC, EF t images. Remove the l

and continue the lines GC, FE to meet in P_1.

Then P and P_1 are conjugate foci. Light proceeding from either to the l proceeds after leaving the lens along lines which converge to the other fo

Ex. 466.—Formation of Images. Draw a section of a concave lens 3241, and its principal axis OF. Mark the principal focus F. Let AB repr the object. From A draw two rays, according to the instructions given in E

cise 462. Then the ray drawn from A parallel to the axis of the lens appears to an eye placed on *the other side of the lens* to proceed from the focus E. The ray proceeding from A through O intersects the dotted line which is the apparent path of the first ray, at *a*, and there accordingly the image of A is formed. A similar construction gives *b* the image of B, and thus *ab*, the image of AB, is obtained.

Fig. 324.—Graphic Construction for the Image of an Object

There is only one case to consider here. Images formed by concave lenses are virtual, upright, and smaller than the object. They are nearer to the lens than the object, and on the same side of it.

Ex. 467.—To find the focal length of a concave lens by help of a convex lens.

(1) Find the focal length (f) of the convex lens alone (Exercise 460).

(2) Place the two lenses in contact, and find the focal length (f') of the compound lens thus formed. The convex lens selected for use must be such that the compound lens is a converging lens.

Then if f'' is the focal length of the concave lens, $\frac{1}{f''} = \frac{1}{f} - \frac{1}{f'}$, whence f'' can be calculated. The quantity f must have a negative sign. See Art. 301.

Ex. 468.—To find the focal length of a convex mirror by the aid of a convex lens.

On the optical bench place in this order (fig. 325) an evenly illuminated light A, a perforated screen B with cross wires, a convex lens C, and a screen D; and obtain on D a clear image of the cross wires. Insert the convex mirror at E, and adjust its position until a clear image of the cross wires is formed on B, close beside the cross wires themselves.

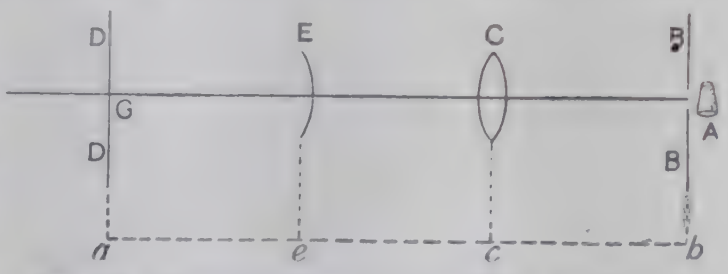

Fig. 325.—Method of Determining the Focal Length of a Convex Mirror

Then since the rays reflected from E have retraced their paths, they must have fallen normally on E; and these same rays, if they had not been reflected, would have converged to G. The point G is therefore the centre of curvature of the mirror. Measure this distance *ea*.

Example.—The following numbers were obtained in three experiments:—

Distance *bc*.		Distance *ce*.		Distance *ea*.
15 cm.	.....	13·5 cm.	.....	11·8 cm.
20 „	.....	6·3 „	.....	11·7 „
25 „	.....	3·4 „	.....	11·6 „

Mean value of r = 11·7 cm., and f = 5·85 cm.

296. THE SIMPLE MICROSCOPE. — The magnifying glass or simple microscope is a converging lens used to produce an enlarged virtual image of an object.

Referring to fig. 321, the object AB is placed nearer to the lens than its focal length; the eye is placed on the side F, and sees the enlarged image.

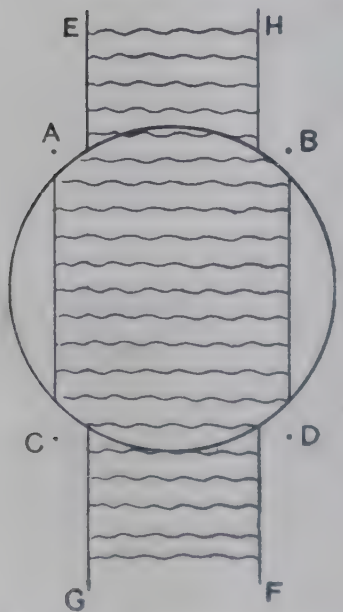

Fig. 326.—Magnifying Power of a Lens

Ex. 469.—To determine the magnifying power of a simple microscope.

Take a strip of newspaper EGFH (fig. 326), or similar object having small details upon it. The width of the strip may be about half that of the lens. Fix it on white paper on a flat board.

Place the lens on the strip. Raise the lens to the highest point at which clear vision of the details is obtained, and fix it there. The appearance is now that shown in fig. 326. Mark on the paper the points A, B, C, D, so that AB and CD are the width of the strip as seen through the lens.

Then the magnification produced under these circumstances is

$$\frac{\text{Apparent width}}{\text{Real width}} = \frac{\frac{1}{2}(AB + CD)}{\frac{1}{2}(EH + GF)}.$$

297. TELESCOPES.

Ex. 470.—(1) On an optical bench by means of a convex lens L (fig. 327) of fairly long focus (20 to 30 cm. is suitable) obtain a clearly defined image of a distant object on a translucent screen placed at A_1B_1. On the other side of the screen place a lens L′ of short focal length in such a position that an eye placed beyond it on the side F′ sees clearly the image on the screen. The distance between the screen and the lens L′ must be less than the focal length of L′.

The screen may now be removed, and the eye looking along the axis of the two lenses sees a magnified inverted image A′B′ of the distant object.

Such an arrangement of lenses is an *astronomical telescope*. The lens L is called the object glass, and L′ the eyepiece.

(2) Replace the convex lens L by a concave lens, and look through this eyepiece as before. No image is seen in that position. Gradually move the eyepiece towards the object glass until the image is seen. It is magnified and upright.

Fig. 328, which should be compared with fig. 327, illustrates the action of the

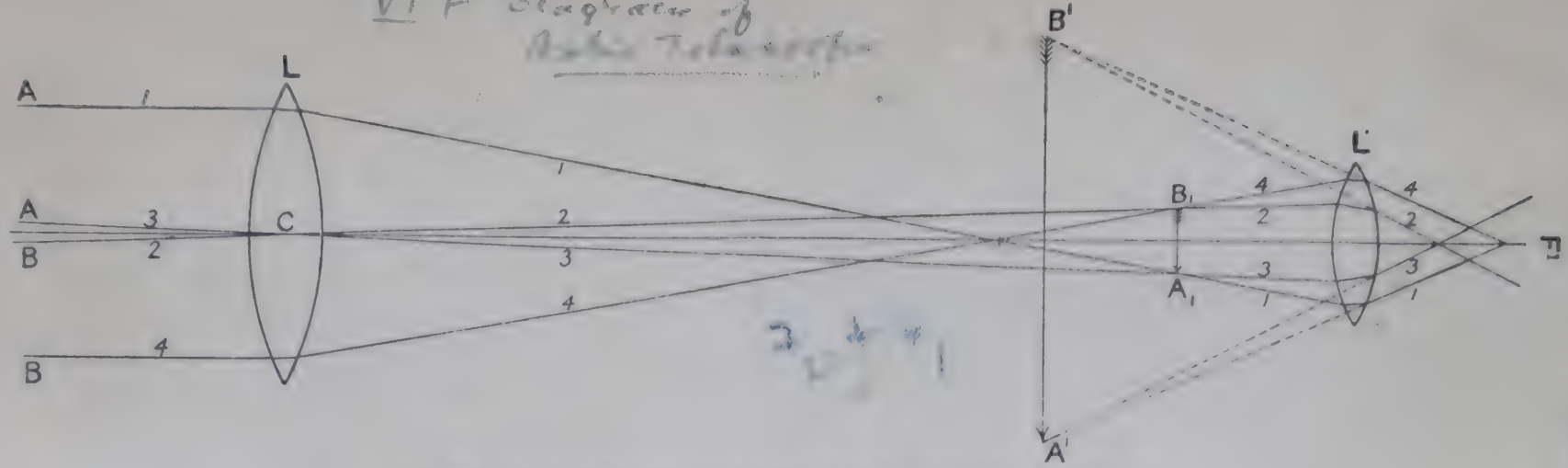

Fig. 327. Mode of Production of an Image by an Astronomical Telescope

Fig. 328. Production of an Image by a Galilean Telescope

lenses. The rays on their way to form the image A_1B_1 are intercepted by the concave eyepiece L′, and are changed from a converging into a diverging beam. Hence the image is seen at A′B′.

Lenses are arranged in this way in the *Galilean telescope*, the *opera glass*, and the *field glass*. It is seen that the employment of a concave eyepiece makes the length of the instrument less than if a convex eyepiece were used.

298. The *photographic camera* is a dark chamber or box (fig. 329) having a converging lens B inserted into one side of it and a screen A on the opposite side. The lens forms an image of a distant object on the screen exactly as was done in Exercise 460. The lens B is mounted in a sliding tube so that its distance from A can be altered as desired. The object is generally large, and a long distance in front of B; the image is smaller, and inverted. The back of the camera A can be taken out. A screen of translucent glass is first placed at A, and by moving B in or out its distance from the screen is adjusted so that a clear image of the object is obtained on the screen, which is viewed from the back, the photographer standing at the back of the camera and covering it and his head in a black cloth. The glass screen is then removed, and in its place is put a photographic plate, i.e. a plate covered with a thin layer of a salt of silver. The light decomposes this salt more or less according to its intensity, which is different in different parts of the image, and thus leaves on the plate a permanent image of the object.

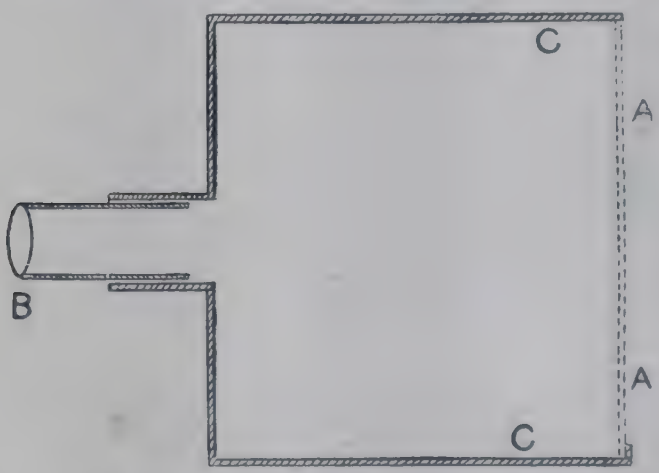

Fig. 329.—Diagram of a Photographic Camera

299. The *magic lantern* or *optical lantern* is illustrated in the diagram of fig. 330. Its object is to throw on a screen B an enlarged image of an object or picture (called a "slide") A.

The apparatus on the left-hand side of A in fig. 330 has for its main object to illuminate A as brightly as possible. A brilliant light D is placed in a blackened box. Between D and A is a large converging lens E, called the condenser, which concentrates the light on to A. Sometimes a concave mirror F at the back of the box

is also used to send back to A the light that has travelled towards the back of the box. But the condenser has also the function of causing the rays that pass through it and through the slide to follow such a path that takes them to a converging lens C, called the objective or focusing lens, which is placed between the slide and the screen. This lens is fixed in a sliding tube so that its

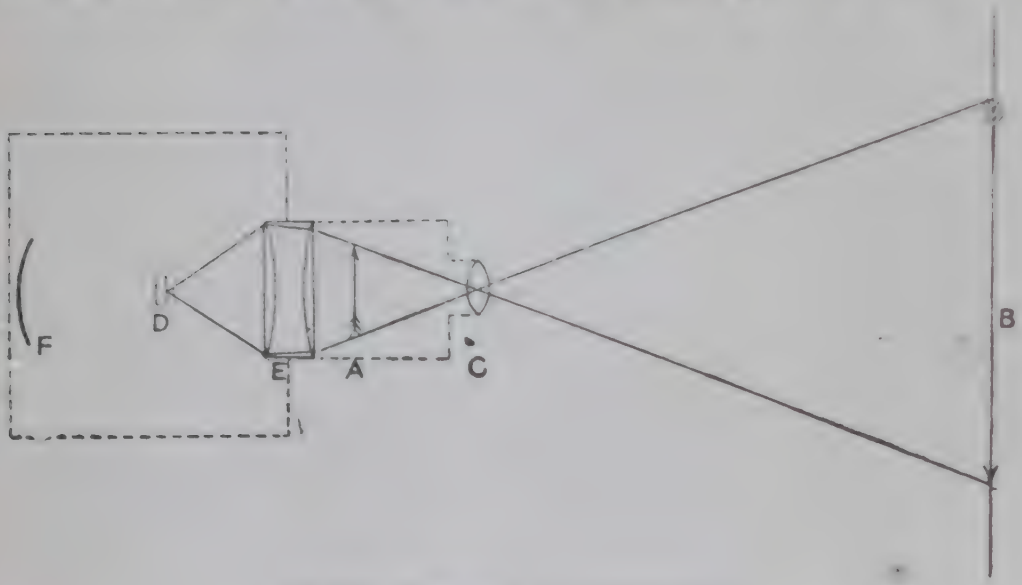

Fig. 330.—Diagram of an Optical Lantern

distance from A can be altered. It forms on B an enlarged inverted image of A exactly as shown in fig. 320.

There are various difficulties which render the use of simple lenses unsatisfactory. Both the condenser and the objective usually consist of a combination of lenses.

300. THE EYE. —Fig. 331 shows in diagrammatic form a section of the eye from front to back. The eye is nearly a sphere. Most of the outer surface consists of a white opaque coat (the sclerotica). The front is transparent and consists of two lenses, the cornea outside, and behind the cornea, inside the eye, the **crystalline lens**. The space between the cornea and the crystalline lens is filled with a fluid called the aqueous humour, and the space behind the crystalline lens is filled with another fluid called the vitreous humour. Just in front of the crystalline lens is an opaque diaphragm called the iris: this is the coloured part of the eye. Through the middle of the iris is an aperture called the pupil, by means of which light enters the eye.

The back of the eye is covered by a delicate coat called the

retina, which is provided with nerve fibres. The light that enters the pupil falls on the retina, and the effect on the nerves is to produce vision.

The lens in front of the eye forms images of objects on the retina in exactly the same way as the lens of a camera forms an image on the sensitive plate at the back of the camera.

Accommodation of the Eye. — The experiments with convex lenses showed that when the object was placed at different distances on one side of the lens, the image was formed at different

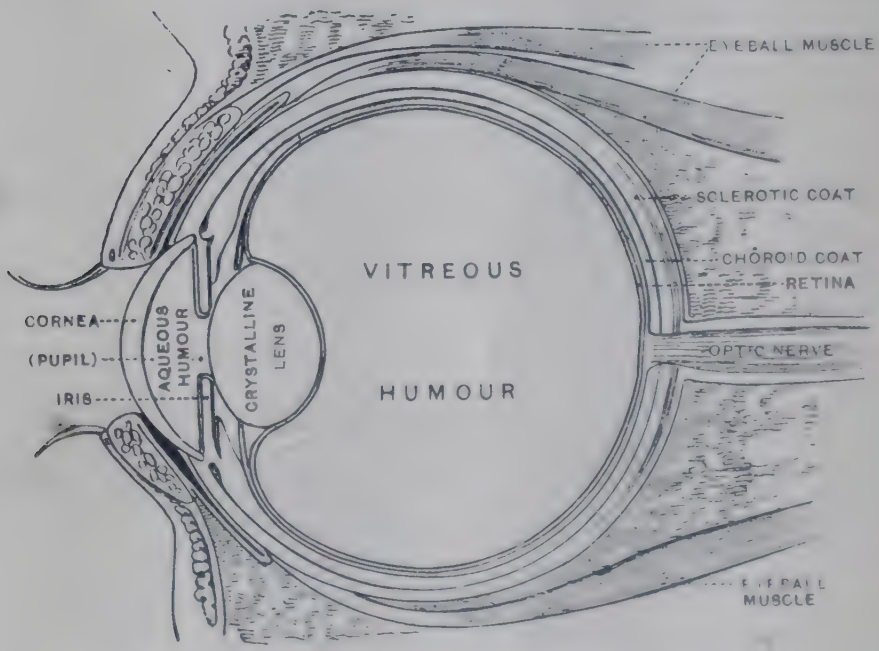

Fig. 331.—Diagram showing the Chief Structures of the Eye

distances on the other side of the lens, and unless the screen were placed in the proper position the image was blurred. Now the distance between the lens of the eye and the retina cannot be varied. Hence if the crystalline lens behaved like a glass lens we should only see objects clearly at one distance and all others would be blurred.

But the crystalline lens is surrounded by muscles which are able to alter its shape and to make it more or less convex. By this means its focal length is altered to suit objects at different distances, so that the image is produced on the retina instead of behind it or in front of it.

There is, however, a limit to this power of accommodation of the eye. An object placed very close to a normal eye, i.e. the

eye of a person whose sight is good, cannot be distinctly seen. The *least distance of distinct vision* for most persons is 25 or 30 cm.

Short Sight.—A person who is shortsighted cannot see distant objects clearly. The image of a distant object is formed, not on the retina, but within the eye, a short distance in front of the retina.

To remedy this defect he wears spectacles which consist of concave lenses. These render the rays of light less convergent, so that they come to a focus on the retina.

Long Sight.—A person who is longsighted cannot see near objects distinctly. The image of such an object is formed at some distance behind the retina. The rays are not sufficiently converged by the lenses of the eye. To remedy this he wears spectacles consisting of convex lenses, which make the rays of light converge more, so that they fall on the retina.

301. GENERAL FORMULA FOR LENSES.—In Exercise 460 it was experimentally verified that a certain relation existed between the distances u, v, and f. But as was pointed out for spherical mirrors in Art. 278, in order that the formula may hold good for the different cases that arise, it is necessary to take into consideration the direction in which the rays of light are travelling, and to call one direction + and the other −. The rule for lenses is the same as that for mirrors. *Distances measured in the direction from the lens towards the object are* +; *those measured in the opposite direction are* −.

Thus in fig. 320, which represents the state of affairs in Exercise 460, the distance u is positive, v is −, and f is −. If we make these changes in the formula shown experimentally in Exercise 460 to be correct, we have—

$$\frac{1}{-v} + \frac{1}{u} = \frac{1}{-f},$$

$$\text{or } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ (A).}$$

This equation A is the general formula for lenses, which if the rule of signs be observed applies to all the cases considered above.

Thus we have u: always +.

f: − for convex lens,
+ for concave lens;

v: + when object and image are both on the same side of the lens,
− when they are on opposite sides.

Examples.—(1) An object is placed 20 cm. from a convex lens, and the image is then 60 cm. from the lens and on the other side of it. If the object be placed 10 cm. from the lens, where will the image be?

In the first position (see fig. 320)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\therefore \frac{1}{-60} - \frac{1}{20} = \frac{1}{f},$$

whence $f = -15$ cm.

In the second position

$$\frac{1}{v} - \frac{1}{10} = \frac{1}{-15},$$

whence $v = 30$ cm.

This is the case shown in fig. 321. The image is virtual.

(2) An object is placed 22·5 cm. in front of a concave lens of 15 cm. focal length. Find the position of the image.

This case is that shown in fig. 324.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\therefore \frac{1}{v} - \frac{1}{22{\cdot}5} = \frac{1}{15},$$

whence $v = 9$ cm.

Ex. 471.—Miscellaneous Exercises.—(1) Draw the section of a double-convex lens; draw an arrow on one side of it, and show by geometrical construction how an image of the arrow is formed on the opposite side of the lens.

(2) Show by drawings how the image is affected when the arrow is caused to approach the lens; and how a magnified image may be obtained.

(3) An arrow 12·5 cm. long is placed 20 cm. in front of a convex lens whose focal length is 7·5 cm. Find the length of the image by calculation and by drawing to scale.

(4) On a sheet of paper placed vertically is written a capital L. If an observer stand 3 ft. in front of the paper, and hold a double-convex lens of 6 in. focal length halfway between his eye and the paper, he will see an image of the letter. Draw a picture of the image as seen, and state whether it is larger or smaller than the object.

(5) A convex lens of 11 cm. focal length is held at a distance of 7·5 cm. from a disk half an inch in diameter. Find the position and size of the image of the disk by a drawing to scale.

(6) Distinguish between convex and concave lenses. How would you obtain an image of the sun by a convex lens? What is the relation between the focal length of the lens and the size of the image?

(7) A candle is placed at a fixed distance opposite a wall. A convex lens held between the candle and the wall, throws on the wall a well-defined magnified image of the candle flame when it is 10 cm. from the candle, and a well-defined diminished image when it is 110 cm. from the candle. Find the focal length of the lens.

(8) An object 3 in. high is placed at a distance of 6 ft. from a lens, and a real image is formed at a distance of 3 ft. from the lens. The object is then placed 1 ft. from the lens. Where and of what height will the image be?

CHAPTER XXXV

The Spectrum

302. DISPERSION OF LIGHT.—Previous experiments have shown that when a beam of light passes obliquely through a refracting substance, it undergoes deviation. It will also have been observed that in some cases coloured fringes were produced. The colours are shown most clearly by a prism.

Ex. 472.—Hold a prism with its refracting edge vertical, and the eye close to one face. Look at various linear objects through the prism, such as—

(i) A knitting needle held vertically between the prism and a window.
(ii) A window bar, or blind cord.
(iii) A lighted taper at some distance.

Move the prism about a little, and slowly rotate it to a small extent until the clearest effect is obtained.

What are the colours seen? In what order?

The colours are shown more clearly in the following manner:—

Ex. 473.—Place a strip of white paper about 2 mm. wide on a black ground and in such a position that a good light falls on the strip.

Look at the strip through a prism held with its refracting edge parallel to the strip, so that the light incident on one face reaches the eye on emergence from the other face.

Rotate the prism until the strip is seen clearly as a multicoloured band (fig. 334, Continuous Spectrum) wider than the strip, in which it is not difficult to distinguish violet, blue, green, yellow, and red. This band is called a **spectrum.**

Fig. 332 illustrates the process. In that illustration light from the sun is shown passing through a small hole in a shutter and falling on the prism P. Passing through the prism, the light falls on a screen VW, and on the screen appears the coloured band VR. The beam must therefore have been widened out and separated by the prism into the different rays that produce the different colours. The beam is said to have been **dispersed.** The rays that produce the different colours were all present, but intermingled, in the light that fell on the prism: the prism had the effect of deviating the

different rays by different amounts, and they left it therefore along different paths.

Thus it is shown that a beam of ordinary white light consists of a number of rays that differ in some respect, with the result that different parts of the beam appear to us to have different colours. Moreover, on account of this difference in their nature, they undergo different amounts of refraction in passing through the

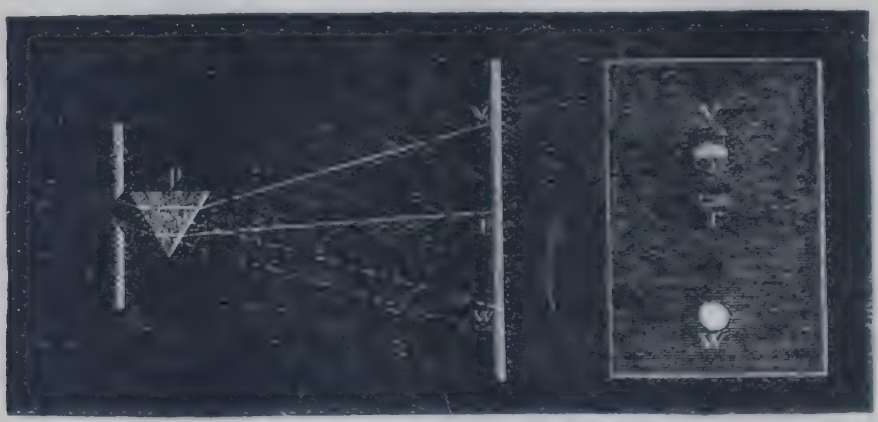

Fig. 332.—Dispersion of Light by a Prism

prism. Some are deviated more than others. Hence result separation and dispersion.

Ex. 474.—Repeat Exercise 473 with strips of various colours, such as may be obtained from the petals of flowers. Note the differences between the spectra obtained from blue, red, and yellow strips, as compared with the white strip.

The experiment shows that light coming to us from such different coloured objects contain the different rays mixed in different proportions.

303. The spectra seen in the previous exercises are **impure**, that is the colours overlap. A purer spectrum may be obtained by the following method.

Ex. 475.—(1) On an optical bench place a perforated screen A (fig. 333), and over the hole fix a piece of metal plate in which there has been sharply cut a slit about 1 cm. high, and not more than 1 mm. wide. The slit must be vertical.

Behind the slit place a bright light B. Sunlight reflected from a mirror into the slit, an incandescent gas mantle, or a gas flame placed edgewise are suitable.

In front of the slit place a lens L and a receiving screen S, and adjust them until a clear image of the slit is formed on the screen.

On a stand near the lens place a prism CDE, with its refracting edge C vertical, so that the light falls on one of the faces of the prism. The light will be deviated. Note in which direction. Measure the distance from the

prism to s. Move the screen to the new position s′ to receive the spectrum, making the distance from the prism the same as before. Adjust the screen and prism until the image is sharply defined.

Place the eye so as to receive the rays leaving the prism. The spectrum is seen to be much clearer and brighter than that obtained in Exercise 473. The colours overlap each other less. The narrower the slit, the less is the overlapping.

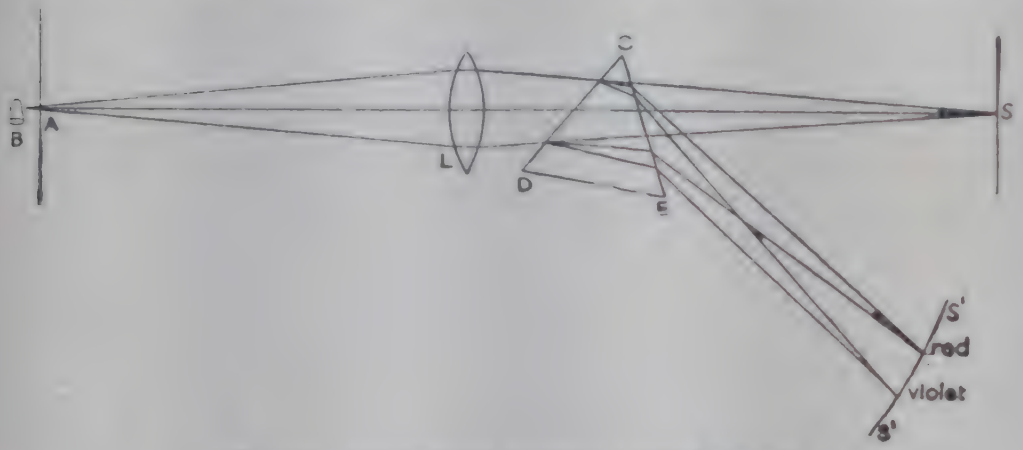

Fig. 333.—Arrangement to obtain a pure Spectrum

Rotate the prism slightly on a vertical axis (without displacing it) first in one direction, then in the other. What are the changes in the length and appearance of the spectrum on the screen?

Choose the position in which the image is sharpest and brightest: this is the position of *minimum deviation*.

Examine the Spectrum (fig. 334, Continuous Spectrum; see frontispiece). Draw it, naming the colours. The colours present in white light are usually described as violet, indigo, blue, green, yellow, orange, red. Which coloured rays have undergone the most deviation?

(2) Cut a narrow vertical slit in the screen s′, so as to let light through of one colour only. Behind the screen look at this slit through another prism. Only one colour is seen. No further colour separation can be obtained.

(3) Place various coloured glasses in front of the slit A, and for each record what colour has disappeared from the spectrum or been diminished in intensity. Some of the rays in white light have been cut off. Substances in the glass cause it to absorb certain rays. The spectra thus obtained are called **absorption spectra**.

(4) If different sources of light are available, place them in turn behind the slit and observe the differences in the spectrum. A candle or a gas jet contain very few of the rays that produce violet: the arc light and sunlight are rich in these rays.

(5) Use a Bunsen burner as the source. Note the dimness of the spectrum. Hold in the Bunsen flame a platinum wire that has been dipped in a strong solution of common salt. The spectrum consists only of a bright yellow line. The sodium contained in the salt gives out yellow light only.

When the spectrum of the sun is formed with more elaborate

apparatus, black lines are seen crossing it as shown in fig. 33 (The Sun).

Fig. 334 (Sodium) shows the effect seen when salt is placed i the flame of a Bunsen burner. Other chemical substances giv bright lines of different colours. Thus besides the ordinary spectrur of white light there are many other spectra. The difference b tween them show that different bodies give out light that differs i character, rays being absent in one case that are present in anothe

304. RECOMPOSITION OF THE RAYS.

Ex. 476.—With the arrangement of Exercise 475—(1) Place a large beak full of water in front of the receiving screen s′, so that all the rays fall on When the right position has been obtained the spectrum disappears, and t white slit reappears. The different rays have been refracted and made to f on the same spot, and their combination gives white light.

(2) Place a second prism beside the first in such a position as would produ deviation in the opposite direction. If the two prisms are exactly alike a scre placed in the original position s receives a white image of the slit.

(3) By means of a narrow strip of cardboard placed between the prisms st some of the rays that have passed through the first prism. The image becom coloured.

305. THE COLOUR DISK.—Exercise 476 has shown th the colours of the spectrum when combined form white light.

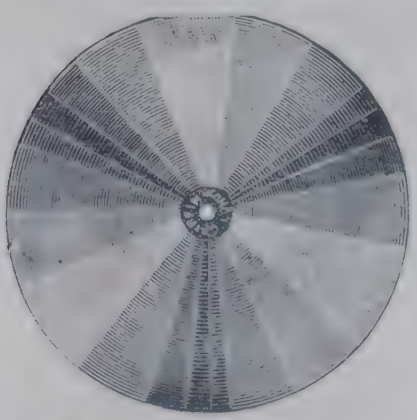

Fig. 335.—A Disk with Coloured Sectors which, when revolved, appears white

This fact may be also demonstrate by the colour disk (fig. 335). T circle is divided into sectors on whi are painted the spectrum colours. T sectors are of different widths. If t widths are correctly proportioned represent the relative intensities of t different coloured lights, then wh the disk is rapidly rotated it appea white or grey.

The impressions received by t eye remain on the retina for a sh period; and when the disk is in ra rotation, before the impression ma by the first sector has passed away the impressions of all the oth have been formed on the same spot. Hence the sensation is t which is aroused by the whole seven colours arriving at the sa time. That sensation is of white light.

PART VII.—MAGNETISM AND ELECTRICITY

CHAPTER XXXVI

Properties of Magnets

306. FORMS OF MAGNETS. — Examine several forms of magnets. All permanent magnets are made of steel. Fig. 336, A shows two **bar magnets.** They are usually kept in pairs with a strip of wood between them, and a piece

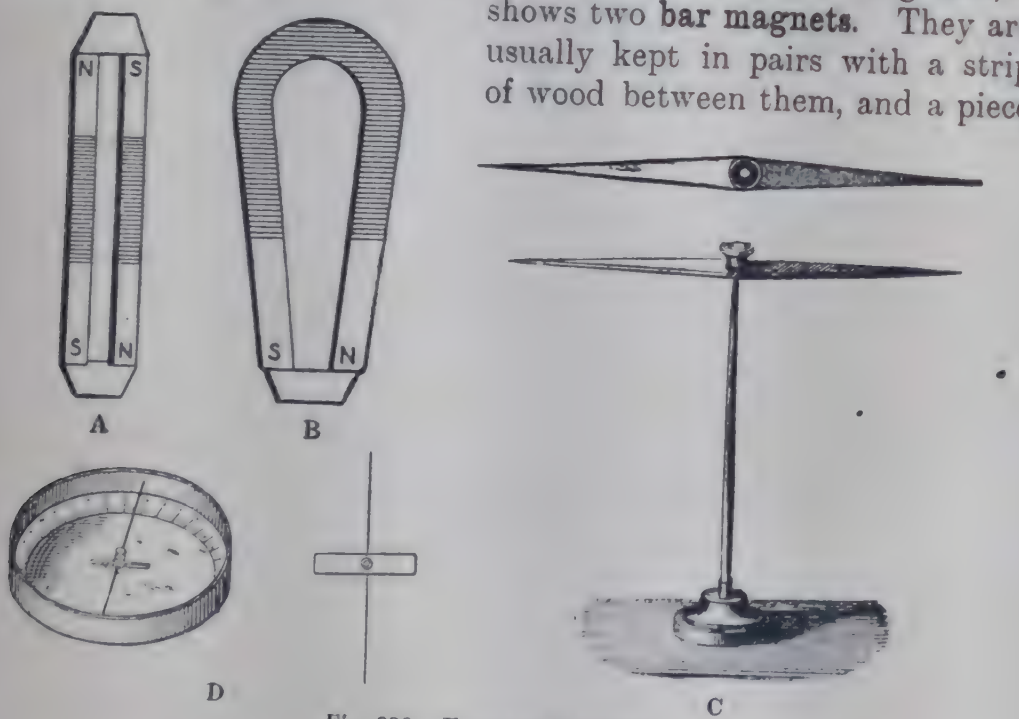

Fig. 336.—Forms of Magnets

of soft iron called a **keeper** joining their ends. The ends are generally marked N and S (north and south **poles**), and the keeper connects a N pole and a S pole.

The shape shown in B is called a **horseshoe magnet**. Note th position of the N and S poles, which are joined by a keeper.

The forms shown in C and D are called **magnet needles**. Th needle C is broad at the middle, where a cup is fixed so that th magnet may be supported on a sharp point.

The form D is better for some experiments. It is a bar wit a cup in the middle so that it may be supported on a shar point; and at right angles to the magnet is a long thin pointe which is not a magnet, but is used merely to detect more easil the movements of the magnet. This form is usually enclosed i a small box, on the bottom of which is a circular protractor compass card round which the pointer moves.

307. METHOD OF MAKING MAGNETS. — Many of th elementary experiments in magnetism may be made with sma magnets made from knitting needles or pieces of steel spring, whi can be magnetized when they are wanted. There are two metho by which any piece of steel may be magnetized by a bar magnet.

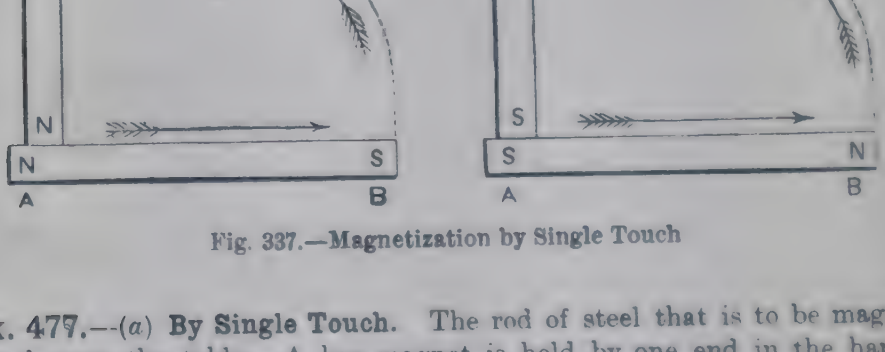

Fig. 337.—Magnetization by Single Touch

Ex. 477.—(*a*) **By Single Touch.** The rod of steel that is to be magneti is placed upon the table. A bar magnet is held by one end in the hand, the other end is drawn all along the steel rod (fig. 337).

The magnet is then lifted up (take care that the rod does not cling to it) carried round through the air into the starting position, and the whole proces repeated twenty or thirty times. Only one end of the magnet is allowed to t the rod, and the magnet must alw be drawn along the rod in the s direction. The motion of the st ing end of the magnet is indic by the dotted lines and arrow fig. 337.

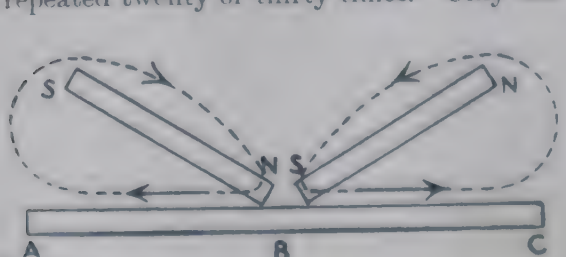

Fig. 338. Magnetization by Divided Touch

Ex. 478.—(*b*) **By Divide Separate Touch.** Place the b (fig. 338) that is to be magnetize

a table, and on it hold two bar magnets in the oblique position shown in fig. 338, with the N pole of one and the S pole of the other at the middle of the bar. Draw the magnets apart along the bar to its ends, raise them, bring them round in the direction of the dotted lines of the figure, and repeat the operation many times. The bar may be kept in position by a piece of wood pressed against each end.

Turn the bar over and repeat the operation. It is then magnetized.

308. RULES FOR THE USE AND KEEPING OF MAGNETS.

1. One magnet must never touch another.

2. A magnet should be kept clean; gently handled; never dropped or knocked.

3. Bar and horseshoe magnets when not in use should be kept with their keepers on, and arranged as shown in fig. 336, with the end marked N of one opposite the end marked S of the other.

4. When making experiments with a magnet, see that no other magnet is near, unless it is being used in the experiment.

309. CHARACTERISTIC PROPERTIES OF MAGNETS.

Ex. 479.—(1) Place one *end* of a magnet in a heap of iron filings, and move it about among them. The filings cling to the end of the magnet (fig. 339). Repeat with the other end of the magnet.

Note that the filings adhere, and are not easy to remove, especially from the angles.

Wipe them all off.

Heap the filings on the *middle* of the magnet. What is the result?

(2) Repeat the experiment with small fragments of other materials, such as copper filings, sand, sawdust, scraps of paper or glass.

What is the result?

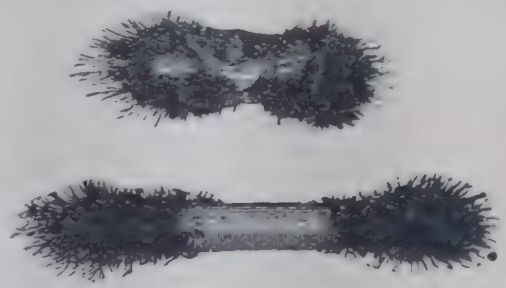

Fig. 339.—Iron Filings attracted by the Poles of a Magnet

(3) Repeat the experiment with the magnet separated from the iron filings by a sheet of glass, paper, brass, copper, tin, wood, lead, and iron. Does attraction take place in all these cases?

Ex. 480.—(1) Support a magnet so that it is free to turn on its middle point. In the case of a bar magnet this may be done by hanging it on a stirrup (fig. 340) made of paper or wire, and suspended by a fine thread;[1] or by placing it on a

[1] In the case of a light magnet suspended by an ordinary piece of twisted thread the movements of the magnet may be interfered with by the torsion, i.e. the twist of the thread. The torsion may be partly got rid of by placing in the stirrup a piece of wood of about the same weight as the magnet and allowing the thread to untwist. The best suspension is unspun silk, which has no torsion.

cork floating on water (fig. 341). A horseshoe magnet may be hung by a thre passing round its middle.

Suspend the various magne in turn in this way, and no that they all place themselv so that a line joining their en always points in the same dire tion.

Note the direction in whi the magnets set themselves.

(2) Repeat Exercises 479 and 480 with an ordinary ir rod, and compare the behavio of the rod with that of the ma nets.

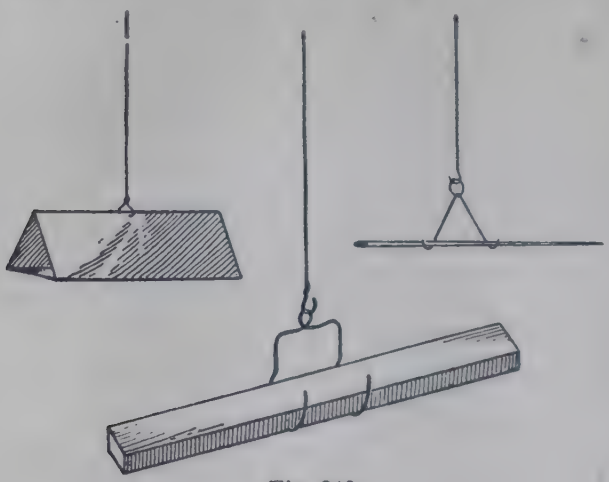

Fig. 340

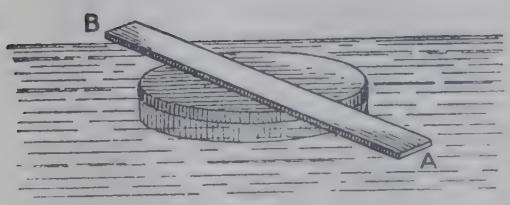

Fig. 341

Modes of supporting Magnets

These experiments sho the distinctive properti of a magnet to be:

(*a*) The ends attra iron filings;

(*b*) The magnet wh suspended places itself a certain direction whi is always the same. Th property is called **orie tation**.

310. ORIENTATION. MAGNETIC POLES.

Ex. 481.—Mark on the table a line which runs north and south. If t direction be unknown, it may generally be found by marking the line of shadow of a window bar or a thin upright rod cast by the sun at midday. S a north-and-south line in any place is called the **geographical meridian** of t place.

Place a suspended magnetic needle with its centre on this line.

The needle points nearly north and south. In some parts of the worl **points exactly north and south.**

Turn the needle slightly out of its direction. On releasing it what happe Turn it completely round. On releasing it what happens?

There is thus a difference between the two ends of the magn One end turns towards the north, the other towards the sou The ends of a magnet are called poles. The end that turns wards the north is called a **north-seeking pole**, and is often mar N; the end that turns towards the south is called the **south seek pole, and is marked S.**

[In dealing with magnets, it is often desirable to mark the poles if they are not already marked.]

The direction along which a suspended magnet sets itself at any place is called the **magnetic meridian** of that place.

The angle between the magnetic meridian and the geographical meridian is called the **magnetic declination**.

Ex. 482.—To measure the angle of declination.

Having marked the true north-south line on a sheet of paper fastened to the table, place a magnet of the form shown in fig. 336D, with its centre over this line.

Gently tap the stand of the magnet to see that the needle does not stick, and take care that the needle if uncovered is not disturbed by any draught of air.

Then the line passing through the pointer is at right angles to the magnetic meridian.

Mark this line (A) on the table, and removing the magnet draw a line B by means of a square at right angles to the line A. Then the line B marks the direction of the magnetic meridian.

By a protractor measure the angle between the line B and the true north-south line.

It will be seen that lines similar to those obtained above can be drawn through any point on the table or any point in the room. When we speak in a general way of the magnetic meridian at a place, we mean the direction in which any line runs that is drawn in the direction in which a magnet points.

311. THE COMPASS.—A magnet mounted at the centre of a circular card on which angles are marked as in a protractor, and letters standing for north (N), south (S), &c., are marked on the diameters, is called a *compass* (fig. 342).

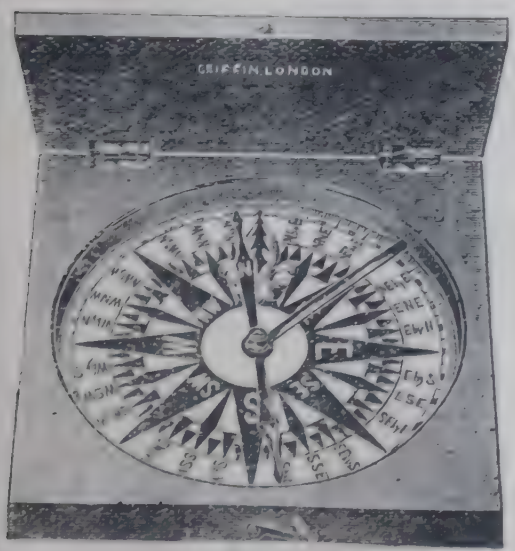

Fig. 342

By means of the compass, travellers in positions in which there is nothing to guide them can tell in which direction they are travelling.

Fig. 343

Forms of Compass

Thus fig. 342 shows, in addition to the magnet needle, a movable hand not connected with the magnet. Then if the compass box be placed so that the magnet points along the NS diameter, and the movable hand be turned so as to point along the traveller's road, the mark on the card to which the hand points tells him in which direction the road leads.

A **ship's compass** (fig. 343) is mounted in a box which is supported on two pivots, and this box is similarly swung in another box, the second line of support being at right angles to the first. This device prevents the magnet from being jerked about as the ship rolls.

312. ACTION BETWEEN MAGNET POLES.

Ex. 483.—Mark (N and S) the poles of a suspended magnetic needle and of a bar magnet.

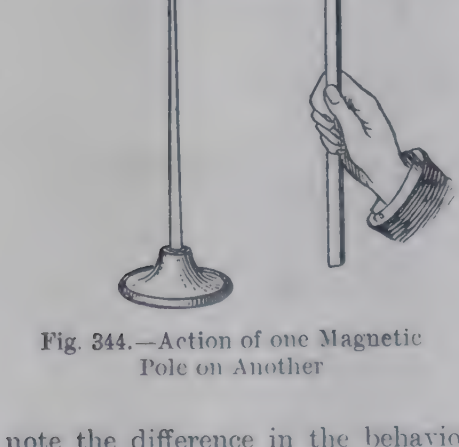

Fig. 344.—Action of one Magnetic Pole on Another

(1) Towards one pole of the suspended magnet needle slowly bring up one pole of the bar magnet, as shown in fig. 344, in a direction sideways to the suspended magnet. Do *not* let the magnets *touch* each other. Note which poles you are bringing near each other, and whether attraction or repulsion takes place between them.

(2) Reverse the magnet in your hand and repeat the operation, observing again which poles repel and which attract.

(3) Repeat both operations on the other end of the suspended magnet.

(4) Repeat all the above operations with an unmagnetized bar of iron. Carefully note the difference in the behaviour of the needle. Can repulsion be got?

The results of this experiment show that—

The N pole of one magnet attracts the S pole of another magnet

S ,, ,, ,, N ,, ,, ,,
N ,, ,, repels N ,, ,, ,,
S ,, ,, ,, S ,, ,, ,,

Or, *unlike poles attract each other: like poles repel each other.*

Either pole of a magnet attracts either end of a bar of unmagnetized iron. This distinction is the test whether a bar of magnetic material is or is not magnetized.

Such effects can only be obtained with a few materials—iron, steel, nickel, cobalt, and one or two others. Such substances are called **magnetic substances**. The effects obtained with iron are

much greater than with any other substance, that iron is regarded as *the* magnetic substance. When a piece of magnetic material is actually magnetized so as to possess a N and a S pole, it is a magnet.

It has already been shown in Exercises 190 and 276 that the magnetic force between two poles is mutual, and varies with the distance between them.

Ex. 484.—With the same arrangement as in the previous exercise—

(1) Hold the bar magnet parallel to the needle in various positions, first with N opposite N, then with N opposite S.

(2) Place two similar bar magnets one on the other, with a strip of thick paper between them, so that the N pole of one is on the S pole of the other. Present the ends of this combination to either pole of the needle. What is the result? Repeat in other positions.

Explain the effects observed.

313. TWO POLES INSEPARABLE.

Ex. 485.—(1) Magnetize a knitting needle or a piece of steel watch spring. Test it by holding its ends near those of a suspended needle: mark its poles N and S.

Fig. 345.—Every Magnet has Two Poles

Break it across the middle. Test both ends of both pieces as before, and mark the poles of each.

Break each piece again anywhere not in the middle. Test both ends of each piece as before.

The process may be continued. However small the pieces may be, the result is the same.

Take all the pieces and gum them end to end on to a small strip of cardboard in any order, but with all the N-marked ends turned the same way.

Test the rod thus re-made in the same manner as before.

(2) Fill a test tube with unmagnetized steel (not iron) filings.

Present its ends to the poles of a suspended magnet, and see that the tube does not behave as a magnet.

Treat the tube as the rod was treated in Exercise 477, and, without shaking it, test its magnetism.

Shake up the filings, and again test the magnetic condition of the tube. If the filings have been thoroughly mixed by the shaking no external magnetic effect is obtained.

The conclusion drawn from these and similar experiments is that every particle or molecule of a magnet is separately magnetized, so that every magnet consists of millions of small magnets,

all having their similar poles turned in the same direction, as shown in the lower portion of fig. 345.

CHAPTER XXXVII

Magnetic Induction

314. It has already been seen that magnets have an effect on other magnets that are near them.

Ex. 486.—(1) Place a heap of iron filings on a board, and hold vertically over them and very close to them an unmagnetized rod of soft iron.

Bring up a bar magnet with one pole over and close to but not touching the iron rod (fig. 346). Note the behaviour of the filings.

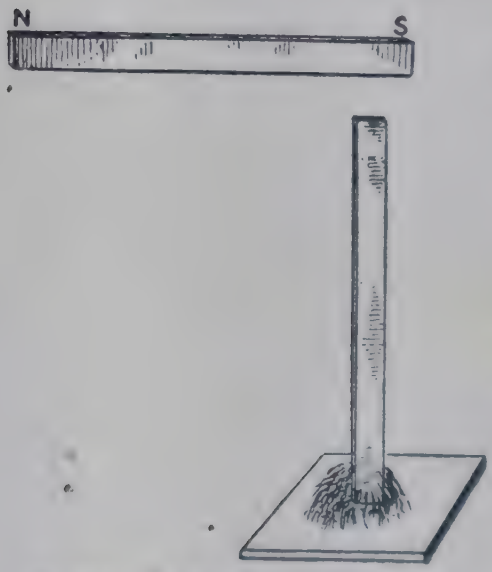

Fig. 346.—Soft Iron Temporarily Magnetized by Induction

Rub the lower end of the rod about among the filings. What is the effect?

To produce this effect what must now be the condition of the rod—magnetized or unmagnetized?

(2) Remove the bar magnet, and knock the iron rod smartly with a mallet. Will it support the filings as before?

What is now the condition of the rod?

(3) Reverse the ends of the magnet and of the iron bar, and repeat the experiment. Is there any difference in the effects observed?

Ex. 487.—(1) At the side of a suspended magnet needle place a rod of soft (unmagnetized) iron in a position at right angles to the needle and in a line with the north pole, but a few centimetres from it (fig. 347). What is the effect?

Place a bar magnet SN in line with the iron rod, with its N pole towards the needle, as in fig. 347. What is now the effect?

Reverse the iron rod end for end. What is the result?

(2) Reverse the bar magnet. What is the effect on the pole of the needle?

Reverse the iron rod. What change occurs?

(3) Remove the bar magnet. What is the effect on the suspended needle?

Reverse the bar. Can you get repulsion of either pole of the suspended needle by means of the iron rod only?

What is the condition of the iron bar now?

(4) According to the test given in Art. 312, what was the magnetic condition of the iron rod in case (1)?

Can anything certain be said about its magnetic condition in case (2)?

Does the result of the removal of the magnet (case 3) help to decide what was the condition of the iron rod in case (2)?

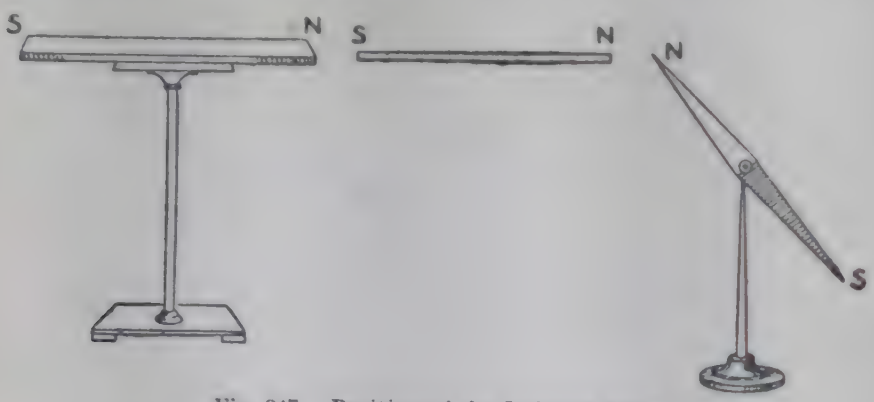

Fig. 347.—Position of the Induced Poles

(5) Repeat the experiments with the bar magnet touching the end of the iron rod; and compare the results with those previously obtained.

The two preceding experiments show that when a bar of soft unmagnetized iron is placed near a magnet, or touching it, the iron bar itself becomes a magnet; and that when the bar magnet is taken away the iron bar ceases to be a magnet.

In such cases the *temporary magnetism* of iron is said to be **induced**, and the process is called **magnetic induction**.

Magnets made of steel when once magnetized remain magnetized for a long period under ordinary usage. Hence they are called **permanent** magnets. The steel possesses a property called **coercive force** or **retentivity** which the soft iron does not possess. This result is due to changes which take place in the steel during its manufacture. A rod of steel can only retain a certain quantity of magnetism. When it has this quantity it is said to be **saturated**. Soft iron is only magnetized when it is under magnetic influence proceeding from something outside itself, as from a magnet or from an electric current (Art. 357).

The experiments also show that the induced magnetism of the iron bar is such that its S pole is nearest to the N pole of the magnet.

315. FURTHER EXPERIMENTS ON INDUCTION.

Ex. 488.—Place some long thin iron nails on a table, with their heads downwards and in contact. Bring the N pole of a bar magnet down upon their points. On raising the magnet, note the direction of the nails.

What is the magnetic condition of the nails? Why do they hang in the directions which they actually take up?

Bring up the heads of the nails near to the N pole of a suspended magnet. Which end of the nails is the N pole?

Repeat the operation with the S pole of the bar magnet.

Ex. 489.—Support several pieces of iron from a magnet pole, as shown in fig. 348.

Find out which kind of pole is at the bottom. Bring up near to the bottom of the chain of pieces a pole of the *opposite* kind. Place it first directly underneath and then a little to one side. In the latter case the chain becomes curved. The induced magnetism has been strengthened by the second magnet.

Bring up a pole of the *same* kind. The pieces of iron fall off because the magnet pole thus brought up has induced in the pieces magnetism in the opposite direction to that produced by the original supporting magnet; and the two opposite polarities thus induced destroy one another.

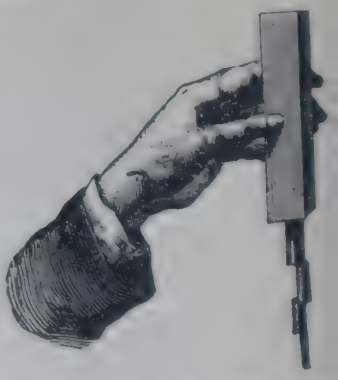

Fig. 348.—Induced Magnetism producing Further Induction

A strong magnet may in this way reverse the polarity of a weak one.

Ex. 490.—(1) Place a magnetic needle on its stand, and allow its N pole to project a short distance through a hole in a piece of cardboard, so that the end of the magnet cannot move far.

Bring up towards this N pole, the N pole of a strongly magnetized bar magnet. The N pole of the needle is repelled. Move the bar magnet up closer to it. At a certain distance the pole of the needle marked N is attracted. What must be its condition for this to occur?

(2) Remove the bar magnet and the card, and allow the needle to take up its position free from any influence.

What does its return to that position show?

(3) Now place the bar magnet horizontally a few centimetres above the needle, and in the magnetic meridian, with its N pole pointing towards the north.

What is the effect on the needle? Rotate the bar magnet very slowly in a horizontal plane. What is the action of the needle? Why does it behave in this way?

316. MAGNETIC FIELDS.—It has been shown that a magnet is able to affect other magnets and soft iron placed at some distance. There is then round a magnet a space within which its influence is felt: this space is called a **magnetic field**.

Certain facts concerning magnetic fields may be learned by observing iron filings placed in the field.

Ex. 491.—Place a bar magnet on a table so that its length is in the magnetic meridian, and cover it with a sheet of thin glass, tin, or smooth cardboard. The sheet should be supported at the sides if necessary, so that it is quite horizontal.

From a pepper-box or similar arrangement sprinkle a small quantity of iron filings evenly over the sheet, and gently tap it. The filings will set themselves in curves, like those shown in fig. 349. Examine the curves, and draw them.

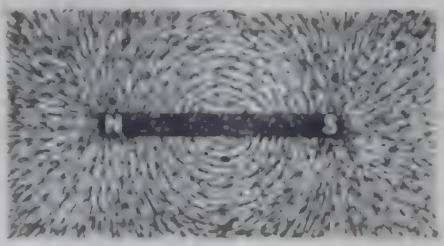

Fig. 349.—Magnetic Field of a Bar Magnet

The filings are moved into the places they occupy by force. Since this force is produced by a magnet it is called **magnetic force**. The filings show that this force acts not in some one fixed direction but along certain curved lines. The lines are called **magnetic lines of force**. The experiments must necessarily be done on a horizontal surface, but the lines of force are exactly the same in all planes round the magnet. Were the force of gravity suspended so that the filings did not fall, they would take up exactly the same positions if the card were vertical or placed at any angle to the horizontal.

The magnetic field of a magnet, then, is full of lines of magnetic force which occupy the space around it. The lines of force are, of course, themselves invisible. The lines of filings indicate how they run, radiating out in all directions from one pole, and passing along curved paths to the other pole of the magnet.

Ex. 492.—Place a horseshoe magnet on the table, and obtain its lines of force as in the previous experiment (fig. 350). Compare them with the curves obtained in the previous experiment.

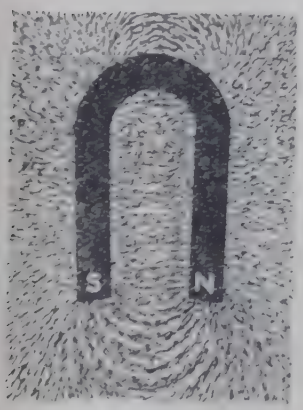

Fig. 350.—Field of a Horse-shoe Magnet

Ex. 493.—Fix a magnet vertical with the sheet of cardboard resting on the end of it. The filings show clearly the lines of force passing outwards from the pole in all directions (fig. 351).

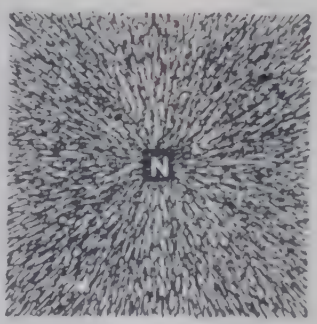

Fig. 351.—Lines of Force radiating from a Single Pole

Ex. 494.—Place two bar magnets on the table in a line, with the N pole of one adjacent to and at a distance of 10 or 15 cm. from the S pole of the other.

Obtain and make a sketch of the lines of force between these two poles. Compare with fig. 349.

Ex. 495.—Proceeding in the same manner as in the preceding exercise,

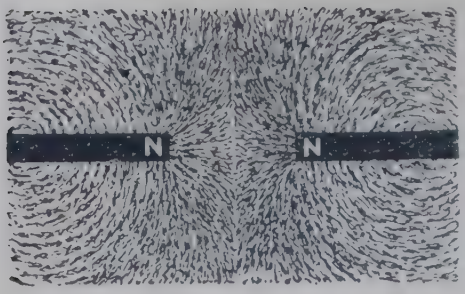

Fig. 352.—Field of Force between Two Similar Poles

obtain the lines of force between two like poles both N or both S (fig. 352).

The lines obtained with two like poles are in striking contrast with those obtained between two unlike poles.

With unlike poles, the lines are in the form of curves running from one pole to the other.

With *like* poles, the curves do not run from one pole to the other. Each pole sends out its own lines of force, and each set behaves as though it repelled and was repelled by the other set. The filings turn off in a direction towards a line making a right angle with the line joining the poles.

Fig. 353

317. TRACING LINES OF FORCE BY A SMALL COMPASS NEEDLE.—Lines of force may be more accurately traced out by a very small compass needle contained in a box (fig. 353).

Ex. 496.—To find the position of the poles of a magnet.

Place on a sheet of paper a bar magnet and a small compass needle in the position shown in fig. 354, and mark the outline of the magnet on the paper.

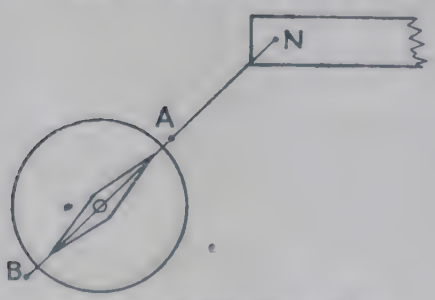

Fig. 354.—Determination of the Position of a Magnet Pole

Rotate the paper until the compass needle is in the magnetic meridian; then mark its line of direction on the paper by the two dots B, A. Then the line BA points to the pole N.

Repeat the operation with the compass needle in several positions round the pole N, and obtain several lines pointing to N. Produce them until they intersect on the paper. The intersecting region is the position of the pole. It is not a point.

In a similar manner find the position of the S pole of the bar magnet.

Mark the positions of the poles on the paper.

Ex. 497.—To find the equatorial line of a magnet.

Use the same sheet of paper with the same magnets as in the last experiment. Place the bar magnet in the magnetic meridian and the compass needle close beside it near its middle. Find a position of the needle in which it is exactly parallel to the bar. Mark on the paper the position of the centre of the needle.

Repeat this operation at three or four points on each side of the bar. Remove the bar and the needle.

Then you have found a number of positions in which the direction of the compass needle is unaffected by the bar magnet.

Join the points thus obtained.

If the bar is uniformly magnetized the line thus drawn will bisect at right angles the line drawn joining the poles. It is the equator of the magnet.

Ex. 498.—To map the lines of force between two magnetic poles.

Place two magnets on a sheet of paper in a line, with a space of 12 or 15 cm. between the N pole of one and the S pole of the other.

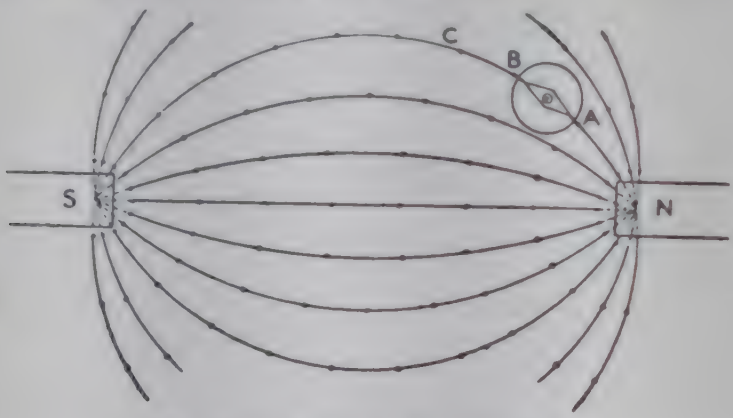

Fig. 355.—Mapping a Magnetic Field

Place the compass needle near one pole. Rotate the paper until the needle is in the magnetic meridian, and mark the positions of its poles AB (fig. 355). Move the compass needle on a step so that one pole is at B and the other at C. In this way trace out the complete line NABCS.

Map out several lines, as in fig. 355.

Ex. 499.—By means of iron filings demonstrate the lines of force in the following cases:—

(1) Two bar magnets, one at right angles to the other.

(2) A piece of steel which has been magnetized and then rubbed by a magnet at two or three points. At these points there are *consequent poles*.

318. EFFECT OF SOFT IRON IN A MAGNETIC FIELD.

Ex. 500.—Demonstrate by filings the lines of force of a horseshoe magnet with the keeper (1) on, (2) near but not on, (3) farther away. Compare the figures obtained with fig. 350.

This experiment shows that when iron is present in a magnetic field the lines of force crowd through it, leaving the surrounding space almost without them.

CHAPTER XXXVIII

The Earth as a Magnet

319. CAUSE OF THE ORIENTATION OF MAGNETS.—In Exercises 496 to 498 it was necessary, in order to obtain a good result, to place the compass needle always in the magnetic meridian. We will examine the result that follows when this precaution is neglected.

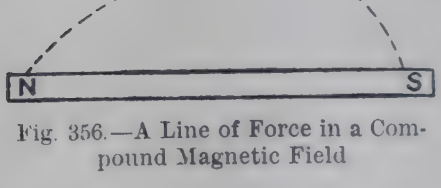

Fig. 356.—A Line of Force in a Compound Magnetic Field

Ex. 501.—On a sheet of paper place a bar magnet about at right angles to the magnetic meridian. Trace out one of its more distant lines of force with a small needle as in the previous exercises, but keep the paper always fixed in the same position. The curve will be found to be distorted, as in fig. 356. This distortion is produced by the presence of another magnet—the earth.

Ex. 502.—(1) Place some small magnet needles on a table. Bring near them a bar magnet. Note the disturbance of their directions.

(2) On a table place a bar magnet in any position. On it near its centre place a magnetic needle. The needle places itself nearly parallel to the bar magnet. Try various positions.

As suggested by the last experiment, the reason why magnets suspended so as to move only in a horizontal plane set themselves in the magnetic meridian is that they are then placing themselves parallel to another larger magnet. The earth behaves as though it had a large magnet in its interior, round which there is a large magnetic field which affects all the magnets and magnetic substances with which we deal. The cause of the earth's magnetism is not certainly known, but the supposition of a large interior magnet helps to understand the results.

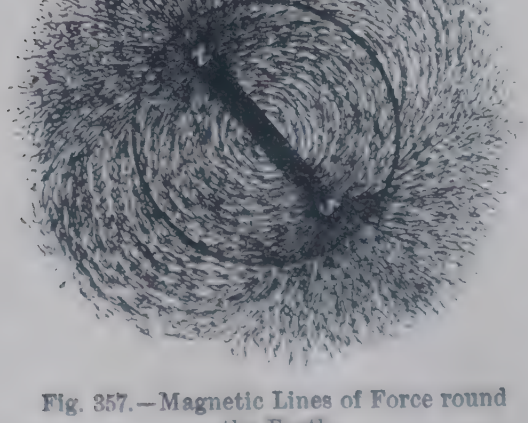

Fig. 357.—Magnetic Lines of Force round the Earth

Thus the circle in fig. 357 may be regarded as a section of the earth passing through the supposed magnet.

The figure shows that the lines of force cut the surface of the

earth in different directions at different parts. At the **magnetic equator** they are parallel to the surface; at the magnetic poles they are perpendicular to the surface, and at intermediate points they are inclined to the earth's surface at different angles.

If a magnet be suspended so as to be quite free to move in any direction it puts itself along the line of force at the place where the magnet happens to be. The magnet does not move as a whole.

320. MAGNETIC INCLINATION.

Ex. 503.—Fig. 358 shows a needle supported so that it rotates not in a horizontal but in a vertical plane. Such a needle is called a *dip needle.*

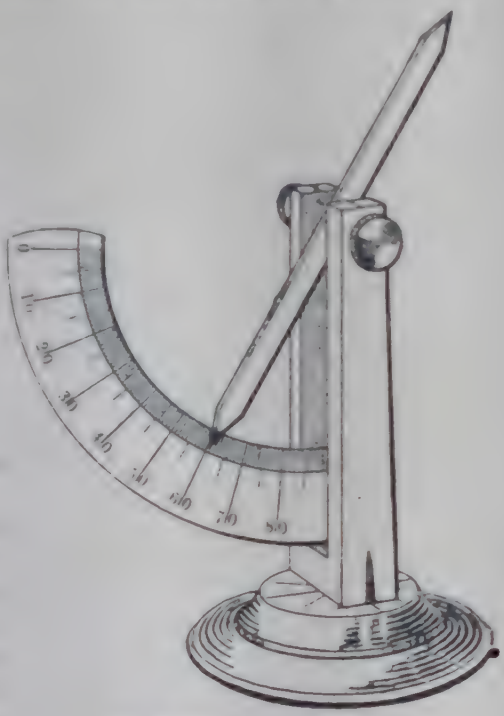

Fig. 358.—A Dip Needle

Set the needle carefully, so that the plane in which it swings—that is, the plane of the graduated arc—is in the magnetic meridian. Then the needle sets itself actually along one of the earth's lines of force. A reference to fig. 357 shows that for most parts of the earth's surface this direction is neither horizontal nor vertical. Read the angle which the needle makes with the horizontal; this is (approximately) the **magnetic inclination** or **angle of dip** for the place where the observation is made.

The result will be more accurate if the needle is taken out, remagnetized *with its poles reversed,* mounted again, and another reading taken. The mean of the two readings is free from error that may be caused by the axle not being in the middle of the magnet.

321. ASTATIC COMBINATION OF MAGNETS.

Ex. 504.—Place two knitting needles side by side, and magnetize them both at the same time.

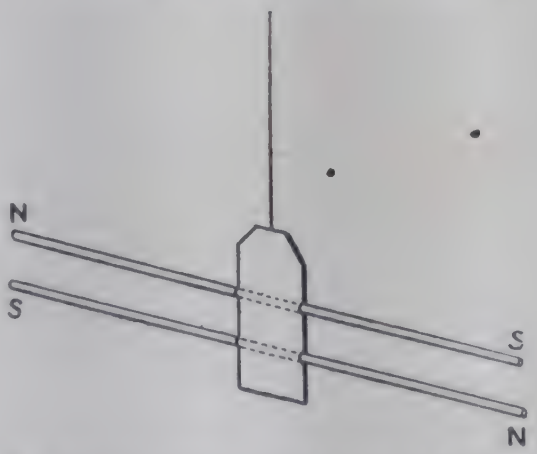

Fig. 359.—An Astatic Combination of Magnets

Gum their middle parts to a piece of paper, so that the two magnets are parallel, with the N pole of one under the S pole of the other (fig. 359). Support the paper by a thread.

Such an arrangement is called an **astatic** combination or astatic magnetic

needle, because if the magnets were exactly alike the combination when suspende would show no orientation, but would remain in equilibrium in any direction.

Test it to see whether this is the case.

Practically it is certain that there is a slight difference between the magnets so that the combination does set itself in the magnetic meridian.

Bring up a weak bar magnet towards the astatic needle. It is deflected whe the bar magnet is at a considerable distance, that is, the needle is *sensitive.* A slight magnetic force causes it to deflect from its position of equilibrium.

Ex. 505.—Miscellaneous Examples.

(1) A horseshoe magnet is brought near a small magnet needle. State hov the needle is affected when the horseshoe is held in various positions.

What would be the effect of putting a keeper on the magnet? Why?

(2) A horseshoe magnet is floated on a large cork on water. How will i place itself? why?

(3) A rod of soft iron is held near a bar magnet in various positions. Stat with diagrams what is its magnetic condition in each position.

(4) The same rod is held in different positions under the influence of th earth's magnetism only. Show by the help of the previous exercise what is it magnetic condition in the various positions.

(5) Magnetize a short needle, suspend it by a thread at its centre. Pass i along over a bar magnet placed a few centimetres below it, and note the dip o the needle at different places; note also which end dips.

(6) A tall iron mast is situated a little in front of the compass on a wooden ship. Explain the nature of the compass error when the ship is sailing in a easterly direction—

(i) in the northern hemisphere.
(ii) in the southern hemisphere.

(7) How would you, by means of a suspended magnet whose poles are un marked, determine—

(i) the magnetic meridian?
(ii) in which direction north lies?

(8) Place a bar magnet so as to deflect a compass needle, and note the deflec tion. Place a bar of soft iron beside the bar magnet. How is the deflection altered? Why?

(9) Magnetize a piece of steel watch spring; bend it so that its poles are i contact. Examine the effect of this magnet on a compass needle.

(10) Magnetize a steel knitting needle; break it into two equal parts, an mark the poles. Place the pieces on a cork at right angles to each other, an float the cork on water. How does the arrangement set itself with regard to th magnetic meridian? Why?

(11) Set up a dip needle in the magnetic meridian. Hold over it a rod of so iron in a vertical position. What is the effect on the dip? Why?

(12) Place the dip needle so that the plane of the graduated arc (fig. 358) i not in the magnetic meridian. Rotate the stand of the instrument, and note ho the dip of the needle changes.

Mark accurately on the table the direction of the graduated arc when the di is least, and when it is greatest. What are the relative positions of these tw lines? When the needle is vertical what is the direction of the plane in which

swings with reference to the magnetic meridian? When the dip is least, in what plane is the needle?

(13) Draw a right-angled triangle the hypotenuse of which makes with the horizontal side an angle equal to the angle of dip. Then, calling the hypotenuse I, the vertical side V, and the horizontal side H, measure and record the lengths of these lines.

The numbers represent the relative values of the total intensity (I), the vertical component (V), and the horizontal component (H) of the earth's magnetic field (Article 130) at the place where the dip was measured.

CHAPTER XXXIX

Magnetic Measurements

The exercises contained in this chapter are of a less elementary character

322. THE METHOD OF OSCILLATION.—It has already been noticed that magnets oscillate when they are placed in the magnetic field produced by another magnet. Such oscillations are performed according to laws which are in the main the same as those of the pendulum. Fig. 360 shows a magnet mounted so that its vibrations may be conveniently studied. The magnet is suspended in a stirrup at the end of a long thin torsionless thread inside a box, along the bottom of which is drawn a line of reference.

Such a **vibration magnetometer** is easily made.

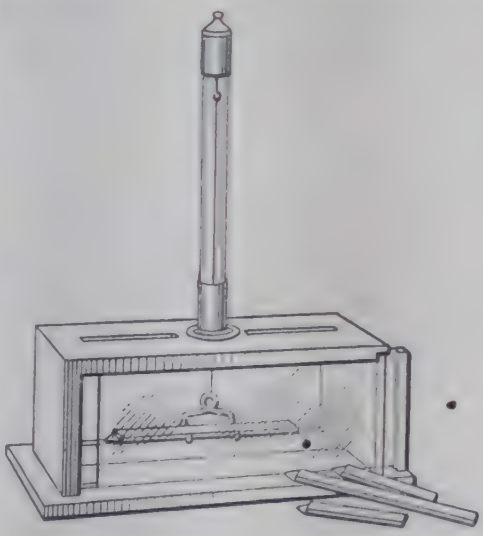

Fig. 360.—A Vibration Magnetometer

Ex. 506.—To determine the circumstances that influence the time of vibration of a magnet: (1) Place a bar magnet (A) in the stirrup, and turn the magnetometer so that the central line of the magnet is exactly over the reference line beneath it on the box.

Set A in vibration by bringing up another magnet (B) for a moment near to one of its poles. Remove B far away.

Find the time of vibration of A by the method of Art. 158. Call it t_1.

(2) Place a brass weight on each end of A, and again determine its period. Call it t_2. The value of t_2 will vary according to the amount of the added masses and their position. Remove the weights.

The difference between t_1 and t_2 shows that the time of vibratio depends on the amount of the swinging mass, and how that mas is arranged round the central supporting fibre. This is expresse by saying that the period depends on the **moment of inertia** of th oscillating bar.

(3) In the two previous experiments A was oscillating in the same magn field—that due to the earth.

Again set A oscillating, and place the magnet B on the table near to, parall to, and in the opposite direction to A. The field of B is now added to that of th earth, so that A is swinging in a stronger field. Find its time of vibration (t_3).

The difference between t_1 and t_3 shows that a magnet vibrat quicker in a stronger field and slower in a weaker field.

(4) Remove A from the stirrup and weaken its magnetism. This may be do after the method of Exercise 477 by drawing magnet B a few times along A the opposite direction to that shown in fig. 337.

Replace A in the stirrup and again find its time of vibration (t_4) in the earth field only.

The difference between t_1 and t_4 shows that the period depen on the strength of the magnetization of the magnet. The distan between its poles also enters into the matter. Call the strength each pole m, and let the distance between them be l, then $m \times l$ called the **magnetic moment** of the magnet (usually written M It is the magnetic moment of the magnet that we have to consid here.

It is thus shown that the period of vibration of a magn depends on—

(a) its moment of inertia (K),
(b) its magnetic moment (M),
(c) the strength of the magnetic field in which it vibrates (

An investigation which we cannot give here shows that if t the time of a single swing, then—

$$t = \pi\sqrt{\frac{K}{MH}}.$$

We can only make some experiments to illustrate this formula.

Ex. 507.—To compare the strengths of magnetic fields (horizontal c ponent).

Fields of varying strengths may be made by arranging one or more magnets in different positions near the magnetometer. As the magnetom magnet swings in a horizontal plane, only the horizontal component of the f in the field affects it.

With the magnet in the chosen position in the first field find t_1. Call the horizontal component of the field H_1.

With the magnet in the second field of strength H_2 find t_2.

Then $H_1 : H_2 :: t_2^2 : t_1^2$.

Thus the ratio between H_1 and H_2 is known.

Ex. 508.—To compare the strength of the earth's field with that of a bar magnet in a certain position.

The method is to make the two observations of Exercise 506 (1) and Exercise 506 (3).

Then t_1 is the time of vibration of magnet A in the earth's field; and t_2 is its time of vibration in the field due to the earth and magnet B together. Then if H is the strength of the earth's field, and H_1 the strength of field due to the magnet B, we have—

$$H : H_1 + H :: (t_2)^2 : (t_1)^2$$
$$\text{or } H : H_1 :: (t_2)^2 : (t_1)^2 - (t_2)^2.$$

Ex. 509.—To compare the strengths of the poles of two magnets.

Take two magnets (A and B) exactly alike in mass and dimensions, but one magnetized more strongly than the other. Call the strengths of their poles m_1 and m_2, and since their lengths are the same $m_1 : m_2 :: M_1 : M_2$.

Place A and B in turn in the magnetometer and find their times of vibration t_1 and t_2.

Then $m_1 : m_2 :: t_2^2 : t_1^2$.

The magnetometer must be in the same field in both cases. The stronger magnet vibrates more quickly than the weaker one.

If the lengths of the magnets are not the same then this experiment gives, not the ratio between the strengths of the poles, but the ratio between the magnetic moments M_1 and M_2.

323. For some purposes it is desirable that the vibrating magnet should be small and light, as in fig. 361, which shows the vibrating magnet supported by a thin fibre, and connected by another fibre to a weight underneath it, which helps to keep it from swinging to and fro as a whole, and makes the vibrations slower.

Fig. 361.—A Vibration Magnetometer

Ex. 510.—To find the distribution of magnetism in a bar magnet.

Note which side of the room is north, and therefore which end of the vibrating magnet is the north pole. The vibrator must be kept in the magnetic meridian throughout the experiment.

Find the number of vibrations made by the vibrator (fig. 361) in one minute in the earth's field only. Call the number x.

Place a bar magnet vertical on the north side of the vibrator so that the south pole of the bar magnet is near to and opposite the north pole of the vibrator.

The field in which the vibrator is now placed is that produced by the additi of the earth's field, and that due to the bar magnet in the position chosen.

Find the number of vibrations made by the vibrator in one minute in th field. Suppose it is n_1.

Then $n_1^2 - x^2$ is proportional to the effect due to the bar alone in th position.

Lower the bar magnet by 1 cm., and again find the number of vibrations one minute (n_2).

Then $n_2^2 - x^2$ represents the effect due to the bar magnet in this seco position.

Continue to lower the bar magnet in steps of 1 cm., and take similar obser tions until the centre of the bar is reached. Tabulate the numbers.

Distance of Vibrator from End of Bar Magnet.	Strength of Field due to Bar Magnet.
0	$n_1^2 - x^2$.
1 cm.	$n_2^2 - x^2$.
2 cm.	$n_3^2 - x^2$.
...	...

Make a graph of the numbers obtained. The curve shows the distribu along the bar of the magnetism which produces its external field.

Note that it is greatest near the end, and falls to nothing at the middle. as the graph is not a straight line the fall is not proportional to the distance f the pole.

324. THE METHOD OF DEFLECTION.—In some measu ments not the time of vibration but the deflection of the magne

Fig. 362.—Deflection Magnetometer

observed, and when this is the case a **deflection magnetomete** used. Fig. 362 shows a simple form. The magnet is *short*, generally carries a long pointer moving over a circular scal degrees. Two long arms bearing millimetre scales pass out at r angles to two sides of the box.

The compass box shown in fig. 336 D, placed on a millim scale, constitutes such a magnetometer.

Ex. 511.—To compare the magnetic moments of two magnets.

(1) Place the magnetometer so that the two arms are at right angles t magnetic meridian, and note the zero position of the magnetometer needle

Call the two magnets A (moment M_1) and B (moment M_2). Place one on each arm at different distances, with their north poles turned towards the needle. Adjust the positions of A and B until the magnetometer needle stands in its zero position, so that it is not affected by A and B. Let the distance of the centre of A from the centre of the magnetometer needle be d_1, and the corresponding distance for B be d_2. Record d_1 and d_2. Reverse the direction of both magnets, and again take d_1 and d_2 as close as possible to the previous distances. Interchange the two magnets and take both observations again with as nearly as possible the same values of d_1 and d_2.

This gives four values of d_1 and of d_2. Take the mean of each set.

$$\text{Then } M_1 : M_2 :: (d_1)^3 : (d_2)^3.$$

The whole set of observations may be repeated for other distances.

The ratio given above is only approximate; a more exact result is obtained by using the result arrived at in Art. 325.

(2) Using the same two magnets as before, remove B far away, put A on one arm with its centre at distance d from the centre of the needle: the magnet is deflected; read the deflection. Changing the direction and side of A as indicated in the previous exercise, but keeping the distance d the same, get four deflections and take their mean; suppose it is Q°.

Repeat the observations with magnet B at the same distance d, and obtain the mean deflection R°.

$$\text{Then } M_1 : M_2 :: \tan Q : \tan R.$$

For the reason of this see Art. 143, where K represents the deflecting force of the bar magnet, and E represents the restoring force due to the earth's magnetism

The experiment may be repeated at a different distance.

Compare the result with that obtained by the different method above.

The strength of magnetic fields may also be compared by means of the deflection magnetometer.

Ex. 512.—To compare the strength of two magnetic fields H_1 and H_2.

Place the magnetometer in the desired position in field H_1, so that its needle (or pointer) is at zero on the scale.

Place a magnet (A) on one of the arms at a distance d, and read the deflection Q°.

Repeat the experiment in field H_2, using the same magnet (A) at the same distance d, and read the deflection R°.

$$\text{Then } H_1 : H_2 :: \tan R^\circ : \tan Q^\circ.$$

325. THE LAW OF INVERSE SQUARES.—The force of attraction or repulsion between two magnet poles of strengths m and m^1 at a distance apart equal to d is—

$$f = \frac{m \times m^1}{d^2}.$$

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The law of distance can be approximately verified by the deflection magnetometer.

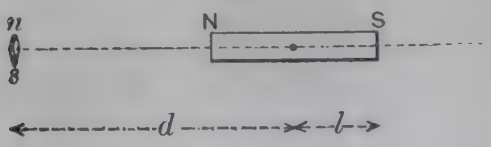

Fig. 363

Let ns (fig. 363) represent the short magnetometer needle, and NS the bar magnet, as set up for Exercise 511 (2). Then assuming the above law to be true—

the repulsion between N and n $= \dfrac{mm^1}{(d-l)^2}$,

the attraction between S and n $= \dfrac{mm^1}{(d+l)^2}$,

and the result of these two reactions $= mm^1\left\{\dfrac{1}{(d-l)^2} - \dfrac{1}{(d+l)^2}\right\}$.

The attraction of N on s, and the repulsion of S on s double this result, so that the total force exercised by NS on ns

$$= f = 2\,mm^1\left\{\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right\};$$

but this is equal to H tan Q where H is the strength of the earth's horizontal field and Q° is the deflection of the magnetometer needle. Hence, omitting the constant quantities m, m^1, and H, we have—

$$\frac{4dl}{(d^2-l^2)^2} \text{ is proportional to tan Q.}$$

Ex. 513.—(1) Take a set of observations as indicated above with the deflection magnetometer. Measure d and l, and read Q: and calculate the value of $\dfrac{4dl}{(d^2-l^2)^2} \div \tan Q$. Using the same deflecting magnet, with different values of d, different values of Q are obtained. Calculate the value of the above expression for each pair.

If the results are approximately constant, the experiment approximately verifies the law $f \propto \dfrac{1}{d^2}$ given above.

(2) If the bar magnet is very long and the vibrating magnet very short the law of inverse squares may also be approximately verified by the method and apparatus of Exercise 510.

Take a similar series of observations to those in that exercise, but instead of moving the bar magnet vertically, move it horizontally away from the vibrator, so as to keep the centre of the vibrator and the south pole of the bar magnet always in the same horizontal line.

Compare the successive values of $(n^2 - x^2)d^2$. If they are constant it indicates that the force between the needle and the south pole of the bar magnet falls off proportionately to the square of the distance.

CHAPTER XL

Electrification by Friction

326. ATTRACTION BETWEEN RUBBED BODIES.—Electrical effects are numerous, and some of these effects may be produced in various ways. One of the main distinctions that arise from experiments is between electricity that appears to us to be stationary and electricity in motion in the form of a current. As in elementary practice the former is usually developed by friction, it is customary to deal with frictional electricity separately from current electricity.

The materials used in these experiments must be warm and dry.

Ex. 514.—(1) Rub a stick of sealing wax on flannel, and hold the rubbed end of the stick near fragments of paper, feathers, pith, gold leaf, or similar light material (fig. 364).

Note the effect on the light bodies.

(2) Repeat the experiment with rods of resin, shellac, sulphur, and ebonite that have been rubbed on flannel or fur, such as catskin, and also with a rod of glass that has been rubbed on silk. (The silk is better undyed.)

(3) Hold the rubbed end of the rod near a loose part of someone's hair.

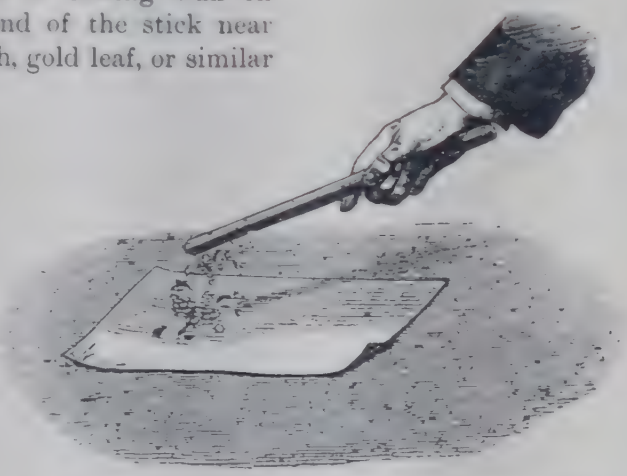

Fig. 364.—An Electrified Rod attracting Light Bodies

In these experiments the light bodies near the rod are attracted by and move towards the rubbed rod.

Ex. 515.—Hold the rubbed end of a rod about 1 mm. from the bent knuckle of one of your fingers.

Note the crackling sound and the small spark.

Ex. 516.—(1) Draw a piece of warm white silk ribbon between the folds of a warm sheet of indiarubber, and quickly place the ribbon loosely against the wall of a room. It will stick to the wall for a time.

(2) Warm a sheet of clean brown paper, place it on a dry board, and rub it with a sheet of indiarubber or a brush. The sheet may then be lifted off by two corners and laid flat on a wall. It will generally stay in that position for some time.

Ex. 517.—(1) Balance a light wooden lath on a suitable glass support, such as a watch glass, the spherical bottom of an inverted flask (fig. 365), or the side of an ordinary flask.

Hold one end of a rod that has been rubbed as in the previous experiments near to but not touching one end of the lath, and at the side of the lath.

By withdrawing the rod before the attracted lath, the latter may be caused to rotate round its point of support.

(2) Place the rubbed end of the rod in a beaker or flask, and see whether the attraction takes place through the glass.

Interpose a sheet of paper or a thin layer of any convenient material. Does the attraction take place through the material?

Fig. 365

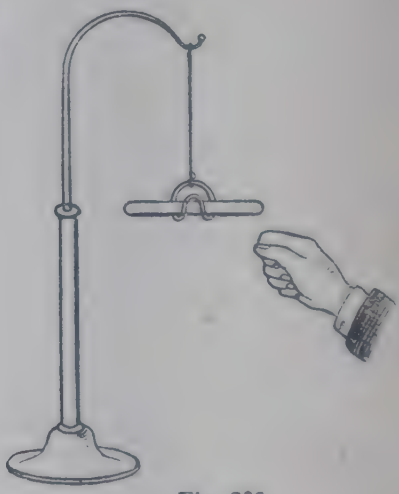

Fig. 366

Attraction between Electrified Bodies

Ex. 518.—(1) By means of thin cotton or silk suspend a stirrup of stou wire (fig. 366).

Rub one of the rods and suspend it in the stirrup. Hold the knuckle nea the rubbed end of the rod. Is the rod attracted?

Hold various objects near the suspended rod. Is the rod attracted?

(2) Repeat the experiment with the different rods rubbed as before.

The preceding Exercises 514 to 518 show that certain ma terials when rubbed with other suitable material acquire the pro perty of attracting bodies. Such bodies are said to be **electrified**. They also show that the attraction between electrified and othe bodies is mutual. Each attracts the other. A force exists betwee them, just as there exists a force between two magnet poles o between two portions of matter.

This electric attraction takes place across space, as did the othe forces mentioned.

[1] The term is derived from the Greek word for amber, *elektron*. Amber was one of t first substances with which the above experiments were made.

On a body that has been thus electrified there is said to reside a **charge of electricity**, and sometimes the body is said to be electrically **excited**.

The electric charge is stationary, not a current such as a voltaic cell causes to pass round a circuit (Art. 346).

327. TWO KINDS OF ELECTRICITY.

Ex. 519.—(1) Suspend a stirrup as in fig. 366, and have at hand dry, warm rods of (*a*) glass, with a silk rubber; and

(*b*) Ebonite, resin, shellac, sulphur or sealing wax, with a rubber of flannel or fur.

Rub the glass rod briskly with the silk rubber throughout nearly its whole length, and without handling it more than necessary place it in the stirrup.

Excite one of the rods (*b*) above, and hold it near to and at one side of the end of the suspended glass rod. Do not allow the rods to touch. The motion of the glass rod may be stopped and reversed by placing the ebonite rod on the other side of it.

Here we have two charged rods acting on each other. Is the result attraction or repulsion?

(2) Repeat the experiment with the charged ebonite or (*b*) rod suspended and the charged glass rod held in the hand. The result is the same.

Ex. 520.—Make an exactly similar experiment to the last, using—

(i) Two glass rods that have been rubbed with silk;

(ii) Two rods and rubbers either alike or different, but chosen from the list (*b*) above.

Is the suspended rod attracted or repelled?

The contrast between the results of Exercises 519 and 520 is important. In one a charged rod attracted another charged rod: in the other a charged rod repelled another charged rod. All the rods would have attracted light bodies as in Exercise 514.

From these facts it is concluded that there are two kinds of electricity: they are called **positive** and **negative**.

When glass is rubbed on silk, the glass becomes positively electrified. When ebonite or sealing wax or any substance in list (*b*) above is rubbed on fur or flannel, the substance becomes negatively electrified.

A positively electrified body is said to have a **positive charge**, and a negatively electrified body a **negative charge**. When two bodies have a charge of the same kind, both positive or both negative, they are said to be **similarly charged**. The results of the preceding experiments may then be stated as follows:—

Similarly charged bodies repel each other: dissimilarly charged bodies attract each other.

Compare this statement with that concerning magnet poles (Art. 312). Note that the electrified rods have no poles.

Ex. 521.—(1) Suspend a positively charged rod (glass rubbed on silk). Rub rods of shellac, resin, sealing wax on silk, and test the charge on each rod by holding it near the positively charged rod.

Suspend a negatively charged rod (ebonite rubbed on fur), and test again.

(2) Rub glass on fur, and find out whether the glass is now positively or negatively electrified.

(3) Rub ebonite on a piece of thin leather which has been coated with electrical amalgam,[1] and find out the nature of its charge.

The substances hitherto employed are those by which such experiments are most easily made. But most substances are capable of being electrified by friction in a similar way to those used above.

Whether the charge on the rod is positive or negative depends on the material of which the rod and the rubber are made.

328. THE ELECTROSCOPE.—Any apparatus by which we can determine whether a body is or is not electrified is called an **electroscope**. The light bodies (fig. 364) and the light suspended lath (fig. 365) are primitive electroscopes.

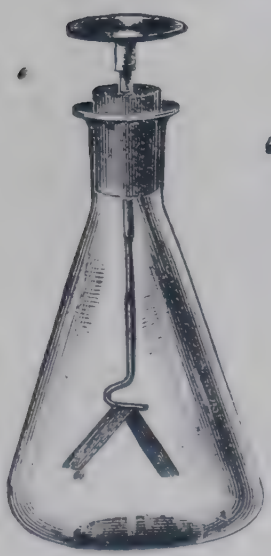

Fig. 367

Fig. 368

Forms of Electroscope

The instrument shown in fig. 367 is called a **gold-leaf electroscope**. A strip of gold leaf is suspended by its middle on a thick metal wire. The wire and leaves are inside a glass flask or similar vessel. The stopper of the flask is made of paraffin wax, or shellac, or ebonite. The wire passes through the middle of this stopper, and ends at the top in a metal plate.

The gold leaves are fragile, and should not be jarred or subject to any violent movement.

[1] To make electric amalgam, heat 1 grm. of tin and 2 grm. of zinc in a crucible until they melt; add 6 grm. of mercury; stir; allow to cool; reduce to powder. Before applying it to the rubber make it into a paste with lard.

Fig. 368 shows another form of electroscope with which some experiments are more easily made. It is merely a rod of ebonite supported at its middle on a sharp point, as are the magnet needles in fig. 336. Exercise 521 shows that such a rod may be electrified either negatively (by rubbing it on flannel) or positively (by rubbing it on leather covered with mercury amalgam); and, if necessary, one end may be electrified positively and the other end negatively.

329. A CHARGE OF ELECTRICITY IS TRANSFERABLE.

Ex. 522.—(1) Excite a rod, and bring it up *near* to the cap of an electroscope. If the leaves diverge too widely wait until their action is moderated. When this is the case rub the rod gently over the cap and withdraw it.

What is now the position of the gold leaves?

If the electroscope be a good one and circumstances are favourable the leaves will remain diverged for a considerable time. The leaves are similarly charged, and are therefore repelling one another.

Thus electricity has been transferred by contact from the rod to the electroscope. Did similar transference take place in Exercise 514?

(2) Repeat Exercise 514, and note whether any of the small bodies fly off after contact. Why do they do so?

(3) Charge an electroscope positively in the above manner. Discharge it by touching the cap with the finger. (Art. 332.)

Charge it negatively, and discharge it in the same way.

Ex. 523.—Charge an electroscope positively by contact. While the leaves are diverging to a moderate extent give the electroscope a small negative charge What is the effect on the leaves?

Make the experiment several times, and endeavour to arrange that the divergence of the leaves shall be just destroyed by the second charge.

These experiments show that both kinds of electricity can pass from one body to another; and that if a negative charge of the right amount be communicated to a positively charged body the result is that the body has no charge at all.

Ex. 524.—Charge a gold-leaf electroscope positively by contact.
Bring up separately *near* to the plate, *but not so as to touch*—

(*a*) A positively charged rod;
(*b*) A negatively charged rod;
(*c*) The hand.

Note the effect on the divergence of the leaves in each case.
Charge the electroscope negatively, and repeat the three observations.

It is seen that when the charge on the rod is the same as that on the leaves, the leaves diverge more widely.

By this means we can always find out what is the kind of charge on a rod, if we know the kind on the electroscope: or if

we know the kind of charge on the rod, we can find what kind of charge is on the electroscope.

330. CHARGING A BODY BY INDUCTION.—It is generally more convenient to charge an electroscope, not by contact, but by induction. The process of induction is explained later (Art. 335).

Ex. 525.—To charge a gold-leaf electroscope by *induction*.

Hold a negatively charged rod *near* to the cap of the electroscope so that the leaves diverge. Then in this order (see fig. 372)—

touch the cap with the finger,
remove the finger,
remove the rod.

The leaves remain diverged, showing that they are charged.

By the method of Exercise 524 show that the charge given by this means to the electroscope is of the opposite kind to that on the rod which induced it.

331. BOTH KINDS OF ELECTRICITY PRODUCED AT THE SAME TIME.

Ex. 526.—(1) Charge with positive electricity an electroscope having a metal cup standing on its cap.

Holding the silk rubber inside a sheet of indiarubber or wearing indiarubber gloves, rub glass on silk, and drop the silk into the can. From the effect on the leaves, can you infer the electrical condition of the silk?

(2) Charge the electroscope with negative electricity and repeat the experiment. From the effect on the leaves infer the electrical condition of the silk.

Both results show that the rubber is electrified as well as the rod.

The difference between the results shows that the rubber was negatively electrified, and we know that the rod itself was positively electrified.

Ex. 527.—(1) Arrange a warm dry rubber round the inside of a warm dry glass beaker. Holding the beaker by the outside, excite a rod on the rubber, and shoot out the rubber into a metal cup standing on the cap of an uncharged electroscope.

What is the effect on the leaves?

Quickly place the rod also in the metal cup.

What is now the effect on the leaves?

(2) Repeat the experiment with other pairs of materials—ebonite and fur, sealing wax and flannel, &c. Endeavour to communicate the charge on the rod and the charge on the rubber at the same time to the electroscope.

Experiments of the kind described in the last two exercises show that when two bodies are rubbed together both become electrified.

one positively and the other negatively, and that the amounts of the charges are equal.

332. CONDUCTORS AND INSULATORS.

Ex. 528.—Charge a gold-leaf electroscope.

(1) Touch it with the finger. The behaviour of the leaves shows that the charge has left them. The electroscope has been discharged.

(2) Charge the electroscope again, and touch it with an unexcited rod of sealing wax. What is the effect on the leaves? The charge remains on them.

(3) Charge the electroscope, and repeat the experiment with the following substances unexcited: rods of glass, ebonite, shellac, resin, metal, wood; rolls of paper, cotton material, silk material; pieces of dry carbon, straw, slate, india-rubber, crockery.

Make a table of the results, stating for each substance the effect on the leaves of the electroscope.

In the cases in which the leaves collapsed the charge left them owing to their being touched. When they remained unaffected it showed that the charge remained on the leaves. If they partly collapsed a part of the charge had left them.

Ex. 529.—(1) Place a reel of thin copper wire on the cap of a gold-leaf electroscope. Fasten one end of the wire to a flannel rubber; holding the flannel inside a sheet of indiarubber, excite a rod of sealing wax. See that the wire does not touch the table or any object.

Are the leaves of the electroscope affected?

(2) Repeat the experiment with a reel of cotton, a reel of silk.

If the leaves of the electroscope diverged, it is clear that some of the electricity produced by the friction passed along the connecting thread or wire.

The results of Exercises 528 and 529, and others of like character, show that some substances conduct frictional electricity well, others conduct it badly, and others practically not at all. Substances of the last class are called **insulators**; the others **conductors**.

The human body is a comparatively good conductor for frictional electricity. The discharge of the electroscope in Exercises 522 (3) and 528 (1) was produced by the passage of the electricity through the body to the earth. The rod of sealing wax prevented this passage. Sealing wax is therefore an insulator.

Exercise 528 (2) showed that glass, ebonite, crockery, silk are insulators, while metals are good conductors. Porcelain insulators are used to support telegraph wires, and wires that are buried in the ground or in the sea are surrounded by indiarubber or some such substance. Parts of electrical apparatus that require to be insulated are usually supported on glass legs (fig. 370), and the glass is often coated with shellac or resin, which are also good insulators.

333. ELECTRIFICATION OF METALS BY FRICTION.

Fig. 369

Ex. 530.—(1) Charge an electroscope, and touch it with a compound rod, half of which is glass and half metal (fig. 369), holding the rod by one end and touching the electroscope with the other end. Try both ends in this way. The behaviour of the leaves shows that the charge remains undisturbed.

(2) With the electroscope uncharged, holding the compound rod by the glass end, rub the metal end with flannel which is held inside a sheet of dry indiarubber, so that the hand does not touch the flannel or metal. Bring up the rubbed metal near to the electroscope, and test whether the rod is charged.

(3) Lightly strike the cap of an electroscope with fur or flannel. The deflection produced shows that electricity was produced by the friction of the metal and the fur.

This exercise shows that metal rods are electrified by friction, but that the reason why we do not generally use them for such experiments as Exercise 514 is that the electricity escapes through the body to the earth. When this escape is prevented, the same results follow as with other substances.

334. ELECTRIC CHARGES RESIDE ON THE SURFACE.

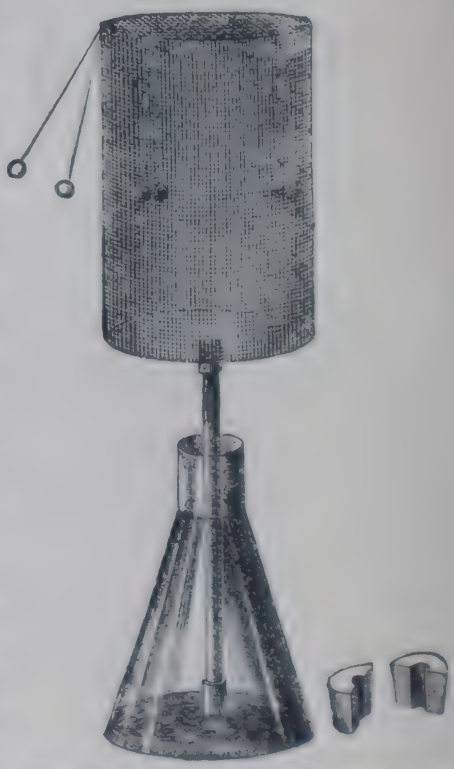

Fig 370.—Electric Charges reside on the Outer Surface

Ex. 531.—(1) Bring up a charged rod *near* to but not touching an uncharged electroscope. The leaves diverge. Touch the rod with the finger. The leaves show no change, indicating thus that the charge remains on the rod.

Wipe the rod along with the hand or a damp cloth. Present it again to the electroscope. The absence of an effect on the electroscope shows that the charge on the rod has been removed by wiping its surface.

(2) On an insulating stand (fig. 370) place a metal calorimeter or a cylinder of wire gauze over the edge of which hang some pith balls on cotton threads. Give a charge to the cylinder. The pith balls that touch the outside of the cylinder are repelled; those that touch the inside are not affected.

335. ELECTRICAL INDUCTION.—In several of the previous exercises it was found that the electroscope was temporarily elec-

trified when a charged rod was simply held near it. A temporary charge produced on a body in this way is called an **induced charge**, and the process is called **electrical induction**.

Ex. 532.—On an insulating stand support a metal rod or stiff wire AB (fig. 371), having suspended from it by cotton threads pairs of pith balls, those of each pair hanging in contact. Bring up near one end of the metal rod a positively electrified body C. The pith balls diverge, showing that they are charged. Keeping C in position, bring up another electrified body underneath the pith balls, and test their charge by the method of Exercise 524. It is found that the end A of the rod is negatively electrified, and the end B positively electrified.

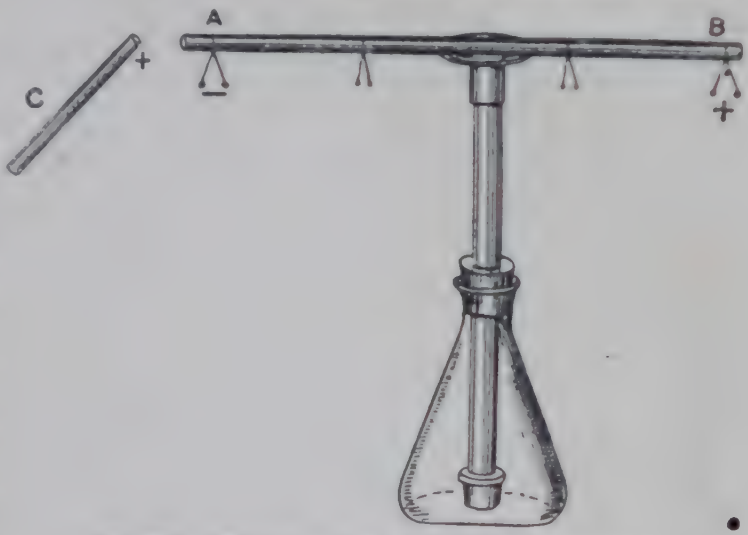

Fig. 371.—Electrification of a Body by Induction

Move the rod C to and fro nearer to and farther from the end A. Note that the amount of the electrification of AB depends on the nearness of C.

With C still in position touch the rod AB for an instant. Note the behaviour of the pith balls. After collapsing they again diverge. Test again the nature of their charge. They are now found to be all negatively electrified. The whole rod AB has now a charge of negative electricity. The positive charge escaped to the earth when the rod was touched.

Remove the rod C. The conductor AB is now negatively charged, and retains its charge for a time.

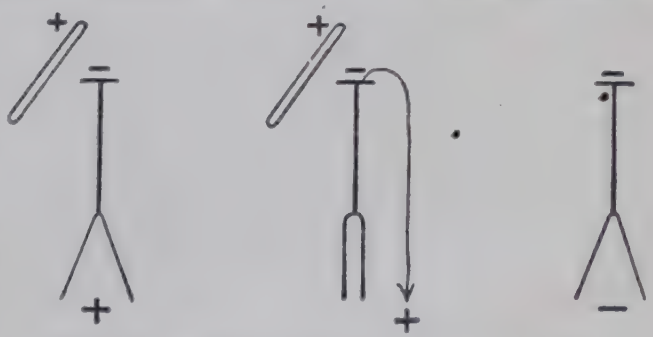

Fig. 372.—Charging an Electroscope by Induction

Ex. 533.—The process in the last exercise was precisely that adopted in Exercise 525 in the charging of an electroscope by induction. Fig. 372 illustrates the stages of the process. Repeat it, and compare the behaviour of the gold leaves with that of the pith balls in the previous exercise.

In Exercise 532 the rod AB was electrified though no electricity passed into it from C. Moreover, both kinds of electricity appeared

on AB in the presence of C, and on the removal of C both these disappear. The conclusion drawn from this is that every piece of matter contains both positive and negative electricity in equal quantities, so that usually no sign of electrification is manifested. But when an electrified body is brought near to a neutral body, the two electricities separate and become manifest.

CHAPTER XLI

Electric Machines. Condensers

336. FRICTIONAL ELECTRICAL MACHINE. — Much greater quantities of electricity than can be obtained by rubbing rods are got by means of machines, of which a common form is shown in fig. 373. A glass plate CC, mounted on a spindle, is caused to rotate by means of a handle A. As it rotates it rubs against two silk rubbers FF′, and the rubbed parts become electrified. A conductor BDEB, made of brass rods, is also supported by the wooden stand that carries the spindle of the plate; and the two U-shaped pieces that curve round the edge of the plate are furnished with a number of sharp needles whose points are close to the plate.

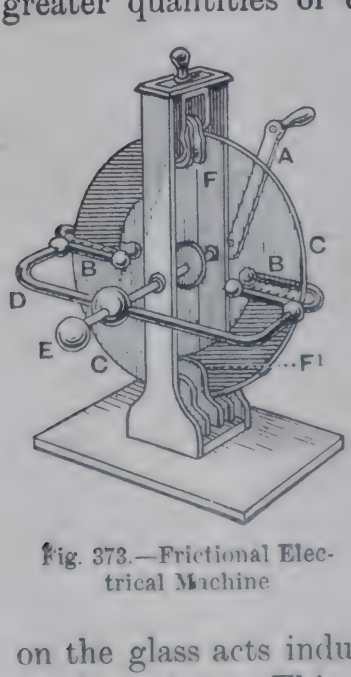

Fig. 373.—Frictional Electrical Machine

When the electrified area of the glass reaches these points the positive electricity on the glass acts inductively on BB, and attracts negative electricity to the points. This negative electricity streams off the points on to the plate, which thus passes on in a neutral condition to the next rubber.

The insulated conductor BDEB is left positively charged, and large sparks can be obtained from the knob E.

The negative electricity produced on the rubbers escapes through the frame of the machine to the earth.

337. THE ELECTROPHORUS. — By using an instrument based on the process of induction many charges may be obtained

with only one excitation. Such an instrument, called an **electrophorus**, is shown in fig. 374. It consists of a plate of ebonite (or shellac) AB having fixed to its underneath side a thin plate CD of metal (tinfoil will do), and a stiff movable metal lid EF, which can be put on AB or raised from it by means of the insulating handle G.

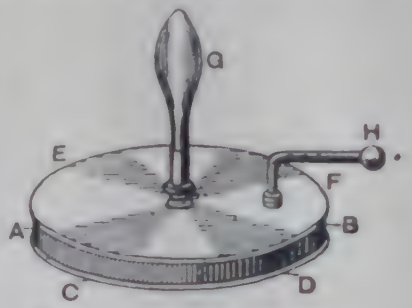

Fig. 374.—The Electrophorus

Ex. 534.—(1) Rub the plate AB with dry flannel; place EF on AB; touch EF with the finger; lift EF by the handle; present the knuckle to the knob H or to the edge of the plate. A spark passes.

Again place EF on AB and repeat the operation several times.

(2) Bring the charged plate EF *near* to a charged electroscope, and ascertain the nature of the charge (Exercise 524). Is the plate charged positively or negatively?

(3) Repeat the operation of (1) above but without touching EF. Test the charge on EF now. If there is any, it is a negative charge.

(4) Instead of discharging EB by the finger, hold it near an insulated conductor (fig. 379). When the spark passes, the conductor becomes charged. Test it. Charge the conductor strongly by allowing several sparks to pass to it. Considerable charges may be got on conductors in this way.

The explanation of the action of the electrophorus can be understood by reference to fig. 375.

The upper surface of the plate AB is electrified negatively by the friction. This electrification produces by induction a positive charge on the sole CD. This charge comes from the earth.

Fig. 375.—Action of the Electrophorus

When EF is placed on AB the two plates only touch at a few points *nn*, and most of the charge on AB remains there, and acts inductively on EF, producing a positive charge on the lower face of EF and a negative charge on the upper face of EF. When the plate EF is touched, the negative charge on it escapes to earth, leaving EF positively electrified.

The charges on AB and CD still remain there when EF is removed.

338. THE WIMSHURST MACHINE.—This machine, illustrated in fig. 376, is based on the principle of the electrophorus. Two glass plates, parallel and close together, are made to revolve in opposite directions. Round the circumference of each plate on its

outer side is fixed a number of tinfoil disks or sectors. Placed diagonally across each plate is a metallic arm, which ends in two brushes that rub against the sectors. Suppose that one of the sectors A has a small positive charge to start with. On rotating the plates induction takes place on the opposite sector B, which is in contact with the diagonal arm, with the result that sector B becomes negatively electrified, and sector C at the other end of the diagonal arm becomes positively electrified.

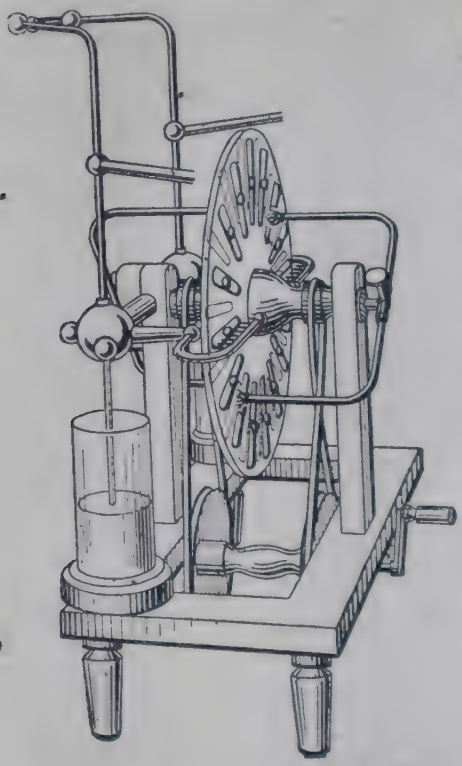

Fig. 376.—The Wimshurst Machine

This takes place with all the sectors in turn, so that the top of the front plate and the bottom of the back plate become positively electrified, while the bottom of the front plate and the top of the back plate are negatively electrified.

The electricity is collected by two horizontal combs that are joined to two knobs forming the terminals of the machine, one of which is thus charged positively and the other negatively.

All electrical machines must be kept quite free from damp and dust.

Ex. 535.—Experiments with a machine.

(1) Examine a frictional or Wimshurst machine working in a dark room.

Note the appearance of the collecting combs and of different parts of the plate, and prime conductor.

Hold a large metal ball a short distance from a knob on the conductor of the machine, or from one of the terminals. Note the *brush* discharge to the ball.

Allow sparks to pass between the terminals, or from the prime conductor to a knob connected with earth, and observe the forms of sparks of various lengths.

Hold a sharp-pointed needle near to various parts of the machine and observe the effect.

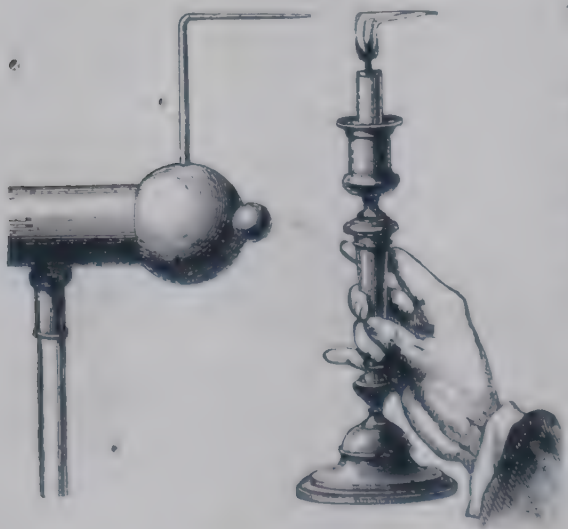

Fig. 377.—The Discharge from Points

Fix a pointed wire or needle to the conductor or to a terminal. Note the altered appearance of the machine. The electricity escapes from the point as fast as it is developed by the machine.

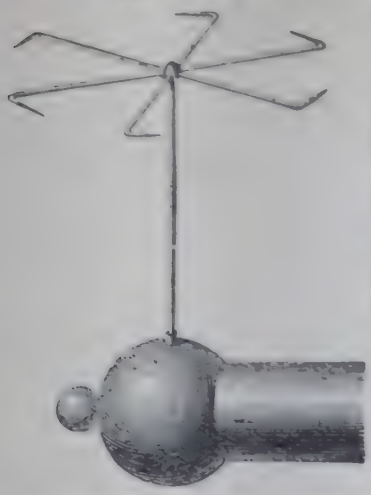

Fig. 378.—Rotation produced by Discharge

(2) Fix a bent pointed wire, as shown in fig. 377. Observe its effect on the flame of a candle.

(3) Replace the bent wire by a whirl (fig. 378). Electrified particles of air are shot off from the points, and the points themselves move backward (Art. 169).

(4) Present the knuckle to the prime conductor or terminal. Note the spark and the slight shock.

(5) Hold by its insulating stand a conductor (fig. 379) near the machine so that a spark passes. The conductor is charged.

Fig. 379.—An Insulated Conductor

Hold a second conductor very near the first; a small spark passes; the charge is shared between the two conductors.

(6) Stand on a board supported on four glass bottles; put a finger of one hand on the conductor of the machine.

Hold the other hand *near* an electroscope. Why are the leaves affected?

Sparks may be obtained from your other hand by another person standing on the ground.

Explain what has taken place.

339. LIGHTNING CONDUCTORS.

Ex. 536.—Stick one or two needles through a small piece of cork, and cover the cork with tinfoil. Drop this little apparatus on the cap of a charged electroscope. What is the effect? Can the electroscope be kept charged while the points are present?

This experiment, together with some of those in Exercise 535, illustrates the action of lightning conductors, which are sharp-pointed rods (fig. 380) projecting over the top of buildings, and extending downward into the ground.[1]

When a cloud charged with, say, positive electricity passes over the building, the earth and buildings immediately beneath it are by induction charged with negative electricity. Through the

Fig. 380. Upper Part of a Lightning Conductor

[1] Such a conductor is of course continuous, not disconnected as shown in the illustration.

pointed rod the two opposite charges quietly neutralize each other, and thus the violent lightning discharge is prevented.

340. CONDENSERS.—The **electric pane** is a plate of glass having a smaller coating of tinfoil on each side and the edges varnished. It is convenient to have two handles of silk cord (fig. 381).

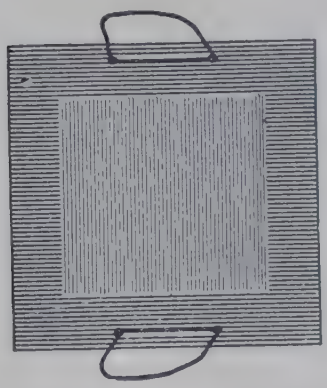

Fig. 381.—The Electric Pane

Ex. 537.—(1) Place the electric pane on the insulating stand shown in fig. 371 or on a large glass bottle, and charge its upper coating from a machine. Disconnect the pane from the machine, and touch the upper plate with the finger. Is there any shock?

(2) Place the pane on the table or connect the bottom coating with the earth. Again charge the plate. Disconnect the top plate from the machine, and touch it. A severe shock may be obtained in this way.

The contrast between the results of these two experiments shows that there was very much more electricity on the pane in the second arrangement than in the first. We say that in the second case the electricity was condensed, and such a

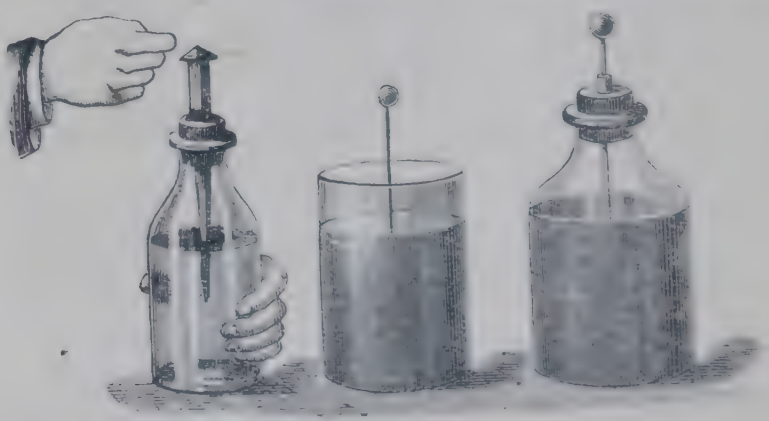

Fig. 382.—Forms of Leyden Jar

plate is a **condenser**. The effect is due to induction. Exactly as with the electrophorus, every small quantity of electricity communicated to the upper plate drew up from the earth an equal quantity of the opposite kind on to the lower plate, and under these circumstances the plates will hold much more than when they are far apart.

The **Leyden jar** is a more convenient condenser than the electric pane. In the elementary form shown on the left of fig. 382 the

water in the bottle forms one coating, the hand forms the other.

In the form shown on the right the bottom part of the bottle is covered inside and out with tinfoil, and a metal rod in contact with the inner coating passes through the stopper. The part of the stopper touching the rod in this and the previous figure must be made of ebonite, paraffin, or other insulating material.

The form shown in the middle is the best. The rod is weighted at the bottom, and touches nothing else, thus hindering leakage of the electricity. The upper part of the glass is covered with shellac varnish for the same reason.

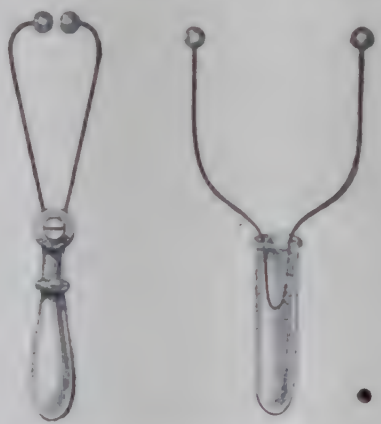

Fig. 383.—Discharging Tongs

Ex. 538.—(1) The jar being held in the hand or placed on a table, allows several sparks to pass between the knob of a machine and the knob of the jar. The jar is charged. Or the jar may be charged by several sparks from an electrophorus.

(2) If it be a small jar, discharge it by bringing up the knuckle near to the knob. Take care not to knock the jar over. If it be a large jar, place one of the knobs of the tongs shown in fig. 383 against the outer coating, and bring up the other knob of the tongs near to the knob of the jar. Hold the tongs by the glass handle only.

341. FREE AND BOUND CHARGES.

Ex. 539.—Charge the jar again, and place it on an insulating stand. Touch the knob; little or no shock is felt.

Discharge the jar by the tongs. The spark shows that the charge had not been removed by touching the knob.

The two charges on the opposite plates of a condenser are said to be **bound**. They attract each other, and one of them cannot be removed unless the other is at the same time removed or can escape. Compare with Art. 337.

An electric charge on a conductor which is not thus bound by another opposite charge is said to be **free**.

342. ATTRACTION PRECEDED BY INDUCTION. — In Art. 327 it was shown that an electrified body attracts another body which is oppositely electrified.

In Exercises 514 to 518 attraction took place between an electrified body (call it A) and another body (call it B), which appeared to be unelectrified. We now see that the body B was really electrified under the inductive influence of A. The attraction resulted

from the fact that the part of B near to A had a charge induced in it of the opposite kind, and an attractive force existed between these oppositely charged bodies.

343. ELECTRIC FIELD.—The facts of induction show that an electrified body exerts an influence on bodies round about it. The space round an electrified body is therefore in a special condition. It is full of **electric lines of force**, just as the space round a magnet is full of magnetic lines of force. The space occupied by electric lines of force is called an **electric field**. Any conductor placed in an electric field becomes electrified. Hence result the phenomena of induction.

Referring to fig. 355, if in place of N and S there be two balls connected to the two terminals of a Wimshurst machine the electric lines of force may be traced out. In place of the magnet AB, a little conductor is used. This may be made of two ordinary small pins tied together head to point, and suspended by a fibre of unspun silk from the end of a glass rod. Cover the stems of the pins with a little wax or shellac. As the little conductor is moved into different positions it takes up positions along the lines, and shows that they are (with a positive and a negative charge) of the same form as those shown in fig. 355.

Ex. 540.—Additional Exercises on Frictional Electricity.

(1) Charge a gold-leaf electroscope. Move the hand near to and away from the electroscope several times. Note the behaviour of the leaves. Regarding the hand as a conductor connected with the earth, explain the action of the leaves.

(2) Place a gold-leaf electroscope near to an electrical machine, and note the movement of the leaves as the machine is worked.

Place a metal plate between the electroscope and the machine, or cover the electroscope with wire gauze (not touching it). The electroscope is now screened from the action of the machine. The effects are due to induction. Explain them in detail.

(3) Explain why nearly all the apparatus connected with frictional electricity is made with rounded edges.

(4) Place a frictional machine standing on plates or blocks of glass. What difference does this produce in its working? Examine the way in which the rubbers are held, and, remembering Exercise 526, explain the effect.

(5) Discharge a Leyden jar with a sheet of thin paper held between its knob and the adjacent knob of the discharging tongs. After the spark has passed, note the small hole in the paper.

(6) Endeavour to make Experiment 532 with a rod of ebonite or other insulator in the place of the conductor AB (fig. 371). The effects of induction are not the same in an insulator as in a conductor.

(7) Place the cork and needles used in Exercise 526 on the top plate of the electrophorus, and see whether it will work as before. What is the cause of the difference?

CHAPTER XLII

Voltaic Cells

344. Elementary current electricity or voltaic electricity is principally concerned with the production of electricity by chemical means, its passage along wires, and the effects it produces during its passage.

The vessels and contents in which the chemical action takes place are called voltaic cells. They usually contain acid.

Acids must be handled with care, poured gently, and not spilled.

345. THE SIMPLE CELL.

Ex. 541.—Place a quantity of water in a glass vessel, and gently pour into the water about one-tenth or one-twelfth of its weight of strong sulphuric acid, stirring the mixture with a glass rod as the pouring slowly proceeds. (Do *not* pour the water into the acid.) Such a dilute solution of acid will be wanted for the ensuing experiments.

Pour dilute acid into a beaker until it is about two-thirds full, and perform the following experiments:—

(1) Place in the liquid a plate of ordinary commercial zinc. Is there effervescence? Do bubbles of gas appear?

(2) The zinc being removed, do the same with a plate of copper. Is there any action?

(3) Put a little mercury on the zinc plate, and rub it about with a brush or rag until the whole plate on both sides has a bright silvery appearance. The surface of the plate is now *amalgamated* with mercury. Place the plate again in the acid. Is there any action?

Sulphuric acid acts chemically on unamalgamated commercial zinc, but not on copper or on amalgamated zinc separately. Pure zinc is unaffected by the acid. This difference is due to the impurities present in commercial zinc, which produce **local action** (Art. 347). In an amalgamated plate the mercury has dissolved some of the zinc and made a surface layer which contains none of these impurities.

Ex. 542.—(1) Place the plates of copper and amalgamated zinc in the acid in the position shown in fig. 384 (1), so that they do not touch. Is there any production of gas?

(2) Allow the tops of the plates to touch, as in fig. 384 (2).

Is there any production of gas? Where does it appear?

(3) Remove the copper plate, rinse it with water, and again placing it in the acid, allow the bottoms of the plates to touch, as in fig. 384 (3). Is there any production of gas?

(4) Having removed the bubbles of gas from the copper plate as before, attach to each plate a **terminal** of the form shown in fig. 385. The plate is placed in the slot at the bottom of the terminal, and the screw is turned up tightly. Fix the plates upright in the liquid so that the liquid covers the plates up to within 2 or 3 cm. from the terminal. The plates should be as close together as is convenient, but not touching. A small piece of wood may be placed between them.

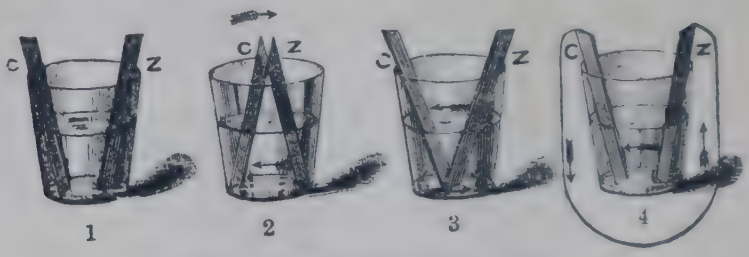

Fig. 384.—A Simple Voltaic Cell

Copper and zinc in liquid—1, No action. 2, Current across liquid and from C to Z. 3, Current across liquid and through the touching ends from C to Z. 4, Current across liquid and through wire.

This apparatus is called a **simple voltaic cell.**

Join the terminals by a piece of copper wire about 2 m. long, as shown in fig. 384 (4). The ends of the wire are placed in the hole at the top of the terminal, and tightly gripped by the screw.

The wire ordinarily used in electrical work is made of copper, and is covered with cotton or silk or gutta percha. The cotton must be scraped off for a length of about an inch at each end of the wire.

Fig. 385.—A Terminal

Fig. 386.—A Connector

If two pieces of wire are to be joined together, a metal connector such as is shown in fig. 386 is used. The connector is a hollow cylinder into which the wires are put and held by screws. In making such connections in electrical work clean metal must be firmly pressed against clean metal.

The plates being thus connected by a wire, bubbles of gas are again seen to form on the copper plate.

Thus it is shown that when a plate of copper and a plate of amalgamated zinc are placed in dilute acid there is no production of gas when they are not joined, but gas is produced when the plates are connected. There is no other directly visible effect; but several effects follow when suitable arrangements are made.

346. ELECTRIC CURRENT AND CIRCUIT.

Ex. 543.—(1) Place a pivoted magnet, such as is shown in fig. 336, on the table near the cell, and allow it to come to rest in its north-south direction. Then taking a length of the wire between the hands, as shown in fig. 387, hold it

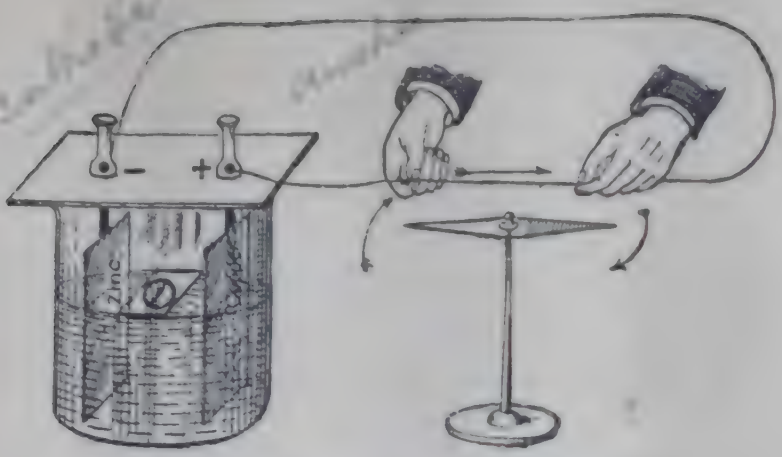

Fig. 387.—Magnetic Action of an Electric Current

that it is stretched straight, just over (but not touching) the magnet, and parallel to it.

When this is ready, to make sure that the cell is in working condition, lift the copper plate out of the acid for a moment, wipe it, and put it back.

If everything is right, the magnet is deflected.

Place the wire in an exactly similar position underneath the magnet. What is the effect?

(2) Place the cell so that the line joining its terminals is in the magnetic meridian. Across the cell place a flat piece of wood. On this piece of wood place a small compass box such as that shown in fig. 353.

Is the magnet deflected? Is the north pole turned towards the east or the west?

Lift the cell up and hold it over a delicately suspended compass needle. What is the effect?

(3) Disconnect one end of the wire from the cell, and see whether any deflection of the magnets can be obtained.

It is more convenient, instead of holding the wire in the hand, to wind it round a frame such as that shown in fig. 405.

Ex. 544.—Join the terminals of the cell to the terminals of a small electric bell by wires, as shown in fig. 388. If the plates of the cell are free from gas bubbles the bell may be set ringing. Should one cell be insufficient two should be used, arranged in series as explained in Art. 355.

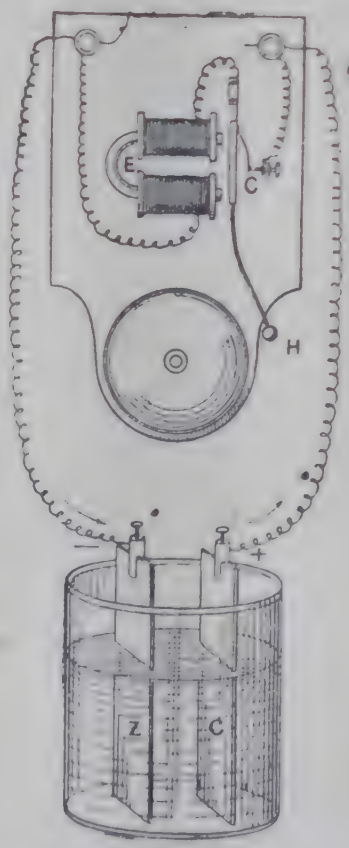

Fig. 388.—Action of an Electric Bell

These experiments show that something is taking place in the wire and in the cell when these arrangements are made, since a magnet is deflected

when it is near the wire or near the cell, and a bell is made to ring. We say that we have a **current of electricity** passing along the wires and through the cell. You may think of a current of electricity as a stream of very small invisible particles of matter which flow through the cell and along the wires as gas flows through pipes; and continuing the comparison, when a great number of such particles are passing along a wire, there is a **strong current** of electricity; when the stream is small, there is a **weak current**.

Such an arrangement as that set up in the last experiment, and shown in figs. 378 and 388, is called an **electric circuit**.

In Exercise 543 we simply joined the two plates of the cell by a single wire. In Exercise 544 we joined each plate by a wire to one terminal of a bell. The terminals of the bell are simply the two ends of wires which are joined to pieces of metal on the bell. So that in this case we had (omitting consideration of the spark gap) one plate of the cell connected with the other plate outside the cell by a continuous succession of wires and pieces of metal, which act just the same as a single wire.

While the bell was ringing, or the magnet was being deflected, along the wires, through the metal vibrator of the bell, and through the liquid in the cell, there passed a current of electricity. The current has its origin in the cell, and is regarded as coming out of the cell at the terminal of the copper plate, which is called the **positive pole** of the cell. Passing along through the wires the current enters the cell again by the zinc plate, which is called the **negative pole** of the cell. Continuing on through the liquid in the cell to the copper plate the current again passes out round the same path as before.

This complete closed path is the electric circuit. The path outside the cell is called the **external circuit**, that inside the cell the **internal circuit**. The external circuit is usually of metal, but may have in it certain other substances.

When we wish to set up a current of electricity a circuit must always be made. In every circuit there must be a cell or some other means of producing an electric current, wires to join the two plates of the cell, and generally there are instruments through which the current passes. These instruments are themselves very often merely coils of wire or vessels containing liquid.

When one of the wires was disconnected in Exercise 543 the effect on the magnet ceased. The current had been stopped.

It is convenient to place in every electric circuit an arrangement

by which the current can be started or stopped at any moment. This is done by means of a **key** or **switch**, of which two common forms are shown in fig. 389. A key consists of two pieces of metal fixed on a base with a gap between them. The wires of the circuit are joined to these pieces of metal, and a movable piece of metal allows the gap to be opened or closed as we please.

When a circuit is completed, so that there is no gap anywhere, it is said to be **made** or **closed**. Where there is a gap the circuit is said to be **broken**.

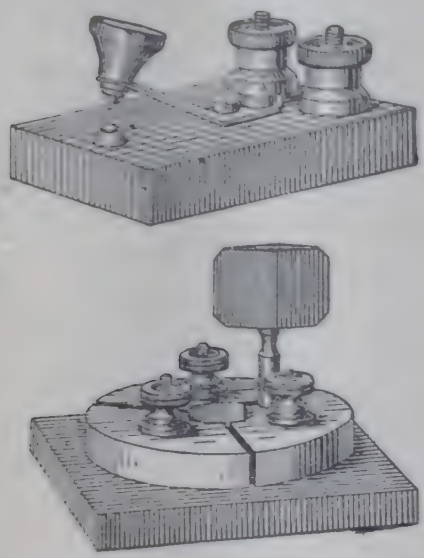

Fig. 389.—Electric Keys or Switches

A circuit is often represented by a diagram, as in fig. 390, where B represents a cell; G a galvanometer (Art. 359); K a key; R a coil of wire often called

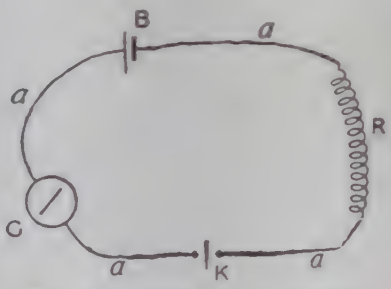

Fig. 390.—Diagram of an Electric Circuit

a resistance (Art. 368), and *aaaa* the wires by which these pieces of apparatus are joined together. When pieces of apparatus are joined together as in fig. 390, so that the current starting from the battery runs through them all one after the other, they are said to be joined **in series**. The order in which instruments are placed in series in a circuit does not matter.

When experiments with electrical currents fail it is often because the circuit is not really made. There is a gap in it. To avoid such failures, *terminals must be clean, and all screws tightly turned.*

347. SIMPLE CELLS OF VARIOUS MATERIALS.

Ex. 545.—Repeat Exercise 543 or 544 with the following pairs of plates: iron and zinc, carbon and zinc, and if possible platinum and zinc. If the plates be close together and not too small, and the magnet light and delicately suspended, deflections of the magnet will be obtained in each case.

Repeat the experiment with plates of any metals that are obtainable.

The results show that a simple cell may consist of various metals.

When the local action mentioned above is taking place in a cell, the small pieces of lead and iron that are present in the zinc form with the acid and the zinc itself a number of tiny cells which send currents in all directions along the zinc and acid surface. These local currents add nothing to the current which the cell sends round the external circuit, and have the harmful effect of using up the materials of the cell.

348. POLARIZATION OF THE SIMPLE CELL.

Ex. 546.—Set up the circuit of Exercise 543, and allow the current to pass round the circuit for several minutes. Note the appearance of the copper plate at intervals, and obtain a deflection of the magnet at intervals.

The deflection becomes smaller as time goes on. Remove the copper plate, rinse it in water, and repeat the observations.

The experiment shows that as the bubbles of gas increase on the copper plate the current given by the cell grows less and less. In time it becomes almost nothing.

This effect—the failure of the cell to continue to send a current—is due to the presence of hydrogen gas on the plate, and is called **polarization.** It renders all simple cells useless for practical purposes. Various cells have therefore been devised which are more or less free from this defect.

Fig. 391.—The Bichromate Cell

349. THE BICHROMATE CELL.

The simplest way of diminishing polarization is to add to the cell some substance which combines with and thus removes the hydrogen which causes the polarization. In the bichromate cell this is done by adding some bichromate of potash to the sulphuric acid.

The ordinary form of the bichromate cell is shown in fig. 391. The plates are of carbon and zinc, and the carbon plate is in two parts, one on each side of the zinc, but as they are connected by a strip of metal at the top they act as one plate. The zinc plate is between them, and is made to slide up and down. When the cell is not in use this plate is always raised up out of the liquid. The carbon is the positive plate.

350. DANIELL'S CELL.

One form of this cell (which

made in various shapes) is shown in fig. 392. The outer glass vessel contains dilute sulphuric acid, in which is placed the negative plate of zinc. In this vessel is placed another vessel made of unglazed porous earthenware, called the porous pot, which is filled with a solution of copper sulphate; and in the copper sulphate solution is placed the positive copper plate, and some solid copper sulphate crystals.

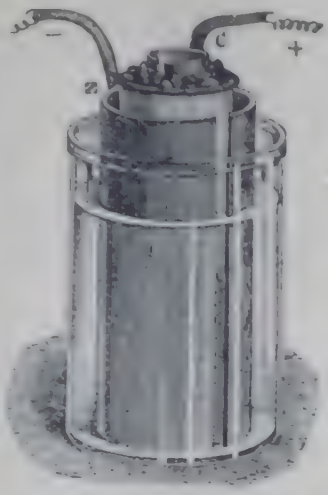

Fig. 392

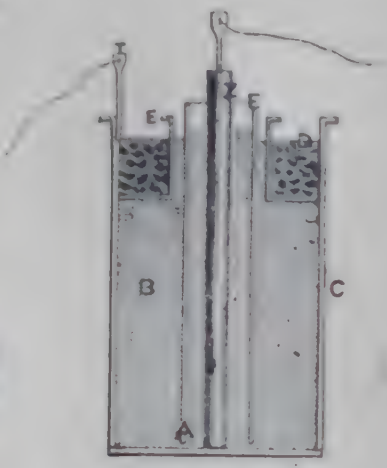

Fig. 393

The Daniell's Cell

When this cell is at work, the hydrogen gas that makes its way towards the copper plate is acted on by the copper sulphate, with the result that not hydrogen gas but solid copper is deposited on the copper plate. Thus polarization is almost completely prevented, and the Daniell's cell is very constant.

Fig. 393 shows a section of a different form of the same cell. In this form the outer vessel is of copper, and contains copper sulphate; the vessel itself being the copper plate of the cell. In this is placed the porous pot containing the dilute sulphuric acid and the zinc.

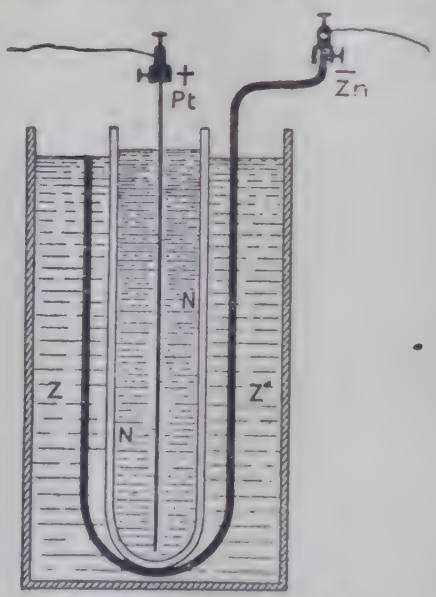

Fig. 394.—The Grove's Cell

351. GROVE'S CELL.—In this cell the metals are platinum and zinc, and there are two liquids—dilute sulphuric acid and strong nitric acid. Fig. 394 shows the cell in section. The outer vessel is made of glass or glazed earthenware, and contains dilute sulphuric acid. In this is placed the U-shaped zinc plate Z. Fitting into the zinc plate is a thin flat porous pot N, which contains nitric acid, and a thin platinum plate is placed in the nitric acid.

The hydrogen gas which is produced in the outer vessel by the action of the zinc on the sulphuric acid passes through the porous pot, and on coming into contact with the nitric acid enters into combination with it and thus ceases to exist as a separate gas. The cell is capable of producing a strong current of electricity, but must be put together every time it is used and taken to pieces when done with.

Fig. 395.—A Battery of Bunsen's Cells

352. BUNSEN'S CELL.—This cell is exactly like the Grove cell, except that a rod of carbon is used in the place of the platinum plate. Fig. 395 shows five Bunsen's cells joined together in series (Art. 355).

353. THE LECLANCHÉ CELL.—This cell, shown in fig. 396, consists of a glass vessel which contains a solution of sal ammoniac (ammonium chloride), and the zinc, which is usually of rod shape. In this is placed a porous pot that contains a rod of carbon packed round tightly with fragments of carbon and powdered binoxide of manganese.

Fig. 396.—The Leclanché Cell

When this cell is at work, the hydrogen gas on its way to the carbon plate is acted on chemically by the binoxide of manganese with the result that water is formed, and the hydrogen does not reach the carbon plate. But this chemical action is slow, so that after a short use this cell requires a rest to allow the chemical action to take place. Hence the Leclanché cell is used only when a current is required for a short time, as in ringing electric bells.

354. DRY CELLS.—Numerous cells are also made called

cells. They are very convenient to use, as there is no acid to spill. Each form of dry cell is made up of substances which act chemically on one another in such a way as to give the effects described in the previous forms.

355. ARRANGEMENT OF CELLS.—When several cells are used, the group is called a battery. The poles of the cells are joined together by clamps, or by wires connecting the terminals.

Fig. 397 shows two Grove's cells with the positive pole of one cell clamped to the negative pole of the next. Cells joined in this way are said to be connected **in series**. Any number of cells may be so connected.

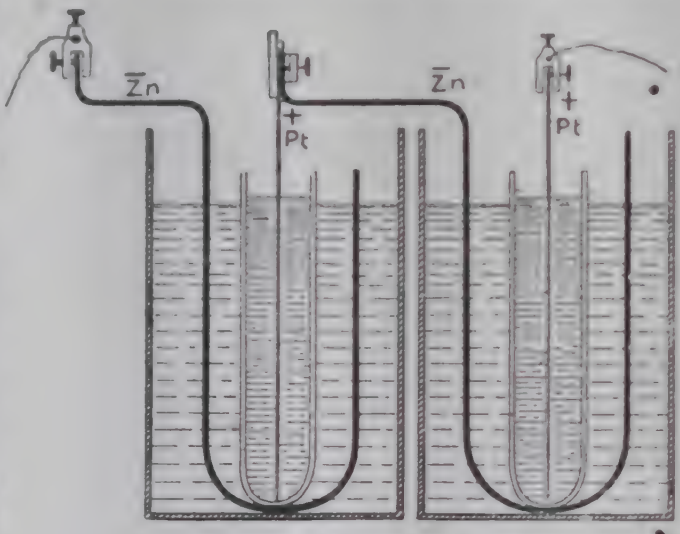

Fig. 397.—Cells joined in Series

When the positive poles of two or more cells are joined to each other, and the negative poles similarly connected to each other, the cells are said to be connected **in parallel**. Fig. 398 shows four Leclanché cells joined in parallel.

The positive and negative terminals of the battery are joined to the circuit in the same manner as a single cell.

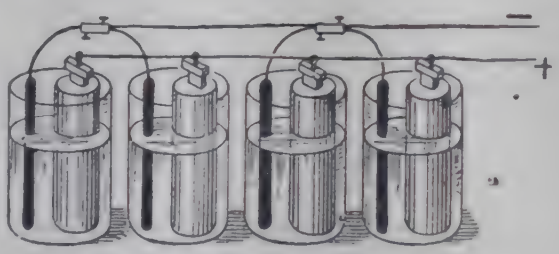

Fig. 398.—Cells joined in Parallel

In every electrical operation there is a certain strength of current that is suitable for it. Combinations of cells in batteries enable us to obtain any desired strength of current.

For the experiments that follow—

(1) A strong current may be obtained from three or four Grove's or Bunsen's cells in series.

A moderate current from two Daniell's or bichromate cells in series.

A weak current from a single cell.

(2) For a steady current use Daniell's cells if possible.

(3) In the use of any cell or battery—

Do not let the terminals touch each other, nor join them by a short wire, unless specially instructed to do so.

Do not leave current running round a circuit when it is not required.

CHAPTER XLIII

Magnetic Effects of an Electric Current

356. MAGNETIC FIELD OF A CURRENT. — The means by which an electric current was detected in Exercise 543 was by its effect on a magnet, since such a current can only be detected by its effects. The relations of currents of electricity to magnetism are of great importance.

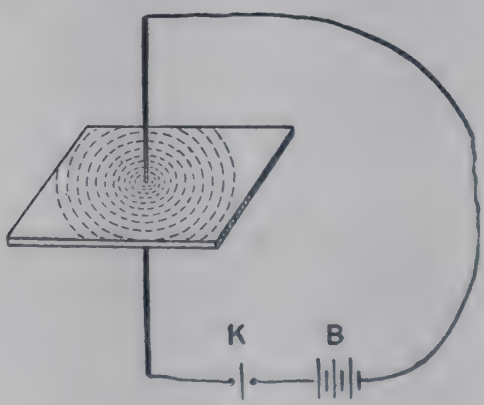

Fig. 399

Ex. 547.—For this experiment a strong current is required.

Arrange in series a battery, a key, and a piece of thick straight wire. Let the thick wire pass vertically through a smooth horizontal board (or a flat piece of stiff cardboard), as shown in fig. 399. Sprinkle a small quantity of fine iron filings on the board near the wire. By means of the key make the circuit, and lightly tap the board with a pencil. If the current is strong enough the filings range themselves in concentric circles round the wire. As soon as they are formed break the circuit.

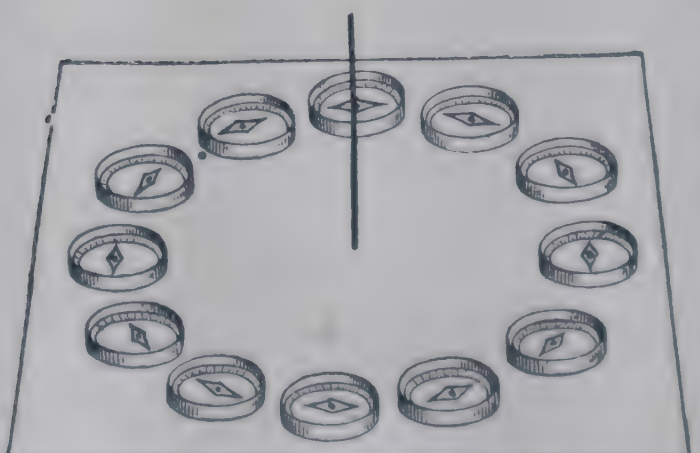

Fig. 400

Magnet Field round a Straight Wire carrying a Current

This experiment shows that round a wire that is carrying a current there is a magnetic field of the same nature as has been shown to exist round about a magnet. The space is full of magnetic lines of force that act on magnets and on soft iron.

Ex. 548.—To trace the lines of force round a wire carrying a current.

(1) With the same arrangement as in the last exercise trace out one or two of the lines of force by means of a very small magnet, as was done in Exercise 498. Fig. 400 shows the position that the magnet takes up in different positions.

The magnetic lines of force are circles round the wire that is carrying the current.

Note the terminals of the battery, and the direction in which the current is flowing through the wire, and in which direction the N pole of the magnet turns.

(2) Interchange the connections of the wires, so that the current flows through the wire in the opposite direction. In which direction does the magnet now turn?

It is found that if the direction in which the current is travelling in the wire be represented by the forward motion of a screw, then the direction in which the N pole of the magnet moves is the direction in which the circumference of the screw rotates. The S pole moves of course in the opposite direction.

Fig. 401 illustrates the effect when a coil of wire is used instead of a single wire for Exercise 547. Owing to the increased effect due to the many turns of wire (each one of which increases the magnetic effect), a very strong battery is not required. The filings arrange themselves in curves around each side of the coil, the lines running near the axis of the coil being practically straight. Compare fig. 401 with fig. 352. The two sets of lines of force are repelling each other. This fact is connected with the fact that in the two sides of the coil the current is travelling in opposite directions—one upward the other downward.

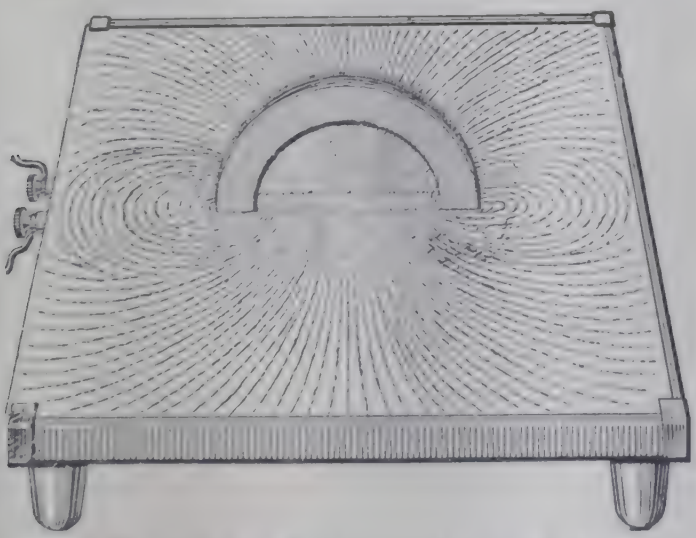

Fig. 401.—Magnetic Field produced by a Coil of Wire

Ex. 549.—Wind a helix of cotton-covered wire on a glass tube (fig. 402). Place the glass tube horizontal half through a piece of cardboard, so that its axis lies in the plane of the card. Pass a current from two or three cells through the wire, and demonstrate the lines of force by filings. Compare them with fig. 349.

Fig. 402.—A Helix of Wire

Push a strip of cardboard having filings sprinkled over it, down the middle of the tube. While the current is passing tap the tube. Note the direction taken up by the filings.

Ex. 550.—Using a constant cell, set up the circuit of fig. 387 with a key in the circuit, and repeat Exercise 543. In each case note the direction in which the current is passing along the wire (leaving the cell at the + terminal), and the direction in which the N pole of the magnet moves, thus:—

Direction of Current.	Position of Wire.	Direction of Motion of North Pole of Magnet.
North to south	Above the magnet	Towards the east
North to south	Below „ „	.....
South to north	Above „ „	.....
South to north	Below „ „	.....

Are the results in accordance with the "corkscrew rule" given above?

357. THE ELECTROMAGNET.

Ex. 551.—(1) Round a rod of soft iron, wind cotton-covered wire, as shown in fig. 402. Pass a current of electricity through the wire.

While the current is running, test the rod for magnetism by the method of Exercise 483. It will be found that the rod is a strong magnet. The electric current produces a magnetic field in the centre of the coil of wire, and any piece of iron in a magnetic field becomes magnetized.

Note the direction in which the current is passing, and which end is the N pole. As you look at the end of the S pole, the current is passing round it in a clockwise direction.

Think of a turn of the wire in the middle of the coil. Is the N pole in the direction given by the corkscrew rule?

(2) Reverse the current. Test the magnetism of the bar. Its polarity is reversed.

(3) Break the circuit and tap the bar. Its magnetism has nearly all disappeared.

Such a bar and coil are called an electromagnet. Electromagnets are much more powerful than steel magnets, but the magnetism is only in existence while a current is passing round the coil.

Electromagnets are often made in horseshoe form, as shown in fig. 403.

If a steel bar, however, be used instead of soft iron, and if while under the influence of the current the bar be tapped, the steel becomes magnetized, and owing to its retentivity keeps its magnetism permanently.

If a strong charge of electricity obtained by means of a frictional machine be passed round a coil of wire containing a thin steel rod, the rod becomes permanently magnetized, thus showing that the electricity produced by friction is really the same as that produced by a cell.

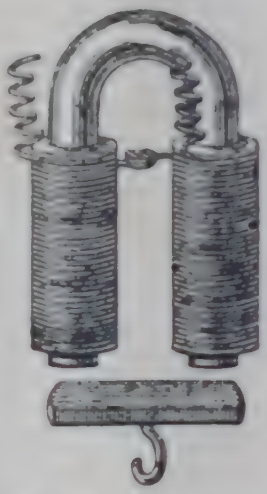

Fig. 403.—An Electromagnet

358. THE ELECTRIC BELL. — We are now in a position to understand the action of an electric bell (fig. 388). Examine an electric bell. The principal parts are the gong, the vibrating hammer, and an electromagnet. On the back of the vibrating hammer is a piece of steel spring which presses lightly against a screw C, and on the front of it is a rod of soft iron.

The current from the battery entering at one terminal passes to the screw along the steel spring to the base of the hammer, thence round the coils of the electromagnet to the other terminal, and so back to the battery. That is the circuit. But immediately the circuit is made, and the current flows, the electromagnet attracts the iron rod and pulls it towards itself. This movement drags away the piece of steel spring from the screw, and thus breaks the circuit at C. When the circuit is broken the vibrating hammer springs back to its first position. There it again makes the circuit, and the whole process is gone through again. Each forward motion of the hammer gives one tap on the gong.

359. GALVANOMETERS.

Ex. 552.—Make a circuit of a constant cell, a key, and a length of 2 or 3 m. of insulated wire in series. Bend the wire into a loop (any shape) round the magnet (fig. 404).

Find out which way the current is passing round the loop, and satisfy yourself that in every part of its circumference, according to the corkscrew rule found above, the current is urging the poles of the needle out into the positions they occupy.

Double the loop, and note the deflection of the magnet, i.e. the angle it has turned through.

If the wire is long enough, make a loop of several turns, and note the increase in the deflection of the magnet.

This arrangement is a simple form of **galvanometer**. Galvanometers consist of a coil of wire with a magnet placed (generally) in the centre of the coil. The magnet does not touch the wire, and is quite independently supported. There is no current of electricity in the magnet. The current passes round the wire, makes a magnetic field across the middle of the coil (fig. 401), and the magnet is deflected.

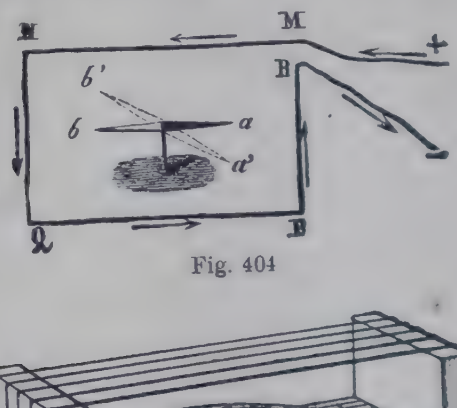

Fig. 404

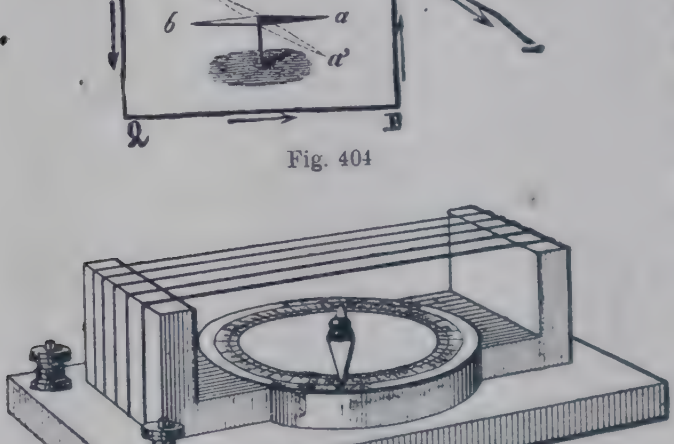

Fig. 405

An Elementary Form of Galvanometer

Galvanometers serve two purposes (1) to detect whether there is or is not a current in a circuit; (2) to measure currents.

Fig. 405 shows a simple form of galvanometer that can be easily made. The coil of wire may consist of one single turn or of several or many turns. If it is to measure or detect very small currents, a large number of turns of wire are required; if for large currents, few turns are necessary. Two or three coils of different lengths may be wound on one framework if necessary, and that coil used which is most suitable for any experiment.

Each end of the coil is fixed to a brass terminal, by which it can be conveniently put in any circuit.

The magnet may be supported on a pivot, or suspended by a thread, as in fig. 407, so that its centre is at the centre of the coil of wire. Underneath it is placed a compass card marked in degrees. The zero line from which the deflection of the magnet is measured is parallel to the turns of wire, and the graduations on the card are from 0° to 90° in each quadrant.

Ex. 553.—Set up a circuit containing a constant cell, a key, and a galvanometer in series. See that no other magnet is near the galvanometer. Place the galvanometer so that the turns of wire are parallel to the needle.

Close the circuit and record the number of degrees through which the galvanometer is deflected. Open the circuit. Rotate the galvanometer so that the needle (no current passing) points to 10°. Close the circuit and again record the deflection of the needle.

Open the circuit: rotate the galvanometer until the needle (no current passing) points to 20°. Close the circuit and record the deflection.

Continue this process for every 10° until the position is found when there is no deflection of the magnet produced by the current. The coils are now at right angles to the magnetic meridian.

Since the deflection obtained depends on the position of the coils, it is always necessary before using any galvanometer to see that *its coils are in the magnetic meridian.*

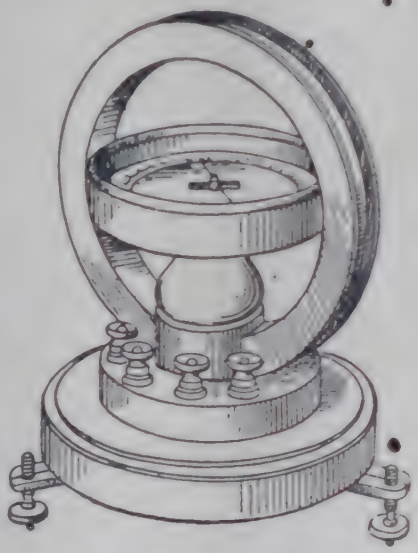

Fig. 406.—The Tangent Galvanometer

360. THE TANGENT GALVANOMETER. — Fig. 406 shows a more elaborate galvanometer, made however in the same way as the simple instrument shown in fig. 405. The coil of wire is wound on a circular ring of wood or brass. The magnet, which is short and carries a pointer, is mounted in a brass box with its centre exactly at the centre of the coil of wire. The ends of the wire are brought down to brass terminals fixed on the base of the instrument.

In the instrument shown in fig. 406 there are three separate coils of wire wound on the ring, with their ends connected to terminals 1 and 2, 2 and 3, 3 and 4 respectively. One coil consists of 1 turn only, the second of 50 turns, and the third of 500 turns. The first is used when a strong current is in action, the second for a moderate current, the third for a weak current.

When the current passes round one of the coils it produces a magnetic force at the centre which is at right angles to the coil.

The earth's magnetism produces a force which is parallel to the coils.

The needle comes to rest under the action of these two forces.

It was shown in Art. 143 that when two forces act at right angles to each other on a body capable of rotation, the deflecting force K is related to the restoring force E by the equation $K = E \tan A^\circ$, where A° is the deflection.

When this galvanometer is in use, K is the deflecting force due to the current, and E is the restoring force due to the earth's magnetism, and E always remains the same.

Hence we may compare different currents by comparing the tangents of the angles of deflection that they produce.

This instrument is therefore called a **tangent galvanometer**. It is the principal instrument for the measurement of currents.

361. THE ASTATIC GALVANOMETER.—Fig. 407 shows a form of galvanometer much more sensitive than that shown in fig. 406, i.e. the needle shows a deflection when a much smaller current is used than is necessary with an ordinary

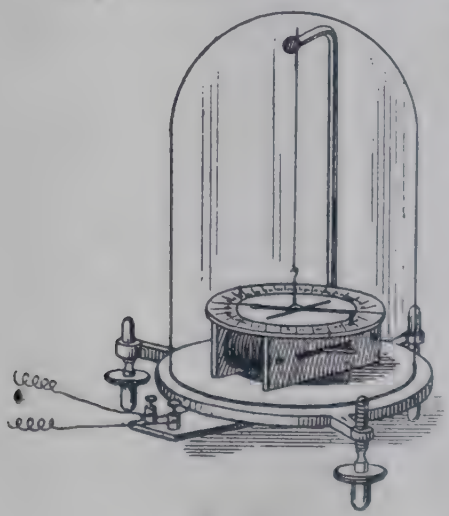

Fig. 407

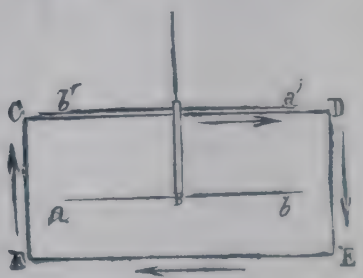

Fig. 408

The Astatic Galvanometer

tangent galvanometer. It is called an astatic galvanometer because an astatic pair of magnets is used (fig. 359). The lower magnet is inside the coil of wire, the upper magnet is on the top of the coil.

Fig. 408 shows how the magnets ab, $a'b'$ are situated with regard to the coils ECDE.

Ex. 554.—From fig. 408, taking the direction of the current in the coils to be that indicated by the arrows, find, by means of the rule arrived at in Art. 356, the direction in which each pole of the magnet ab is urged by the magnetic field produced by the current in each part of the wire.

The magnet $a'b'$ is affected almost entirely by the current in the part CD; the other parts of the wire may be neglected in thinking of this magnet.

It will be found that the forces acting on ab, and those acting on $a'b'$ work together to produce a deflection in the same direction. Thus a small current will deflect the magnet needle and the astatic galvanometer is sensitive.

Sensitive galvanometers must be used cautiously. They are not used to measure currents, but (1) to detect whether there is an

current at all in a circuit; (2) to compare very small currents. With very small deflections, currents may be regarded as proportional to the deflections which they produce.

RULES FOR THE USE OF GALVANOMETERS.

1. The plane of the coils must be in the magnetic meridian.
2. The needle or pointer must be at the zero of the scale when the circuit is open.
3. In reading deflections avoid parallax. Read both ends of the needle, taking the mean as the true deflection.
4. Too large a current must never be sent through any galvanometer, or it will probably be damaged.

362. THE ELECTRIC TELEGRAPH. —The sending of messages by means of a current of electricity is in its simplest form

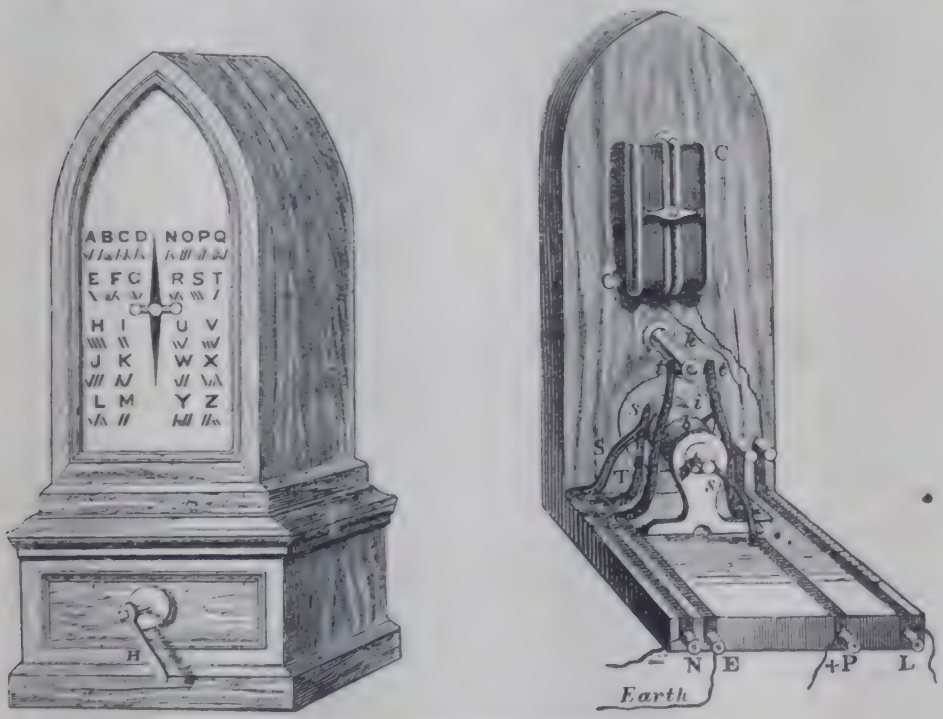

Fig. 409.—Telegraph Instrument

effected by a circuit that contains a battery, a switch, and two galvanometers of special shape shown in fig. 409.

The arrangement of the circuit is indicated in fig. 410, where CZ is the battery, one at each station, and S and R the two galvanometers joined by a telegraph "line". Suppose the operator at S

moves his switch so that the needle of his instrument is deflected as shown in the figure; a current passes along the line, moves the needle of R into the same position as his own, passes down into the

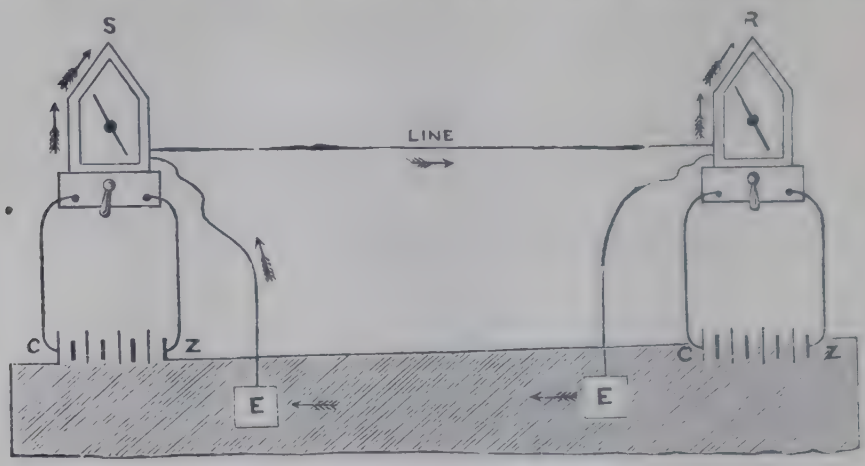

Fig. 410.—A Telegraph Circuit

earth to a plate E, along the earth to the other plate E, and so completes the circuit.

Different combinations of movements of the galvanometer needle stand for different letters, and so the words of the message are spelled out.

CHAPTER XLIV

Induction of Electric Currents

363. It was shown in the last chapter that a current of electricity passing along a wire produces a magnetic field round the wire. We have now to show that the motion of a magnetic field produces electrical effects in a wire.

Fig. 411.—Current induced in a Coil by the Motion of a Magnet

Ex. 555.—(1) Take a coil of wire of small diameter and of many turns of wire, such as is shown in fig. 411, and by long wires connect its terminals to those of a delicate galvanometer, standing some

distance away from the coil. Push quickly into the middle of the coil one pole of a strong bar magnet into the position shown in the figure. The galvanometer needle gives a quick deflection. Keep the bar magnet in the same position. The needle swings to and fro and returns to its zero position.

Rapidly pull the bar magnet straight away from the coil. The galvanometer needle is instantly deflected in a direction opposite to the first. Again it soon returns to its zero.

At the moment when the magnet pole was thrust into the coil, a current of electricity rushed round the circuit. This current lasted only a fraction of a second, and ceased when the motion of the magnet stopped.

At the moment when the magnet pole was withdrawn, another current rushed round the circuit in the opposite direction, and ceased when the magnet was a short distance from the coil.

(2) Keeping the magnet fixed in position, quickly move the coil up to the magnet; then, after the needle is still, away from the magnet. The same effects follow.

(3) Take two strongly magnetized bars and support them on blocks a few inches above the table in the same straight line with the north pole of one a short distance from the south pole of the other, so as to produce a magnetic field like that shown in fig. 350.

Hold the coil of wire in this field vertically so that the lines of force pass through it. Placing a hand at each side of the coil, rapidly turn it top to bottom. The galvanometer in its circuit shows a deflection.

After an interval complete the turn. The current now passes in the opposite direction round the coil.

(4) Fix the magnet in the position shown in fig. 411. Quickly bring a mass of soft iron up to the pole that is in the coil. Observe the direction of motion of galvanometer needle. Quickly remove the soft iron. In which direction is the deflection now?

Such momentary currents produced in a coil of wire when a magnet is moved in its neighbourhood are called **induced currents**.

Remembering that a magnet is surrounded by a magnetic field full of lines of force, we see that the movements of the magnet or coil caused these lines of force to move across the turns of wire in the coil: this process is called cutting the lines of force. And using this language we may say that *whenever magnetic lines of force cut a coil of wire, a momentary current is induced in the coil.* The action of several important electrical machines is based on this fact.

364. The *telephone* is shown in fig. 412 in section. A permanent steel magnet A has round one end of it a coil of wire B. The ends of this wire are connected to terminals C. In front of the coil of wire is placed a thin disk of soft iron D fixed by its edges. These parts are mounted in a wooden framework E, in which a conical opening F is left facing the disk, coil, and magnet. When one talks into the mouthpiece F the disk D is caused to vibrate in

and out like the membrane of a drum when it is struck. By each one of these changes in the position of the middle part of D, the lines of force of the magnet A are altered in position (Exercise 555 (4)). Every alteration of the lines of magnetic force inside the coil B causes some of them to cut the coil and thus produces an electric current through the wire. The current passes out at one

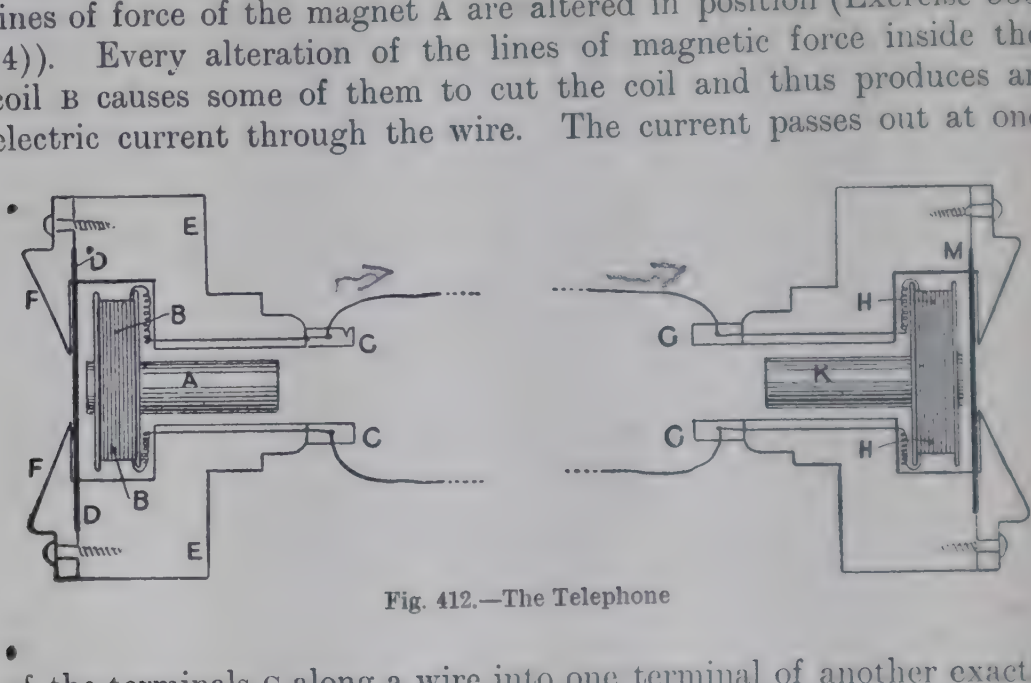

Fig. 412.—The Telephone

of the terminals C along a wire into one terminal of another exactly similar telephone G, round the coil H of this telephone, and so round the circuit. As the current passes round H it alters the magnetism of the magnet K, and this alteration causes an attraction or repulsion of the disk M. Thus every motion communicated to the disk D is repeated by the disk M.

In this way the drum of an ear placed opposite M receives all the motions set up by the voice at

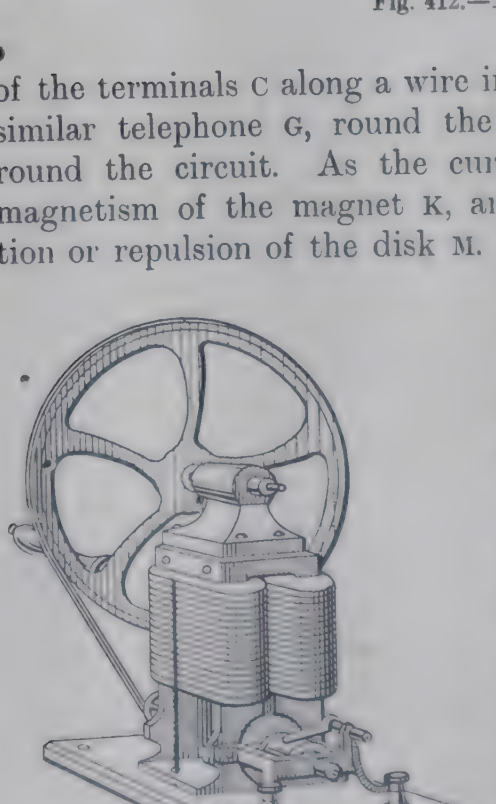

Fig. 413.—Dynamo

365.—The *dynamo*, which is a machine for producing very strong currents, acts also by induction. Very large coils of wire are rotated rapidly in a strong magnetic field and the currents of electricity in duced in the coils are collected and taken to the terminals of the machine, whence they pass into the external circuit.

Fig. 413 shows a simple laboratory machine. An electromagnet has its poles curved so as to

a hollow cylindrical space across which the magnetic lines of force run horizontally. In this field is placed a coil of wire called the armature, whose construction is shown in fig. 414. The turns of wire run parallel to the spindle of the armature, and the ends are brought out each to a piece of brass fixed on the spindle, but insulated from it. These two pieces of brass form the commutator. The spindle of the armature is mounted on a framework, so that the coil of wire just fits into the cylindrical space.

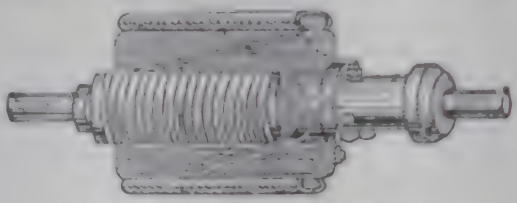

Fig. 414.—Armature of a Dynamo

By means of the wheel the armature is caused to rotate rapidly.

As was shown in Exercise 555 (3), during every complete revolution of the armature two brief currents pass round the coil in opposite directions. As the commutator turns, it rubs against two metal brushes (one of which is shown in fig. 413), whose position is so arranged that at the instant when the current reverses its direction in the coil the commutator slips on to the other brush. Thus the current always comes out at one brush, passes to one of the screw terminals shown on the base, round the external circuit, and back to the other brush.

366. THE INDUCTION COIL.

Ex. 556.—On a rod of soft iron wind a helix of insulated wire (fig. 402) of many turns. Call this the *primary* coil.

Over this wind another helix of as many turns as conveniently possible. Call this the *secondary* coil.

Connect the ends of the primary coil through a key to a battery of three or four cells in series. Join the ends of the secondary coil to a galvanometer. By means of the key make the primary circuit. Note the effect on the galvanometer. Let the needle come to rest.

Break the circuit and again observe the galvanometer.

Every time the primary circuit was made, the iron core became an electromagnet, and its lines of force rapidly crossed the turns of the secondary coil; hence the momentary current in the secondary circuit.

Every time the primary circuit was broken the lines of force crossed the secondary in the opposite direction. Hence, the momentary current in the opposite direction round the secondary.

The **Ruhmkorff coil** shown in fig. 415 is constructed on this principle.

The core and coils, of which the secondary is very long, are mounted on a stand, which also supports a vibrating spring A, and an upright carrying a screw B. On the top of A is a mass of soft iron. Inside the base of the instrument is a condenser C.

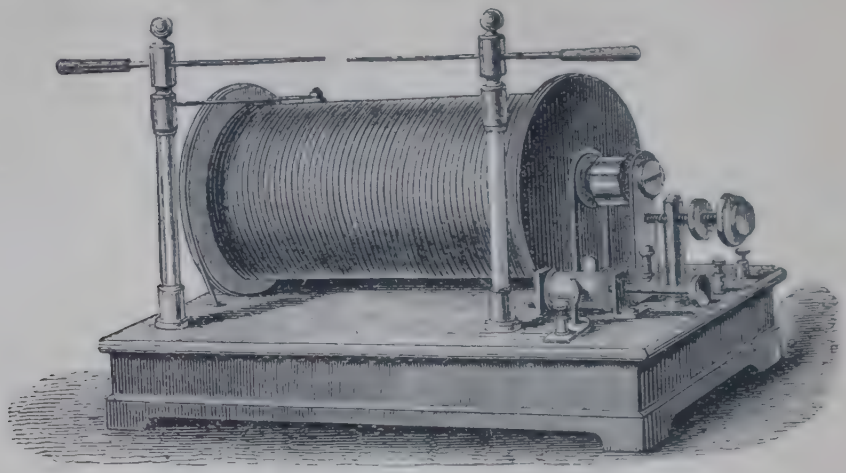

Fig. 415.—A Ruhmkorff Coil

The electrical connections in the primary circuit are as indicated in fig. 416. The secondary terminals D are left unconnected.

The screw B being in contact with A, on turning the switch a current passes round the primary, charges the condenser, and magnetizes the core. Thereupon the head of A is attracted to the core, the connection between A and B is broken, the current ceases, and the core becomes demagnetized. This process is repeated with every vibration of the spring, as in the electric bell.

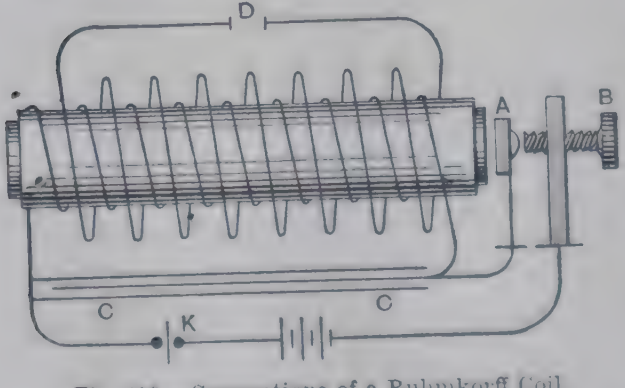

Fig. 416.—Connections of a Ruhmkorff Coil

Lines of force being thus alternately introduced into and withdrawn from the secondary coil, momentary currents flash round it and produce a stream of sparks between its terminals D.

The key K is usually fixed on the base of the instrument.

A small coil, such as gives a $\frac{1}{4}$-inch spark, is suitable for school use. Larger coils will give sparks many inches long, but *such coils are dangerous, and should only be used by persons of some experience*

Ex. 557.—Using a *small* coil—(1) See that the switch is turned off. Attach a battery of two or three cells to the terminals; see that the screw B *lightly* touches the head of the spring A, and place the terminals D 3 or 4 mm. apart. Turn on the switch. Observe the spark discharge.

(2) Place a finger on one of the secondary terminals. A jarring sensation is experienced. Small alternating currents of this character are sometimes used for medical purposes, but if too strong they are harmful.

CHAPTER XLV

Resistance

367. CONDUCTORS AND INSULATORS.—In the previous exercises a current of electricity has been caused to pass through copper wires and other pieces of metal, and also through dilute sulphuric acid. Any material through which a current of electricity will pass is said to conduct the current, and is called a **conductor**.

When an air-gap was interposed in the circuit the current ceased. It could not cross the gap, and ceased altogether to flow. Materials through which a current of electricity will not pass are call **non-conductors** or **insulators**.

The next experiment is to try a few substances to see whether they are conductors or non-conductors.

Ex. 558.—Set up a circuit consisting of a battery B, a galvanometer G, a key K, and the substance to be tested S, all in series as indicated in fig. 417.

(1) Let the battery be a single Daniell's cell, and use any form of galvanometer that is not sensitive. Place in turn between the two terminals AC in fig. 417 a length of wire—iron, German silver, platinoid, manganin, tin are suitable. Note the deflection of the galvanometer. All metals are conductors.

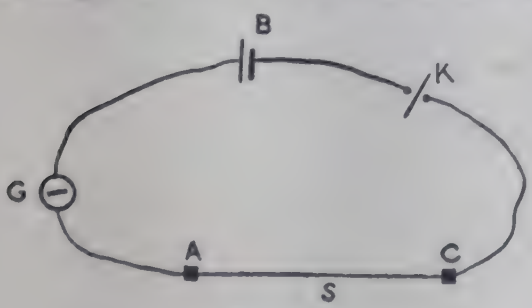

Fig. 417

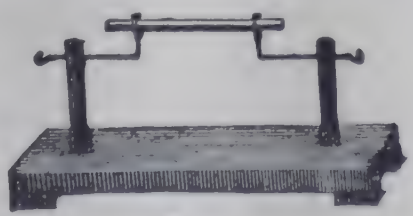

Fig. 418

(2) Using a battery of several cells in series, and as sensitive a galvanometer as is available, place between the terminals AC short pieces of substances such as the following:—dry string or cotton; silk; string moistened with dilute acid; rods of wood, glass, sulphur, resin, ebonite, carbon, sealing wax; a piece of

earthenware. See that the materials make good contact with the terminals. This may be done by supporting them on metal terminals, as indicated in fig. 418.

With some of these substances in the circuit no deflection whatever of the galvanometer magnet can be obtained. They are insulators.

With others a small deflection is obtained. They conduct a little. There is no perfect insulator. Under certain circumstances all ordinary substances conduct electricity more or less.

The wires used in electrical work are covered with cotton, silk, gutta percha, or other insulator, to prevent the electric current from passing from one part of the wire to another that may touch it, as it is always desired that the current should run lengthwise along the wire, and not out at the side.

The fact that different substances placed in a circuit with a battery allow more or less current to flow round the circuit is expressed by saying that the substances offer **resistance** to the passage of the current, some more, some less. The substances classed as insulators offer a very great, or as it is usually called a **high resistance** to the passage; those classed as conductors offer a small or **low resistance**. All conductors offer some resistance to the passage of a current through them.

A comparison with Art. 332 shows that on the whole substances that conduct the electricity produced by a cell also conduct the electricity produced by friction. This is also an indication that the electricity developed by such different means is really the same.

The differences between the two lists of conductors and insulators are due to the fact that the electricity developed by friction is in a different state to that produced by a cell. This difference of state is called a difference in **potential**.

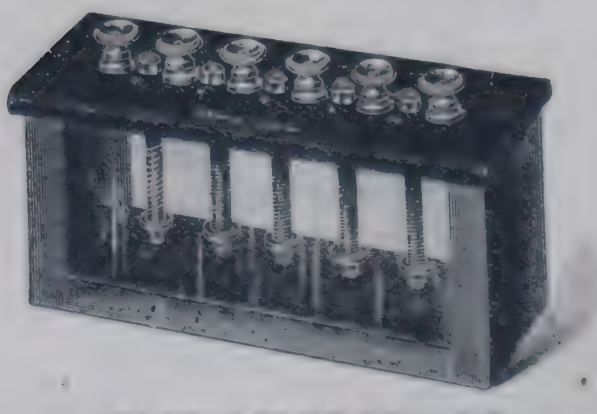

Fig. 419.—A Set of Resistance Coils.

368. RESISTANCE COILS.—It is convenient to have measured length of insulated wire wound each on a reel. These are called *resistance coils*. Fig. 419 shows a set mounted on a board, with their ends connected to terminals for convenience.

Ex. 559.—(1) Take a resis-

ance coil on which there is wound 2 or 3 m. of thin wire. Place it in series with a battery and a galvanometer; and read the deflection of the magnet of the galvanometer.

(2) Repeat the experiment with a resistance coil on which there is 20 or 40 m. of the same wire, and record the deflection.

The second deflection is smaller than the first. This means that the current was weaker, and this weakening was produced by the greater length of wire in the circuit.

Thus a long wire offers more resistance to the passage of a current through it than does a short wire of the same kind.

By putting coils of wire into a circuit in this way we can alter the current within limits as we please.

Ex. 560.—Set up a circuit containing a cell, a sensitive galvanometer of low resistance, and a resistance coil of fairly high resistance. Suppose the deflection is $x°$. By putting in more resistance reduce the deflection to $\frac{1}{2}x°$. Then it will be found that the second resistance is nearly double the first.

Numerous experiments made in this way show that the current in a circuit is inversely proportional to the resistance of the circuit when other conditions remain the same.

369. RESISTANCE OF A CELL.

Ex. 561.—Make up a simple cell consisting of a plate of carbon, a plate of zinc, and some bichromate solution; or the simple cell used in Exercise 542. Let there be very little liquid in the cell. Arrange the plates so that their distance apart can be varied, either as indicated in fig. 420, or in any other convenient way.

Join the cell in series with a key and a low-resistance galvanometer.

(1) With the plates wide apart, and the liquid covering only a small part of the plates, make the circuit and read the galvanometer.

Add more liquid, and again read the galvanometer.

Continue to add liquid and take readings until the cell is nearly full.

The deflections show a steady increase. This is due to the decreasing resistance of the cell as the area of the working part of the plates is increased.

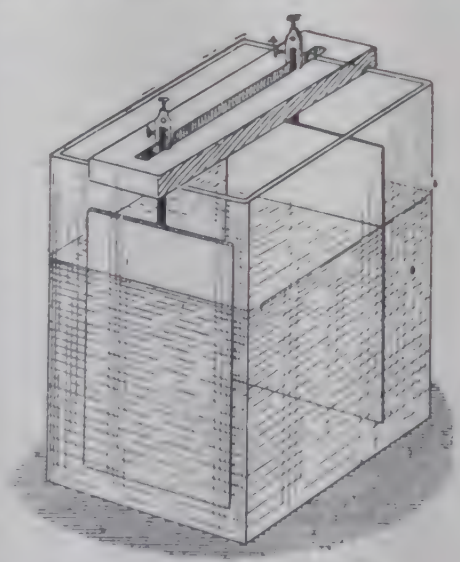

Fig. 420.—To show the Resistance of a Cell

(2) Bring the plates nearer together in successive steps, and note the further increase in the deflection.

Thus a cell offers resistance to the passage of a current in the

same way as a wire does, the resistance being proportional to the distance apart of the plates, and inversely proportional to the sectional area of the cell. The resistance of a Grove's cell is small, because the plates are large and close together.

370. REGULATION AND COMPARISON OF CURRENTS.

Ex. 562.—Set up a circuit consisting of a constant battery, a key, a tangent galvanometer, and a set of resistance coils in series.

(1) With a high resistance in the circuit obtain a deflection of, say, 30°.

(2) Lessen the resistance in the circuit so that the deflection is, say, 45°.

Compare the values of the currents in the two cases—tan 30° = ·577; tan 45° = 1. The currents were in the ratio of ·577 to 1.

(3) By altering the resistances obtain currents of various strengths, and compare their strengths. (For table of tangents, see p. 408.)

Ex. 563.—Arrange a circuit consisting of a single constant cell, a key, a sensitive galvanometer, and a set of resistance coils in series, and repeat the operations of Exercise 545. Arrange for all the deflections to be small—5°, 10°, 15°. Then approximately the currents are proportional to these deflections.

In the two preceding exercises it must be remembered that the galvanometer wire, the cell, and the other wires are offering resistance, so that if the resistance of the coils be halved, the whole resistance in the circuit has not been halved.

CHAPTER XLVI

Heating and Electrolytic Effects of a Current

371. The passage of an electric current through bodies affects the bodies themselves in various ways. One effect is to make them warmer.

Ex. 564.—Place three small slips of wood about the size of a match round the bulb of a thermometer, and wind closely round the slips about half a metre of the thinnest cotton-covered wire available. If necessary bind the coil on with tape, leaving its bare ends projecting.

To the ends of the thin wire join lengths (called leads) of thicker wire.

Place the thermometer in a test tube, and read the temperature.

Connect the leads to the terminals of a cell, and observe the thermometer while the current continues to run for a few minutes.

Ex. 565.—Wind a spiral of thin wire, preferably of iron, tin, or platinoid, on a piece of thin glass tube, and pass the tube through the stopper of a test tube into a little liquid, as shown in fig. 421. The arrangement is an air thermometer. See that the stopper is air-tight.

By leads of ordinary copper wire connect the spiral to the terminals of a cell, in whose liquid is placed a mercury thermometer.

As the current passes, note both thermometers.

Considerable heat is developed in the coil, and the cell also will in a few minutes rise in temperature.

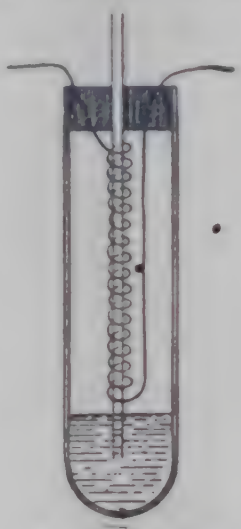

Fig. 421.—Heating Air by an Electric Current

Suitable arrangements show that heat is developed in every part of the circuit through which the current passes, and that the temperature rises most where the resistance is greatest, and rises least where the resistance is least.

Ex. 566.—(1) Join in series a battery of two or three Grove's cells, a key, leads of copper wire, and a piece of fine wire of platinoid, iron, or tin about 6 or 8 cm. long. Support the terminals of the fine wire so that it touches nothing.

When the current passes, the wire may be made red-hot. Thin tin wire may be melted.

(2) Remove the fine wire, and for an instant bring the ends of the copper leads within about ·2 mm. of each other. A spark leaps across a narrow gap of this kind. This spark, exactly like that obtained from the electrified rod in Exercise 515, again indicates that frictional and voltaic electricity are the same.

If the ends of a short chain, consisting of alternate links of fine silver and platinum wire, be connected to the terminals of a battery or two or three Grove's cells, the platinum links become white-hot, while the silver links, although warm, are not hot enough to give out light. The reason of the difference is that the platinum wire offers much more resistance than the silver, while its specific heat is less, thus more heat is produced in the platinum, which also requires less heat than the silver to make it hot.

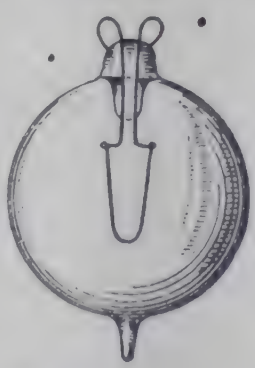

Fig. 422.—An Incandescent Lamp

372. THE ELECTRIC LIGHT.—An incandescent electric lamp merely consists of a very thin wire made of metal or carbon enclosed in a vacuous bulb.

Fig. 422 shows a 4-volt Osram lamp. When the terminals of a battery of three Grove's cells are joined to the terminals of this lamp the wire inside becomes white-hot and emits a brilliant light.

The leads of the lamp are made of thicker wire, which keeps comparatively cool.

The electric lamps ordinarily used in houses contain a longer wire and require a stronger battery.

Two carbon rods mounted on a stand, as shown in fig. 423, form a simple **arc lamp**. Each rod is in a metal holder, which is joined by thick wire to one of the terminals on the base of the instrument. A strong current, such as is produced by forty Grove's cells in series, passed

Fig. 423.—A Primitive Arc Lamp

Fig. 424.—The Carbons of an Arc Lamp in Action

through these carbons produces an **arc light**. The carbons ar first placed in contact, then the current is started. The ends o the pointed carbon rods get white-hot. The rods are then draw a short distance apart. A spark leaps across the gap and volatiliz some of the carbon, which glows and forms a very hot flame, bu does not give out much light. The ends of the carbons becon intensely hot and bright, and give out a brilliant light.

Fig. 424 shows the appearance of the carbons after the lamp h been burning for some time. The current is passing from th upper to the lower carbon, and from the upper carbon particl are volatilized and torn away so that it forms a **crater**. Mo of the light is emitted from this crater.

The current required for electric lighting is usually supplied by a dynamo.

373. FUSES.—In houses where electricity is used for lighting, heating, or to drive machinery, it is necessary to protect the building from risk of fire or accidents, which might be caused if from some disturbance of the circuit there were a sudden great rush of current. This is effected by using a *fuse*. In its simple form a fuse is a short piece of tin wire (fig. 425), which is placed in the circuit where the current enters the building, so that all the current that comes in passes through the fuse. Tin melts at about 235 C. If, then, from any cause the current becomes too strong, the fuse wire quickly becomes hot, melts, and breaks. (Exercise 566.) The current is thus stopped altogether.

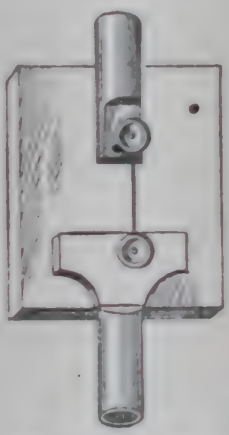

Fig. 425.—A Fuse

374. ELECTROLYSIS.—Since currents pass through the cells that produce them, some liquids are conductors of electricity.

In Exercise 542 it was noticed that bubbles of gas appeared on the copper plate of the cell.

Anyone who has used a Grove's cell has noticed the smell and the gaseous fumes arising from it.

We now have to examine the effect when a current of electricity is passed through liquid contained in a separate vessel. Such a vessel fitted up for this purpose is called a **voltameter**.

Fig. 426 shows a water voltameter joined to two Bunsen cells.

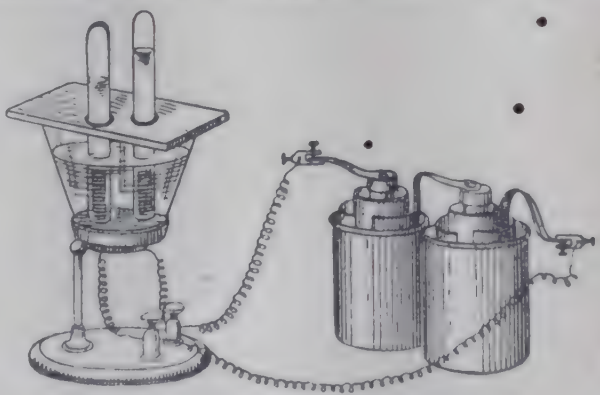

Fig. 426.—The Water Voltameter

The water voltameter consists of a glass vessel standing on a base; two wires, each connected to a terminal on the base, lead to two small platinum plates called **electrodes** at the bottom of the vessel: over each of the electrodes is placed a large test tube or a burette. It is better for the tubes to be graduated.

Ex. 567.—To prepare for use, the voltameter is filled with dilute sulphuric

acid (see Exercise 541) to a level above the top of the electrodes. The two tubes are *filled* with the acid and then placed mouth downwards so as partly to cover the electrodes. They may be kept in position by putting a rubber ring round each and passing them through two holes in a piece of wood, or by fixing them in a retort stand. There should be no air left in the tubes.

The battery must consist of not less than two Grove's, or Bunsen's, or bichromate cells; or of three other cells arranged in series. It is better to use three or four cells.

Arrange the battery, voltameter, and a key in series to form a circuit. Note which is the positive pole of the battery. The electrode joined to this pole is called the *anode*; the electrode joined to the negative pole of the battery is called the *cathode*. When all is in order, close the circuit by means of the key and note the time. Which way is the current passing through the voltameter? From anode to cathode or the opposite?

On watching the electrodes bubbles of gas are seen on each. Note the difference between the appearances of the electrodes. From which electrode does the most gas come off?

When sufficient gas has collected, note the time the current has been passing and the comparative quantities of gas in the two tubes.

Repeat this observation three times, until one of the tubes is nearly full of gas. Is this the cathode tube or the anode tube?

Remove the cathode tube, keeping it mouth downward, and apply a lighted match to the end of the tube. The gas that burns thus is hydrogen.

Remove the anode tube, invert it, and apply to its mouth a piece of wood that has been lit and then blown out, so that it is still glowing. The gas that thus acts on a glowing ember is oxygen.

By the passage of the current through it, water has thus been split up into hydrogen and oxygen. The process is called **electrolysis**. Any liquid that behaves in this way is called an **electrolyte**. The two substances that result from the decomposition are called **ions**.

The observations made indicate that, estimated by volumes, water contains twice as much hydrogen as oxygen.

They also show that the quantity of gas produced is proportional to the time during which the current was passing.

375. ELECTROPLATING.

Ex. 568.—Take two strips of copper plate about 10 cm. long and 5 cm. wide so that they will conveniently stand in a beaker, as the two plates stand in fig. 384 (1).

Clean the plates; scratch a distinguishing mark on each; weigh them; attach a terminal to each.

Prepare a battery of two cells in series.

Pour into the beaker some copper sulphate. Place the two plates in the liquid so that they do not touch. This is a copper voltameter. Connect the voltameter to the battery. Allow the current to run for about half an hour.

Note which plate is the cathode.

Take out the plates and examine them; do not touch the parts that have been in the solution. The new deposit on one plate is copper. Is the deposit on the anode or cathode? (Compare Art. 350.)

Allow the plates to dry slowly. When quite dry, weigh again. Which plate has gained in mass? How much? Has the other plate lost in mass? How much?

Thus when a current of electricity passes through a liquid which is, or which contains, a chemical compound, the liquid is decomposed by the passage of the current, and there appear at the electrodes entirely different substances. When one of these substances is a metal it is deposited on the cathode. This process is electroplating. The cathode in Exercise 568 was electroplated with copper. If silver nitrate had been used as the liquid the cathode would have been electroplated with silver.

CHAPTER XLVII

Ohm's Law

376. ELECTROMOTIVE FORCE.

Ex. 569.—(1) Set up a circuit consisting of a Daniell's cell, a sensitive galvanometer, and a *high* resistance in series.

Alter the value of the resistance if necessary until the deflection of the galvanometer magnet is about 15°. Suppose it is $a°$.

(2) Leaving the remainder of the circuit exactly as it was, replace the Daniell's cell by a bichromate cell. The deflection now is $b°$.

Proceed in an exactly similar manner with a cell of each type that is available.

Each cell produces a different strength of current. But there was no difference in the external circuit, so that the difference was due to the difference in the cells. They possess different current-producing powers *under the same conditions.* This is expressed by saying that they have different **electromotive forces**. The electromotive force of a cell depends entirely and only on the materials of which the cell is made, and not on its size.

Ex. 570.—Set up a circuit consisting of a high resistance, a galvanometer, a key, and a cell. The deflection is $a°$.

To the single cell add another cell of the same kind in series with it, making no other difference in the circuit.

The deflection $b°$ is now greater. From the deflections compare the currents. The second current is twice as great as the first.

Use three cells in series, and make a further comparison.

As the only change is in the battery, the increased current must be due to the greater current-producing power of the battery. This is expressed by saying that the electromotive force of cells in series is the sum of their separate electromotive forces.

Ex. 571.—Using the same circuit as in the last exercise, arrange the two cells *in parallel*. The deflection is now $a°$.

Repeat the experiment with three cells in parallel.

The electromotive force of cells in parallel is the same as that of a single cell.

377. THE POTENTIOMETER.—A potentiometer in its simplest form is a long wire of high resistance fixed on a board beside

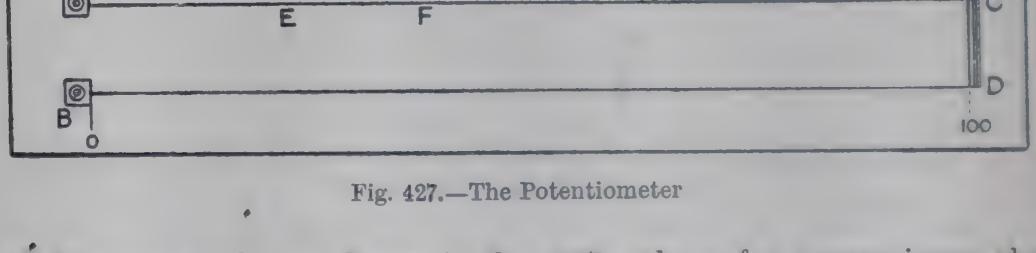

Fig. 427.—The Potentiometer

a millimetre scale, as shown in fig. 427, where for convenience the wire is shown in two parts, each 100 cm. long. The two parts of the wire are shown connected by a thick piece of metal whose resistance is so small that it need not be considered.

Platinoid or manganin is the best material for the wire.

The ends of the wire are fastened to two terminals A, B, by means of which it can be placed in any circuit.

Ex. 572.—Arrange in series (fig. 428) a steady battery B, a key K, and the potentiometer wire AZ.

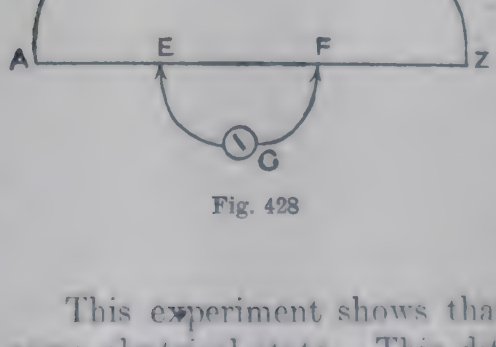

Fig. 428

To the terminals of a sensitive galvanometer G attach two wires each about 1 m. or 2 m. long. Taking the free ends of these wires in your hands, press them on the potentiometer wire, one at E and the other at F, a few centimetres apart. Arrange the distance EF such that there is a readable deflection of the galvanometer needle.

Try the same length EF at different parts of the potentiometer wire. Is the deflection always the same?

This experiment shows that the points F and I are not in the same electrical state. This difference of electrical state is such as to cause a current to flow from one point to the other when the points are joined by a conductor, and is called a **difference of**

potential. Difference of potential is the same kind of property as electromotive force.

Ex. 573.—Make the length EF successively several different values and read the current in the galvanometer for each of the lengths EF.

Compare the different values of the length EF with the different deflections.

They show that the difference of potential between the two points E and F is proportional to the length EF, that is to the resistance of the wire between the points E, F, as the wire is exactly the same all along its length. If E and F are at the same point no current flows through the galvanometer, because there is no difference of potential between its terminals.

These facts are used to compare electromotive forces and resistances.

Ex. 574.—To compare the electromotive forces of cells.

Arrange in series (fig. 429) a battery C of two or three constant cells, a key K, and the potentiometer wire AZ. To this attach, as shown, another circuit consisting of a single cell E, a sensitive galvanometer G, a key H, and a wire HF, of which the end F is left free.

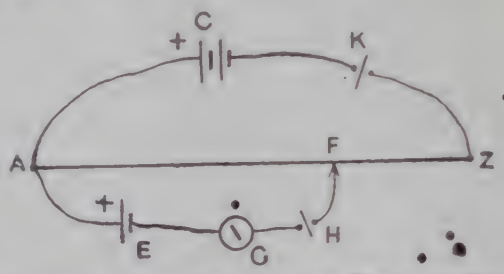

Fig. 429.—Comparison of the Electromotive Forces of Cells

The positive poles of both batteries must be joined to the same point A; and the circuits must only be kept made when actually required.

Close the key K, then the key H; then place the free end F of the wire HI on the potentiometer wire. Try F in different positions along AZ until the galvanometer G shows no deflection. Read the length AF.

Repeat the operation with a different kind of cell at E, and obtain similarly a length AF′.

Then the electromotive forces of the cells are as AF to AF′. The reason for this is easily seen. Since the galvanometer in each case showed no deflection, the electromotive force of the cell E was just sufficient to counterbalance that which the battery C produced at the two points.

The values of the electromotive forces of cells are expressed in volts. The **volt** is the unit of electromotive force. It is very nearly equal to $\frac{10}{11}$ of the electromotive force of a Daniell's cell.

The electromotive forces of cells are approximately as follows:—

Daniell	...	1·08 volts	Bunsen	...	1·9 volts
Grove	...	1·9 „	Bichromate	...	1·9 „

The *standard cell* by which others are measured is the Clark cell, whose electromotive force is 1·434 volts when made in a special manner.

378. MEASUREMENT OF THE RESISTANCE OF WIRES.

Ex. 575.—To obtain an idea of the unit of electrical resistance, cut off one of the following lengths of wire, silk or cotton covered:—

	Size.	Diameter.	Length.
German silver ...	22	·71 mm.	175 cm.
Copper	28	·37 „	693 „
Manganin	22	·71 „	92 „

using whichever wire is available. Wind the wire on a reel and affix terminals to its ends.

The resistance offered by the wire thus cut off is (approximately) **the unit of resistance**. Its name is the **ohm**. Wires whose resistance is exactly 1 ohm are sold by makers. All other resistances are measured by comparing them with the resistance offered by such a wire, and are expressed in ohms. The comparison is best made by means of the apparatus called a Wheatstone bridge. The method of making the comparison is precisely the same, whether the resistance chosen as the unit or **standard** is an ohm or some other resistance which it may be more convenient to use.

379. THE WHEATSTONE BRIDGE. — A simple form is shown in fig. 430. A long wire is stretched in front of a metre scale, and is joined at the ends to thick copper bands. Along the other side of the metre scale is a long copper band, between which and the end pieces are two gaps. The resistance of these copper bands is so small that it may be neglected.

The two resistances to be compared are placed in the two gaps.

Along the wire moves a slider, by means of which contact can be made with the wire at any point, as was done in Exercise 574.

Fig. 431 shows the arrangement of the circuits of a Wheatstone bridge set up for the measurement of resistance, B representing a constant battery, G a galvanometer, R the standard resistance, and r the resistance to be measured. A comparison of figs. 430 and 431 will show the points of the junctions of the wires to the bridge.

Suppose the circuit to be made, but the galvanometer contact A not made. Then current enters at C and runs along both paths CRED and CAD back to the battery.

Then, as shown in Exercise 573, the potential at E is lower than at C and higher than at D, and $\frac{\text{the fall of potential between C and E}}{\text{the fall of potential between E and D}}$ which we will call $\frac{p}{q} = \frac{\text{resistance R}}{\text{resistance } r}$.

Fig. 430

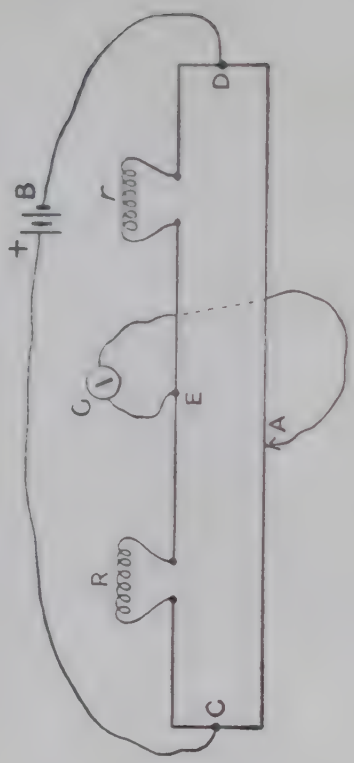

Fig. 431

The Wheatstone Bridge, and its Circuit

Now there must be some point along the wire CAD where the potential is the same as at E. Suppose it is found to be at A.

Then

$$\frac{\text{the fall of potential between C and A}}{\text{the fall of potential between A and D}} = \frac{p}{q} = \frac{\text{resistance R}}{\text{resistance } r}.$$

But also $\dfrac{\text{length CA}}{\text{length AD}} = \dfrac{p}{q}$.

Hence $\dfrac{\text{resistance R}}{\text{resistance } r} = \dfrac{\text{length CA}}{\text{length AD}}$.

One terminal of the galvanometer is connected to E, and the

other to A. The correct point A is found by trial, when no curren flows through the galvanometer.

Ex. 576.—(1) Using the Wheatstone bridge and a suitable standard re sistance at R, place at r successively different measured lengths of the sam kind and size of wire. Measure their resistances, and show that the resistance of these wires are *proportional to their lengths.*

(2) Place at r successively wires of the same material and the same lengt but of different measured diameters. Measure the resistances of the wires an show that they are *inversely proportional to their sectional areas.*

(3) Place at r successively wires of different material, but of the same lengt and diameter. Measure their resistances and compare them. Each materi offers a different amount of resistance to the passage of the current; that i each has a *specific resistance.*

Ex. 577.—Place a coil of wire in cold water, and by the method of Exerci 576 measure its resistance in ohms. Make the water hot and measure again th resistance of the wire. It has increased.

Ex. 578.—Measure the resistance of two wires separately. Suppose they a r_1 and r_2. Fasten the wires together in parallel (fig. 432), and measure th resistance of the combination. Suppose it is R. Show that $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$.

Fig. 432.—Arrangement of Wires in Parallel

380. OHM'S LAW.—In Art. 377 was given a practical idea the volt, and in Art. 378 of the ohm. The exact definition of t ohm is the resistance offered by a column of mercury whose leng is 106·3 cm. and sectional area 1 sq. mm.; and of the volt that it $\frac{1}{1{\cdot}434}$ of the electromotive force of a Clark standard cell.

When a difference of potential of 1 volt is caused to exist the terminals of a wire whose resistance is 1 ohm, then the curre that passes through the wire has a strength of 1 **ampere**. T ampere is the unit of current.

In Exercise 569 a rough comparison between the electromot forces of cells was made by means of a galvanometer. A spec form of galvanometer called a **voltmeter** is made which measu electromotive forces accurately in volts.

In Exercise 562 the strengths of different currents were co pared by means of the tangent galvanometer. A special form galvanometer, called an **ampere-meter** or **ammeter**, is also made which currents are at once measured in amperes.

It was shown in Exercise 570 that the greater the electromotive force in a circuit the greater was the current. Exact experiments show that the strength of the current that flows round a circuit is proportional to the electromotive force in the circuit. Also exact experiments made on the principle of Exercise 560 show that the current in a circuit is inversely proportional to the resistance of the circuit.

When current is measured in amperes, electromotive force in volts, and resistance in ohms, these two important principles are summed up in Ohm's law:—**The current in any circuit is equal to the electromotive force divided by the resistance.** $\mathbf{C = E \div R}$.

Example.—A circuit consists of two Daniell's cells in series, each having an electromotive force 1·08 volts and resistance ·9 ohm; a galvanometer whose resistance is 27 ohms; and a coil whose resistance is 43 ohms. Find the strength of the current in the circuit.

Electromotive force $= 1{\cdot}08 \times 2 = 2{\cdot}16$ volts.

Resistance $= (.9 \times 2) + 27 + 43 = 71{\cdot}8$ ohms.

$\therefore$ Current $= \dfrac{2{\cdot}16}{71{\cdot}8} = {\cdot}03$ ampere.

Ex. 579.—Additional exercises in Current Electricity.

(1) Make a circuit consisting of a constant battery, a key, a galvanometer, and two resistance coils, in series. Alter the order of the arrangement, so as to get the galvanometer in different positions in the circuit. Is the strength of the current the same all round the circuit?

(2) Show experimentally that the deflection of the needle of a galvanometer, when a current passes round the coils, does not vary if the strength of the needle's magnetism is changed.

The reason is that the deflecting force and the restoring force (Arts. 143 and 360) both depend on the strength of the magnetic poles of the needle; and both these forces are altered to the same extent when the needle's magnetism is altered.

(3) Endeavour to perform Exercise 567 with a single Daniell's cell. The result shows that it requires an electromotive force greater than 1·08 volts to electrolyse water. Try whether the electrolysis can be effected with a single Grove's or Bunsen's cell.

(4) To make a small accumulator. Take two lead plates, and place them opposite each other in a solution of sulphuric acid, as in fig. 384 (4). Connect each plate to one of the terminals of a battery of two or three cells. Electrolysis proceeds as in Exercise 568. Note the direction of the current, and mark the cathode plate. Allow the current to run for an hour or more, and then remove the battery and put a galvanometer in its place. The two lead plates in the acid now act as a cell, and send a current. This form of cell is called an **accumulator.** Find out whether the cathode is the positive or the negative plate.

(5) Calculate the resistance of a single wire which is equivalent to an arrangement of two wires of resistances 5 ohms and 10 ohms respectively, placed in parallel.

(6) In a room A is a battery. From its terminals wires are led into another

room B. How could a person in room B determine which wire was connected to the positive pole of the battery? (Arts. 356, 374.)

(7) A short iron rod is laid across the ring of a tangent galvanometer through which a current is passing. What is the magnetic condition of the rod? Explain.

(8) To show the damping effect of induced currents. Set the magnet of a vibration magnetometer swinging through an arc of convenient size (say 30°), and find the total time that it takes in coming to rest (t_1).

Place underneath and close to the magnet a plate of copper or brass; or, if more convenient, place beside one of its poles a coil of wire which has its ends joined. Set the magnet swinging through *the same arc*, and find the time that it takes in coming to rest (t_2).

The reason why t_2 is smaller than t_1 is that the currents induced in the plate or coil by the motion of the magnet exercised a force which opposed that motion (Art. 169).

(9) Make a circuit consisting of a battery of two similar cells, a key, and a galvanometer all in series; note the deflection of the galvanometer needle. Turn one of the cells round, so that the two are arranged thus: Copper: zinc joined to zinc: copper. What deflection does this battery give? Why?

(10) To compare the electromotive forces of dissimilar cells.

Take two cells, suppose a Grove and a Daniell. Arrange them in series with a suitable resistance, a key, and a tangent galvanometer.

(a) so that the cells help each other: deflection A°;

(b) so that the cells oppose each other: deflection B°.

Then if E_1 and E_2 are the electromotive forces of the cells,

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan A}{\tan B}.$$

$$\therefore \frac{E_1}{E_2} = \frac{\tan A + \tan B}{\tan A - \tan B}.$$

(11) Make, and explain the action of, a floating battery. (Refer to fig. 34 and Exercise 549.)

Pass a strip of amalgamated zinc and a strip of copper through a large cork. To the upper ends of the strips attach the ends of a helix of wire of as many turns as possible. Float the cork on dilute acid contained in a large vessel so that the plates are in the acid and the helix in the air. Bring up a pole of a bar magnet close to each end of the helix in turn. Explain the effects observed.

(12) To measure the resistance of a galvanometer, that is, of its coil of wire.

Proceed as in Exercise 576. At R put a standard coil whose resistance in ohms is known. At r put the galvanometer coil, placing the instrument so that its coils are at right angles to the magnetic meridian. The adjustments and calculations are as in Article 379.

(13) To shunt a galvanometer.

Knowing the resistance r of the tangent galvanometer, prepare (from the information of Art. 378) a coil of wire whose resistance is $\frac{r}{n}$.

Connect the galvanometer to a battery, and read the deflection (A°). Find tan A.

Leaving the arrangements otherwise as they are, connect the ends of the prepared coil to the terminals of the galvanometer; read the deflection (B°). Find tan B.

Compare tan A with tan B. What is the ratio between them?

The prepared coil thus used is called a **shunt**, and the galvanometer is said to be shunted. Only $\frac{1}{n}$ of the whole current now passes round its coil.

By shunting a galvanometer, it may be used to measure much stronger currents than it could carry when unshunted.

(14) To find approximately the reduction factor of a tangent galvanometer.

Set up the circuit of Exercise 572. Let the battery B be a single clean Daniell's cell freshly made up.

From the information given in Art. 377, find two points E and F on the potentiometer wire whose difference of potential is 1 volt. Put the terminals of the galvanometer whose resistance is known (r ohms) in connection with E and F. The deflection is A°; find tan A. From Ohm's law, the current flowing round the galvanometer is $\frac{1}{r}$ amperes, and this gives a deflection of A°. Calculate the value of $\frac{1}{r}$. Then $\frac{1}{r} = x \tan A$, whence x can be calculated. Find x. The value of any other current flowing round this galvanometer and giving a deflection of, say B°, is found in amperes by multiplying tan B by the value of x.

(15) You have three cells, the resistance of each of which is 2 ohms, and a coil of wire whose resistance is 12 ohms. Calculate the current which this battery would send through the wire (*a*) with the cells connected in series, (*b*) with the cells connected in parallel.

(The resistance of the battery (R) when its cells are joined in parallel is found from the formula obtained in Exercise 578, viz.

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r}, \text{ where } r = 2 \text{ ohms.})$$

(16) A circuit consists of a battery, a tangent galvanometer, and leads, its total resistance being 4 ohms. The galvanometer then shows a deflection of 48°. A coil of wire is then put into the circuit, and the galvanometer deflection diminishes to 24°. Explain why the current is now smaller, and calculate by Ohm's law the resistance of the coil of wire.

(17) To show that the quantity of heat developed in a wire by the passage of a current is proportional to the resistance of the wire.

Prepare two coils of fine wire (A and B) of the same material and diameter, but making B twice as long as A.

Place A and B in separate beakers or calorimeters that contain the same mass of water and a thermometer. The water must cover the coils.

Make up a circuit consisting of a battery of three or four cells, a key, and the two coils, all in series. Read the thermometers.

Let the current flow for several minutes: read the thermometers, and compare the quantities of heat produced in the two beakers.

Make a set of experiments with different values of the current.

(18) To show that the heat developed is proportional to the square of the current.

Take one of the coils prepared for the previous exercise, and place it in a known mass (m grm.) of water. Put the coil in series with a battery of two or three cells, a resistance coil (r), and a tangent galvanometer. Let the current run for a measured time t; read the galvanometer deflection (A°), and find the heat (H_1) developed in the wire.

Using the same coil of wire in a fresh mass of m grm. of water, alter the battery or the resistance r so as to obtain a different current. Let this current run for the same time t as before, and again find the heat (H_2), and note the deflection ($B°$) of the galvanometer.

Repeat the operation with another different value of the current, and obtain H_3 and the deflection $D°$.

Find the ratios: $\frac{H_1}{(\tan A)^2}$, $\frac{H_2}{(\tan B)^2}$, $\frac{H_3}{(\tan D)^2}$.

Careful experiments show that these ratios are equal. The heat developed in a wire by the passage of a current is proportional to the square of the strength of the current.

(19) To show that H is proportional to C^2R. Take a coil of wire and measure its resistance R. Set up the circuit as in experiment 18 above, and determine H_1 and tan $A°$.

Find the ratio $\frac{H_1}{R\,(\tan A)^2}$.

Make the same measurements with different coils of wire, and find the corresponding ratio for each experiment.

If the work is accurately done the ratios are very nearly the same.

(20) To determine approximately the resistance of a battery.

Make a circuit consisting of the battery whose resistance we will call a ohms, a key, a tangent galvanometer whose resistance is known (r ohms), and a coil of wire whose resistance is known (r_1 ohms). Note the galvanometer deflection ($A°$).

Replace the coil r_1 by another whose resistance is known (r_2 ohms). Note the deflection ($B°$).

Then, if E is the electromotive force of the battery

$$x \tan A = \frac{E}{r + r_1 + a},$$

$$\text{and } x \tan B = \frac{E}{r + r_2 + a};$$

$$\therefore \frac{\tan A}{\tan B} = \frac{r + r_2 + a}{r + r_1 + a},$$

whence all the quantities except a being known, a can be calculated.

The experiment only gives satisfactory results when the resistances are in suitable proportions. Select a battery whose resistance is not very small, and try several values of r_1 and r_2.

(21) Make an electromagnet (Art. 347), and remagnetize the steel magnets. Use a strong battery.

Determine the poles of the electromagnet (Art. 312). Put the north pole of the bar on the north pole of the electromagnet, and draw it steadily along the whole length of the bar. Then draw the bar across the other pole of the electromagnet in the opposite direction. Repeat both operations many times, on both sides of the bar.

APPENDIX

1. *Force of Magnetic Attraction.*—Measure in grammes weight the force of attraction between a magnet and its keeper.

Attach a dynamometer to the keeper of a magnet (see fig. 1). Fix the magnet firmly, and see that the line of pull of the dynamometer is at right angles to the end faces of the magnet. Pull steadily until the keeper leaves the magnet. Take several readings of the dynamometer, and find their mean.

Place a piece of paper between the magnet and the keeper, and again find the mean attractive force.

Increase the thickness of the paper layer, and repeat the observations.

Fig. 1.—Measurement of the Force of Magnetic Attraction

The readings show that the force of attraction between a magnet and its keeper diminishes as the distance between them increases.

2. *Action and Reaction.*—When one body acts forcibly on another we often direct our attention only to one of the bodies. But both are affected, and forces are called into play in opposite directions.

Repeat exercise, par. 1 above, first attaching the dynamometer to the keeper, and having the magnet fixed; and secondly having the keeper fixed, and applying the dynamometer to the magnet. Compare the pulls in the two cases. If gravity is acting on one of the bodies, its effect (i.e. the weight of the keeper or magnet hanging on the dynamometer) must be subtracted from the pull registered by the dynamometer, in order to get the magnetic force alone.

Thus not only does the magnet attract the iron, but the iron attracts the magnet exactly as much. The attraction is mutual.

When one body acts forcibly on another we often direct our attention only to one of the bodies. But both are affected, and forces are called into play in opposite directions.

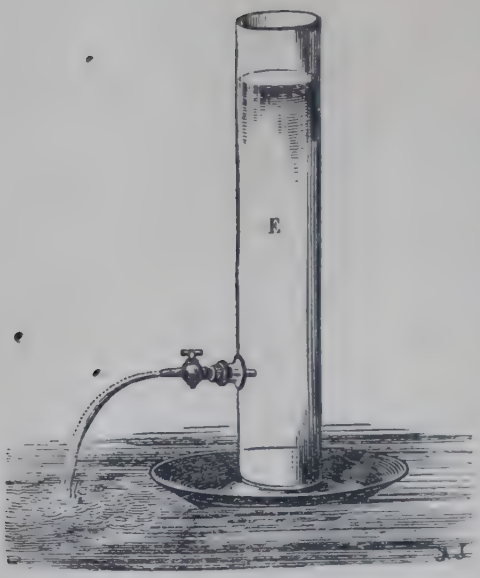

Fig. 2.—Recoil of a Discharging Vessel

Take a tall vessel with a hole closed by a cork or tap near the bottom of the vessel, fig. 2.

Fill it with water and float it upright in a larger vessel of water.

On opening the tap, water flows out, and the vessel moves backward in a direction opposite to the flow of the water.

The pressure of the fluid in the vessel forces out a jet of water, and at the same time a pressure is produced in the opposite direction.

This result is similar to that which occurs in the firing of a gun loaded with ball. The ball is propelled forward and the rifle backward. The "kick" back of a rifle is very decided. In large guns such as those used on war ships, the recoil of the gun is very great.

If you strike an object with your hand you feel that the effect is the same as though the object struck you.

If a glass flask be knocked against a stone the effect is the same as if the stone were knocked against the flask.

When a force is exerted on the spring of a dynamometer so as to extend it, the spring exerts a force in the opposite direction.

In many cases it is not so obvious that there is a second force acting against the first. A book placed on a table presses on the table. But the table must press back against the book: otherwise the book would fall under the influence of the force of gravitation. If a man stand on a piece of board, upward from which project two handles, he cannot by lifting vertically raise himself and the board. The upward pull which he exercises with his arms is exactly counteracted by an increase in the downward pressure transmitted through his feet.

3. To take times of vibration accurately, it is better that there should be two observers—one to count the oscillations of the swinging body, the other to record the time.

The oscillation observer makes a sharp tap on the table when the vibrating body passes the middle point of its swing. Both observers then commence counting—one the oscillations, the other seconds. In the case of a swinging magnet the counting should continue until the oscillations are quite small. The oscillation observer then makes another tap when the magnet is passing the middle

point of its swing in the same direction as at first. Then the number of seconds is divided by the number of complete double oscillations. This gives *t*.

4. *The Tangent Law.*—There are several examples occurring in physics in which a body is caused to take up a position under the action of two forces or couples at right angles to each other. This case should be studied.

At the centre of a circle fixed with its plane vertical, and having one quadrant divided into degrees (fig. 3), mount a strip of wood by a pin B so that the strip or "needle" can rotate easily.

At a point D suspend from a pin a weight E.

To the same pin fasten another thread which passes over a pulley L and supports a weight K.

Place a small weight at K. See that the two threads are exactly at right angles to each other; the arm AH is deflected from the vertical. Measure the angle ABC. The moments round B are the force K multiplied by the arm BF, and the force E multiplied by the arm DF. These are equal. Record thus:—

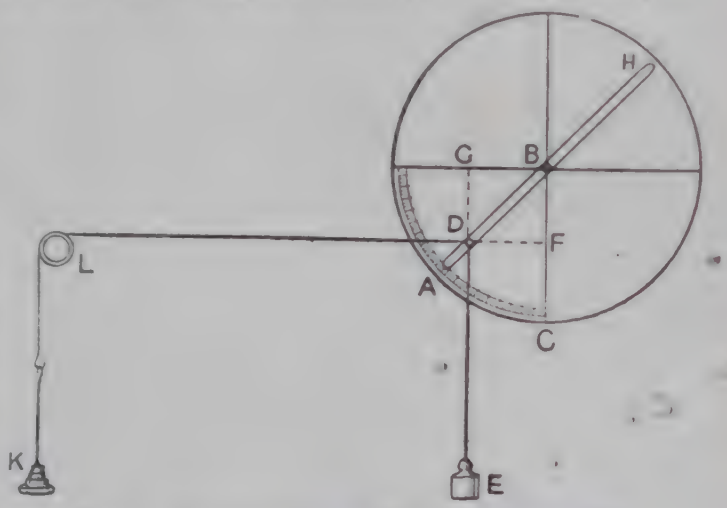

Fig. 3.—Two Forces at Right Angles to each Other turning a Lever through an Angle

Horizontal Force.	Vertical Force.	Deflection of Needle.	Ratio of Forces.	Tangent of Angle ABC.
K	E	Angle ABC	$\frac{K}{E}$	...

Take several observations of this character with different values of K, so as to obtain different values of the deflection ABC, and for each observation compare $\frac{K}{E}$ with the tangent of ABC, which is $\frac{DF}{BF}$.

The observations show that—

$$\frac{K}{E} = \tan ABC; \text{ or } K = E \times \tan ABC.$$

This result is the basis of measurements made by the tangent galvanometer (Art. 360).

TABLES

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TABLES

Mensuration

$\pi = 3{\cdot}1416$; $\pi^2 = 9{\cdot}87$; $\frac{1}{\pi} {\cdot}318$; $\sqrt{\pi} = 1{\cdot}77$.

Circumference of circle $= 2\pi r = \pi d$.

Areas

Triangle, base b, perpendicular height h	$= \frac{1}{2}bh$
Parallelogram, base b, perpendicular height h ...	$= bh$
Circle	$= \frac{1}{4}\pi d^2 = \pi r^2$
Sphere	$= 4\pi r^2$
Curved surface of right cylinder, radius r, height h	$= 2\pi rh$
Curved surface of right cone, radius r, slant height h_1	$= \pi rh_1$

Volumes

Rectangular body, edges a, b, c	$= abc$
Pyramid, area of base a, perpendicular height h ...	$= \frac{1}{3}ah$
Prism and cylinder, area of base a, altitude h ...	$= ah$
Cone, radius of base r, altitude h	$= \frac{1}{3}\pi r^2 h$
Sphere	$= \frac{4}{3}\pi r^3$

Reciprocals of Numbers

Numbers whose reciprocals are obvious are omitted.

No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.	No.	Reciprocal.
6	·1667	21	·0476	35	·0286	49	·0204
7	·1429	22	·0455	36	·0278	51	·0196
8	·1250	23	·0435	37	·0270	52	·0192
9	·1111	24	·0417	38	·0263	53	·0189
11	·0909	26	·0385	39	·0256	54	·0185
12	·0833	27	·0370	41	·0244	55	·0182
13	·0769	28	·0357	42	·0238	56	·0179
14	·0714	29	·0345	43	·0233	57	·0175
15	·0667	30	·0333	44	·0227	58	·0172
16	·0625	31	·0323	45	·0222	59	·0169
17	·0588	32	·0313	46	·0217	60	·0167
18	·0556	33	·0303	47	·0213	61	·0164
19	·0526	34	·0294	48	·0208	62	·0161

Sines and Tangents of Angles

Angle.	Sine.	Tangent.	Angle.	Sine.	Tangent.
6°	·1045	·1051	42°	·6691	·9004
8°	·1392	·1405	44°	·6947	·9657
10°	·1736	·1763	45°	·7071	1·000
12°	·2079	·2126	46°	·7193	1·035
14°	·2419	·2493	48°	·7431	1·110
16°	·2756	·2867	50°	·7660	1·191
18°	·3090	·3249	52°	·7880	1·279
20°	·3420	·3640	54°	·8090	1·376
22°	·3746	·4040	56°	·8290	1·482
24°	·4067	·4452	58°	·8480	1·600
26°	·4384	·4877	60°	·8660	1·732
28°	·4695	·5317	62°	·8829	1·880
30°	·5000	·5774	64°	·8988	2·050
32°	·5299	·6249	66°	·9135	2·246
34°	·5591	·6745	68°	·9272	2·475
36°	·5878	·7265	70°	·9397	2·747
38°	·6157	·7813	72°	·9511	3·077
40°	·6428	·8391	74°	·9613	3·487

For the values for the intermediate degrees take the mean of the preceding and succeeding values.

Density—Mass in Grammes of 1 Cub. Cm.

Alcohol (absolute)	·79	Iron	7·4
Water at 4° C.	1	Lead	11·2
Air	·001293	Marble	2·6
Aluminium	2·6	Mercury	13·59
Beeswax	·96	Oil of turpentine	·87
Brass	8·4	Olive oil	·92
Brick	2·1	Paraffin	·9
Copper	8·7	Petroleum	·88
German silver	8·5	Platinum	21·5
Glass (crown)	2·6	Sand (dry)	1·5
„ (flint)	2·9–5·9	Silver	10·6
Glycerine	1·26	Sulphur	2·0
Gold	19·3	Tin	7
Ice	·91	Zinc	7·1

Many of the values are approximate: different specimens vary

Coefficients of Expansion, per Degree Centigrade

Linear.		Cubical.	
Aluminium	·000022	Air	·00366
Brass	·000018	Alcohol	·00108
Copper	·000018	Glycerine	·00053
Glass	·000008	Mercury	·00018
Iron	·000012	Olive oil	·0007
Platinum	·000009	Turpentine	·00105
Silver	·000019	Water	·00045

Specific Heats

Air	·24	Glass	·16	Olive oil	·47
Alcohol	·55	Glycerine	·37	Paraffin	·64
Aluminium	·21	Ice	·5	Petroleum	·51
Beeswax	·64	Iron	·11	Steam	·48
Brass	·094	Lead	·031	Turpentine	·47
Copper	·093	Marble	·22	Zinc	·093
		Mercury	·033		

Some of these values are approximate.

Pressure of Water Vapour, in Millimetres of Mercury

Temperature.	Pressure.	Temperature.	Pressure.	Temperature.	Pressure.
6° C.	6·971	17° C.	14·395	28° C.	28·065
7°	7·466	18°	15·330	29°	29·744
8°	7·991	19°	16·319	30°	31·510
9°	8·548	20°	17·363	35°	41·784
10°	9·140	21°	18·466	40°	54·865
11°	9·767	22°	19·630	50°	91·978
12°	10·432	23°	20·858	60°	148·88
13°	11·137	24°	22·152	70°	233·31
14°	11·884	25°	23·517	90°	525·47
15°	12·674	26°	24·956	95°	633·66
16°	13·510	27°	26·470	100°	760·00

Melting-points and Latent Heats of Fusion

Beeswax	62° C.	42		Mercury	−39° C.	2·8
Benzene	5·3°	31		Paraffin	52°	35
Ice	0°	79·4		Sulphur	115°	9·4

Boiling-points and Latent Heats of Vaporization

Alcohol (ethyl) ...	78° C.	216	Mercury	357° C	62
Benzene	80°	93	Sulphur	444°	362
Ether	35°	85	Turpentine ...	156°	74
Glycerine ...	290°	—	Water	100°	540

Indices of Refraction (from Air)

Alcohol	1·36	Diamond ...	2·5	Glycerine ...	1·47
Benzene	1·5	Ether	1·35	Ice	1·31
Canada balsam ...	2·5	Glass (flint) ...	1·6	Water	1·33
		Glass (crown)	1·5		

Relative Electrical Resistances

Silver	1	Platinoid, about	23
Copper	1·04	Manganin, about	29
Iron	6·6	Mercury	64
German silver, about ...	16	Carbon, about	3000

Water more than 1000000.

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